Constraints over Lambda-Structures in Semantic Underspecification

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Abstract
We introduce a first-order language for semantic underspecification that we call Constraint Language for Lambda-Structures (CLLS). A λ-structure can be considered as a λ-term up to consistent renaming of bound variables (α-equality); a constraint of CLLS is an underspecified description of a λ-structure. CLLS solves a capturing problem omnipresent in underspecified scope representations. CLLS features constraints for dominance, lambda binding, parallelism, and anaphoric links. Based on CLLS we present a simple, integrated, and underspecified treatment of scope, parallelism, and anaphora.

1 Introduction
A central concern of semantic underspecification (van Deemter and Peters, 1996) is the underspecification of the scope of variable binding operators such as quantifiers (Hobbs and Shieber, 1987; Alshawi, 1990; Reyle, 1993). This immediately raises the conceptual problem of how to avoid variable-capturing when instantiating underspecified scope representations. In principle, capturing may occur in all formalisms for structural underspecification which represent binding relations by the coordination of variables (Reyle, 1995; Pinkal, 1996; Bos, 1996; Niehren et al., 1997a). Consider for instance the verb phrase in

(1) Manfred [vP knows every student]

An underspecified description of the compositional semantics of the VP in (1) might be given along the lines of (2):

(2) \[ X = C_1 (\forall x (\text{student}(x) \rightarrow C_2 (\text{know}(Z, x)))) \]

The meta-variable \( X \) in (2) denotes some tree representing a predicate logic formula which is underspecified for quantifier scope by means of two place holders \( C_1 \) and \( C_2 \) where a subject-quantifier can be filled in, and a place holder \( Z \) for the subject-variable. The binding of the object-variable \( x \) by the object-quantifier \( \forall x \) is coordinated through the name of the object-variable, namely ‘\( x \)’. Capturing occurs when a new quantifier like \( \exists x \) is filled in \( C_2 \) whereby the binding between \( x \) and \( \forall x \) is accidentally undone, and is replaced with a binding of \( x \) by \( \exists x \).

Capturing problems raised by variable coordination may be circumvented in simple cases where all quantifiers in underspecified descriptions can be assumed to be named by distinct variables. However, this assumption becomes problematic in the light of parallelism between the interpretations of two clauses. Consider for instance the correction of (1) in (3):

(3) No, Hans [vP knows every student]

The description of the semantics of the VP in (3) is given in (4):

(4) \[ Y = C_3 (\forall y (\text{student}(y) \rightarrow C_4 (\text{know}(Z', y)))) \]

But a full understanding of the combined clauses (1) and (3) requires a grasp of the semantic identity of the two VP interpretations. Now, the VP interpretations (2) and (4) look very much alike but for the different object-variable, namely ‘\( y \)’ instead of ‘\( x \)’. This illustrates that in cases of parallelism, like in corrections, different variables in parallel quantified structures have to be matched against each other, which requires some form of renaming to be done on them. While this is unproblematic for fully specified structures, it presents serious problems with underspecified structures like (2) and (4), as there the names of the vari-
ables are crucial for insuring the right bindings. Any attempt to integrate parallelism with scope underspecification thus has to cope with conflicting requirements on the choice of variable names. Avoiding capturing requires variables to be renamed apart but parallelism needs parallel bound variables to be named alike.

We avoid all capturing and renaming problems by introducing the notion of $\lambda$-structures, which represent binding relations without naming variables. A $\lambda$-structure is a standard predicate logic tree structure which can be considered as a $\lambda$-term or some other logical formula up to consistent renaming of bound variables ($\alpha$-equality). Instead of variable names, a $\lambda$-structure provides a partial function on tree nodes for expressing variable binding. A graphical illustration of the $\lambda$-structure corresponding to the $\lambda$-term $\lambda x . \text{like}(x,x)$ is given (5).

