Proof of $f(f(fx)) \leftrightarrow fx$ in $S(HB)$

The proof is by repeated case analysis using

BA $\vdash$ $x = (y \leftrightarrow 0)\,x + (y \leftrightarrow 1)\,x$ and

HB $\vdash$ $x \leftrightarrow y \wedge gx = x \leftrightarrow y \wedge gy$ (BRep)

The proof is shown in decision tree style on the next slide.
It is completely straightforward.

Acknowledgements. The problem was posed by Mark Kaminski in Fall 2004.
We couldn't prove it then. Chad Brown proved it with TPS
on June 24, 2005. Hence it was clear that there is a proof in $S(HOL)$.
We didn't see the proof earlier since we discovered BRep only
in May 2005.

HB $\vdash$ $f(f(fx)) \leftrightarrow fx$