Guido Tack
PS Lab, Saarland University, Germany

Christian Schulte
ICT, KTH - Royal Institute of Technology, Sweden

propagators

Propagator $p \in \mathrm{Dom} \rightarrow \mathrm{Dom}$

- $p(d) \subseteq d$ (contracting)
- $d \subseteq d' \Rightarrow p(d) \subseteq p(d')$ (monotonic)

Induced constraint:
$p \equiv c_p = \{\, a \mid p(\{a\}) = \{a\} \,\}$

Complete:
$p(d) \subseteq \mathrm{dom}(c_p \cap d)$

Bounds($\mathbb{Z}$)-complete:
$p(d) \subseteq \mathrm{bnd}(c_p \cap \mathrm{bnd}(d))$

Simple propagator

$$p(d)(z) = \begin{cases} d(x) \cap \{-\infty, \ldots, \max(d(y))\} & \text{if } z = x \\ d(z) & \text{otherwise} \end{cases}$$

$p(\{(x \mapsto 1, y \mapsto 2)\}) = \{(x \mapsto 1, y \mapsto 2)\}$ ✓
$p(\{(x \mapsto 2, y \mapsto 1)\}) = \{\}$ ✗

$c_p = \{\, (x \mapsto a, y \mapsto b) \mid a \leq b \,\}$

derived

Variable view $\varphi_x \in \mathrm{Val} \rightarrow \mathrm{Val}'$, injective

View $\varphi \in \mathrm{Dom} \rightarrow \mathrm{Dom}'$ (lift φ_x to domains)

Derived propagator:
$\hat{\varphi}(p) = \varphi^{-1} \circ p \circ \varphi$

Derived constraint:
$\varphi^{-1}(c)$

Arithmetic views

$p \equiv \{\, (x,y,z) \mid x + y + z = 0 \,\}$
$\varphi_x(v) = 2v$
$\varphi_y(v) = v - 3$

$\hat{\varphi}(p) \equiv \{\, (x,y,z) \mid 2x + y + z = 3 \,\}$

Boolean views

$p \equiv \{\, (x,y,z) \mid x \wedge y \leftrightarrow z \,\}$
$\varphi_x(v) = 1 - v$
$\varphi_y(v) = 1 - v$
$\varphi_z(v) = 1 - v$

$\hat{\varphi}(p) \equiv \{\, (x,y,z) \mid x \vee y \leftrightarrow z \,\}$

perfect

$\hat{\varphi}(p)$ is

- contracting
- monotonic

$\Rightarrow$ a propagator

$\hat{\varphi}(p)$ is correct:
$c_{\hat{\varphi}(p)} = \varphi^{-1}(c_p)$

p complete
$\Rightarrow \hat{\varphi}(p)$ complete

p bounds($\mathbb{Z}$)-complete, φ interval-bijective
$\Rightarrow \hat{\varphi}(p)$ bounds($\mathbb{Z}$)-complete

p bounds($\mathbb{Z}$)-complete, φ interval-injective
$\Rightarrow \hat{\varphi}(p)$ bounds($\mathbb{R}$)-complete

Linear equations

$$p \equiv \left\{ (x_1, \ldots, x_n) \;\middle|\; \sum_{i=1}^{n} x_i = 0 \right\}$$ bounds($\mathbb{Z}$)-complete

$\varphi_{x_i}(v) = a_i v$ interval-injective

$$\hat{\varphi}(p) \equiv \left\{ (x_1, \ldots, x_n) \;\middle|\; \sum_{i=1}^{n} a_i x_i = 0 \right\}$$ bounds($\mathbb{R}$)-complete

Applicability: Saves 100.000 lines of code (70%) in Gecode

Efficiency: Implementation without overhead

Examples: Linear (in-)equations, min/max, all-different, Boolean connectives, reified constraints, set operations/relations, channeling, specialized versions using constant views, ...