![Diagram of $\lambda$-structure](image)

Formally, the binding relation of the $\lambda$-structure in (5) is expressed through the partial function $\lambda^{(5)}$ defined by $\lambda^{(5)}(v_2) = v_0$ and $\lambda^{(5)}(v_3) = v_0$. We propose a first-order constraint language for $\lambda$-structures called CLLS which solves the capturing problem of underspecified scope representations in a simple and elegant way. CLLS subsumes dominance constraints (Backofen et al., 1995) as known from syntactic processing (Marcus et al., 1983) with tree-adjoining grammars (Vijay-Shanker, 1992; Rogers and Vijay-Shanker, 1994). Most importantly, CLLS constraints can describe the binding relation of a $\lambda$-structure in an underspecified manner (in contrast to $\lambda$-structures like (5), which are always fully specified). The idea is that $\lambda$-binding behaves like a kind of rubber band that can be arbitrarily enlarged but never broken. E.g., (6) is an underspecified CLLS-description of the $\lambda$-structure (5).

\[
\begin{align*}
X_0 & \equiv X_1 \land \lambda(X_1) = X_4 \land X_1 ; \text{lam}(X_2) \land X_2 ; \text{lam}(X_3) \land X_3 ; \text{like}(X_4, X_5) \land X_4 ; \text{var} \land X_5 ; \text{var}.
\end{align*}
\]

The constraint (6) does not determine a unique $\lambda$-structure since it leaves e.g. the space between the nodes $X_2$ and $X_3$ underspecified. Thus, (6) may eventually be extended, say, to a constraint that fully specifies the $\lambda$-structure for the $\lambda$-term in (7).

\[(7) \quad \lambda y . \lambda z . \text{and}(\text{person}(y), \text{like}(y, z))\]

$\lambda z$ intervenes between $\lambda y$ and an occurrence of $y$ when extending (6) to a representation of (7) without the danger of undoing their binding. CLLS is sufficiently expressive for an integrated treatment of semantic underspecification, parallelism, and anaphora. To this purpose it provides parallelism constraints (Niehren and Koller, 1998) of the form $X/X' \sim Y/Y'$ reminiscent to equality up to constraints (Niehren et al., 1997a), and anaphoric bindings constraints of the form $\text{ante}(X)=X'$.

As proved in (Niehren and Koller, 1998), CLLS extends the expressiveness of context unification (Niehren et al., 1997a). It also extends its linguistic coverage (Niehren et al., 1997b) by integrating an analysis of VP ellipses with anaphora as in (Kehler, 1995). Thus, the coverage of CLLS is comparable to Crouch (1995) and Shieber et al. (1996). We illustrate CLLS at a benchmark case for the interaction of scope, anaphora, and ellipsis (8).

\[(8) \quad \text{Mary read a book she liked before Sue did.}\]

The paper is organized as follows. First, we introduce CLLS in detail and define its syntax and semantics. We illustrate CLLS in sec. 3 by applying it to the example (8) and compare it to related work in the last section.

2 A Constraint Language for $\lambda$-Structures (CLLS)

CLLS is an ordinary first-order language interpreted over $\lambda$-structures. $\lambda$-structures are particular predicate logic tree structures we will introduce. We first exemplify the expressiveness of CLLS.

2.1 Elements of CLLS

A $\lambda$-structure is a tree structure extended by two additional relations (the binding and the linking relation). We represent $\lambda$-structures as graphs. Every $\lambda$-structure characterizes a unique $\lambda$-term or a logical formula up to consistent renaming of bound variables ($\alpha$-equality). E.g., the $\lambda$-structure (10) characterizes the higher-order logic (HOL) formula (9).
(9) \( (\text{many} \text{(language)})(\lambda x. \text{speak}(x))(\text{john}) \)

Two things are important here: the label ‘@’ represents explicitly the operation of function application, and the binding of the variable \( x \) by the \( \lambda \)-operator \( \lambda x \) is represented by an explicit \textit{binding relation} \( \lambda \) between two nodes, labelled as \texttt{var} and \texttt{lam}. As the binding relation is explicit, the variable and the binder need not be given a name or index such as \( \lambda \).

We can fully describe the above \( \lambda \)-structure by means of the constraints for immediate dominance and labeling \( X : f(X_1, \ldots, X_n) \), (e.g., \( X_1:\@ (X_2, X_3) \) and \( X_3: \text{lam}(X_4) \) etc.) and binding constraints \( \lambda(X) = Y \). It is convenient to display such constraints graphically, in the style of (6). The difference of graphs as constraints and graphs as \( \lambda \)-structures is important since underspecified structures are always seen as descriptions of the \( \lambda \)-structures that satisfy them.

**Dominance.** As a means to underspecify \( \lambda \)-structures, CLLS employs constraints for \textit{dominance} \( X \text{d}\@ Y \). Dominance is defined as the transitive and reflexive closure of immediate dominance. We represent dominance constraints graphically as dotted lines. E.g., in (11) we have the typical case of undetermined scope. It is analysed by constraint (12), where two nodes \( X_1 \) and \( X_2 \), lie between an upper bound \( X_0 \) and a lower bound \( X_3 \). The graph can be linearized by adding either a constraint \( X_1 \text{d}\@ X_2 \) or \( X_2 \text{d}\@ X_1 \), resulting in the two possible scoping readings for the sentence (11).

(11) Every linguist speaks two Asian languages.

(12)

**Parallellism.** (11) may be continued by an elliptical sentence, as in (13).

(13) Two European ones too.

We analyse elliptical constructions by means of a \textit{parallellism constraint} of the form

(14) \( X_s / X_p \sim Y_t / Y_p \)

which has the intuitive meaning that the semantics \( X_s \) of the \textit{source clause} (12) is \textit{parallel} to the semantics \( Y_t \) of the elliptical \textit{target clause}, up-to the exceptions \( X_p \) and \( Y_p \), which are the semantic representations of the so called \textit{parallel elements} in source and target clause. In this case the parallel elements are the two subject NPs.

(11) and (13) together give us a ‘Hirschbühlere sentence’ (Hirschbühlere, 1982), and our treatment in this case is descriptively equivalent to that of (Niehren et al., 1997b). Our parallellism constraints and their quality up-to constraints have been shown to be (non-trivially) intertranslatable (Niehren and Koller, 1998) if binding and linking relations in \( \lambda \)-structures are ignored.

For the interaction of binding with parallellism we follow the basic idea that binding relations should be isomorphic between two similar substructures. The cases where anaphora interact with ellipsis are discussed below.

**Anaphoric links.** We represent anaphoric dependencies in \( \lambda \)-structures by another explicit relation between nodes, the \textit{linking relation}. An anaphor (i.e. a node labelled as \texttt{ana}) may be linked to an antecedent node, which may be labelled by a name or \texttt{var}, or even be another anaphor. Thus, links can form chains as in (15), where a constraint such as \texttt{ante}(\textit{X}_3) = \textit{X}_2 is represented by a dashed line from \textit{X}_3 to \textit{X}_2.

The constraint (15) analyzes (16), where the second pronoun is regarded as to be linked to the first, rather than linked to the proper name:

(15)

(16) John\(^i\) said he\(^j\) liked his\(^j\) mother.
In a semantic interpretation of \(\lambda\)-structures, analogously to a semantics for lambda terms,\(^1\) linked nodes get identical denotations. Intuitively, this means they are interpreted as if names, or variables with their binding relations, would be copied down the link chain. It is crucial though not to use such copied structures right away: the link relation gives precise control over strict and sloppy interpretations when anaphors interact with parallelism.

E.g., (16) is the source clause of the many-pronouns-puzzle, a problematic case of interaction of ellipsis and anaphora. (Xu, 1998), where our treatment of ellipsis and anaphora was developed, argues that link chains yield the best explanation for the distribution of strict/sloppy anaphors interacting with parallelism. The basic idea is that an elided pronoun can be linked to its denotation. In turn, would be copied down the link chain. It is crucial over strict and sloppy interpretations when anaphors interact with parallelism.

The notion of parallelism is the notion of \(\lambda\)-structure \(L\) is a tree structure with two (partially functional) binary relations \(\lambda_L(\cdot)=\cdot\) for binding, and \(\text{ante}_L(\cdot)=\cdot\) for anaphor-to-antecedent linking. We assume that the following conditions hold: (1) binding only holds between variables (nodes labelled var) to \(\lambda\)-binders (nodes labelled lam); (2) every variable has exactly one binder; (3) variables are dominated by their binders; (4) only anaphors (nodes labelled ana) are linked to antecedents; (5) every anaphor has exactly one antecedent; (5) antecedents are terminal nodes; (6) there are no cyclic link chains; (7) if a link chain ends at a variable then each anaphor in the chain must be dominated by the binder of that variable.

The not so straightforward part of the semantics of \(\lambda\)-structures is the notion of parallelism, which we define for any given \(\lambda\)-structure \(L\) as follows:

\[ \nu_1 \downarrow \nu'_1 \sim_L \nu_2 \downarrow \nu'_2 \]

if there is a path \(\pi_0\) such that:

1. \(\pi_0\) is the “exception path” from the top node of the parallel structures to the the two exception positions: \(\nu_1 \downarrow \nu_1 \downarrow \nu_2 \downarrow \nu_2 \pi_0 \)
2. the two contexts, which are the trees below \(\nu_1\) and \(\nu_2\) up to the trees below the exception positions \(\nu'_1\) and \(\nu'_2\), must have the same structure and labels:
3. there are no ‘hanging’ binders from the contexts to variables outside them:
4. binding is structurally isomorphic within the two contexts:

\[^1\text{We abstain from giving such a semantics here, as we would have to introduce types, which are of no concern here, to keep the semantics simple.}\]
\[\forall \pi \forall \pi' \quad \pi_0 \leq \pi \land \forall_1 \pi \downarrow L \land \forall \pi_0 \leq \pi' \land \forall_1 \pi' \downarrow L \Rightarrow \lambda L(\forall_1 \pi) = \forall_1 \pi' \Leftrightarrow \lambda L(\forall_2 \pi) = \forall_2 \pi'\]

5. two variables in identical positions within their context and bound outside their context must be bound by the same binder:
\[\forall \nu \forall \pi (\nu \forall_1^2 \forall_1 \nu_1 \land \nu \forall_1^2 \forall_2 \nu_2) \land \forall \pi_0 \leq \pi \land \forall_1 \pi \downarrow L \Rightarrow \lambda L(\forall_1 \pi) = \nu \Leftrightarrow \lambda L(\forall_2 \pi) = \nu\]

6. two anaphors in identical positions within their context must have isomorphic links within their context, or the target sentence anaphor is linked to the source sentence anaphor:
\[\forall \nu \forall \pi \quad \pi_0 \leq \pi \land \forall_1 \pi \downarrow L \land \vartriangle L(\forall_1 \pi) = \nu \Rightarrow \exists \pi' (\nu = \forall_1 \pi' \land \forall \pi_0 \leq \pi' \land \vartriangle L(\forall_2 \pi) = \forall_2 \pi') \lor \vartriangle L(\forall_2 \pi) = \forall_1 \pi\]

3 Interaction of quantifiers, anaphora, and ellipsis

In this section, we will illustrate our analysis of a complex case of the interaction of scope, anaphora, and ellipsis. In the case (8), both anaphora and quantification interact with ellipsis.

(8) Mary read a book she liked before Sue did.
(8) has three readings (Crouch, 1995 for a discussion of a similar example). In the first, the indefinite NP a book she liked takes wide scope over both clauses (a particular book liked by Mary is read by both Mary and Sue). In the two others, the operator before outscop es the indefinite NP. The two options result from the two possibilities of reconstructing the pronoun she in the ellipsis interpretation, viz., ‘strict’ (both read some book that Mary liked) and ‘sloppy’ (each read some book she liked herself).

The constraint for (8), displayed in (18), is an underspecified representation of the above three readings. It can be derived in a compositional fashion along the lines described in (Nehren et al., 1997b). \(X_s\) and \(X_t\) represent the semantics of the source and the target clause, while \(X_{16}\) and \(X_{21}\) stand for the semantics of the parallel elements (Mary and Sue) respectively. For readability, we represent the semantics of the complex NP a book she liked by a triangle dominated by \(X_2\), which only makes the anaphoric content \(X_{12}\) of the pronoun she within the NP explicit. The anaphoric relationship between the pronoun she and Mary is represented by the linking relation between \(X_{12}\) and \(X_{16}\). (\(X_{20}\) rep-
4 Related Work

CLLS allows a uniform and yet internally structured approach to semantic ambiguity. We use a single constraint formalism in which to describe different kinds of information about the meaning of an utterance. This avoids the problems of order dependence of processing that for example Shieber et al. (1996) get by interleaving two formalisms (for scope and for ellipsis resolution). Our approach follows Crouch (1995) in this respect, who also includes parallelism constraints in the form of substitution expressions directly into an underspecified semantic formalism (in his case the formalism of Quasi Logical Forms QLF). We believe that the two approaches are roughly equivalent empirically. But in contrast to CLLS, QLF is not formalised as a general constraint language over tree-like representations of meaning. QLF has the advantage of giving a more direct handle on meanings themselves - at the price of its relatively complicated model theoretic semantics. It seems harder though to come up with solutions within QLF that have an easy portability across different semantic frameworks.

We believe that the ideas from CLLS tie in quite easily with various other semantic formalisms, such as UDRT (Reyle, 1993) and MRS (Copestake et al., 1997), which use dominance relations similar to ours, and also with theories of Logical Form associated with GB style grammars, such as (May, 1977). In all these frameworks one tends to use variable-coordination (or coindexing) rather than the explicit binding and linking relations we have presented here. We hope that these approaches can potentially benefit from the presented idea of rubber bands for binding and linking, without having to make any dramatic changes.

Our definition of parallelism implements some ideas from Hobbs and Kehler (1997) on the behavior of anaphoric links. In contrast to their proposal, our definition of parallelism is not based on an abstract notion of similarity. Furthermore, CLLS is not integrated into a general theory of abduction. We pursue a more modest aim at this stage, as CLLS needs to be connected to "material" deduction calculi for reasoning with such underspecified semantic representation in order to make progress on this front. We hope that some of the more ad hoc features of our definition of parallelism (e.g. axiom 5) may receive a justification or improvement in the light of such a deeper understanding.

Context Unification. CLLS extends the expressiveness of context unification (CU) (Niehren et al., 1997a), but it leads to a more direct and more structured encoding of semantic constraints than CU could offer. There are three main differences between CU and CLLS.

1) In CLLS variables are interpreted over nodes rather than whole trees. This gives us a direct handle on occurrences of semantic material, where CU could handle occurrences only indirectly and less efficiently. 2) CLLS avoids the capturing problem. 3) CLLS provides explicit anaphoric links, which could not be adequately modeled in CU.

The insights of the CU-analysis in (Niehren et al., 1997b) carry over to CLLS, but the awkward second-order equations for expressing dominance in CU can be omitted (Niehren and Koller, 1998). This omission yields an enormous simplification and efficiency gain for processing.

Tractability. The distinguishing feature of our approach is that we aim to develop efficiently treatable constraint languages rather than to apply maximally general but intractable formalisms. We are confident that CLLS can be implemented in a simple and efficient manner. First experiments which are based on high-level concurrent constraint programming have shown promising results.
5 Conclusion

In this paper, we presented CLLS, a first-order language for semantic underspecification. It represents ambiguities in simple underspecified structures that are transparent and suitable for processing. The application of CLLS to some difficult cases of ambiguity has shown that it is well suited for the task of representing ambiguous expressions in terms of underspecification.

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