# Choice Operator

Choice Constants $\quad C_T : (T \to B) \to T$

Choice Axioms $\quad f(Cf) = \exists f \qquad$ **Choi**

**HOL** $\overset{def}{=}$ BQ ∪ Ext ∪ Choi

Our choice operator is a higher-order formulation of Hilbert's epsilon operator (Hilbert 1923)



# BQ ∪ Choi ⊢ Sko

**Sko** $\quad \forall x \exists y.\, gxy = \exists h \forall x.\, gx(hx)$

Proof "→"

$$\begin{array}{lll} & \forall x \exists y.\, gxy & \\ = & \forall x.\, \exists(gx) & \eta \\ = & \forall x.\, gx(C(gx)) & \text{Choice} \\ \vdash_\circ & \exists h \forall x.\, gx(hx) & \exists I \;\; h = \lambda x.\, C(gx) \end{array}$$

"←"

$$\begin{array}{lll} & (\exists h \forall x.\, gx(hx)) \to \forall x \exists y.\, gxy & \\ \dashv\vdash & (\forall x.\, gx(hx)) \to \exists y.\, gxy & \exists\to, \to\forall \\ {}_\circ\dashv & gx(hx) \to gx(hx) & \forall I, \exists I, W \\ = & 1 & \end{array}$$

□

# Duality

$$\boxed{BQ \cup Choi \vdash \forall f = f(C(\lambda x.\overline{fx}))}$$

Proof

$$\begin{array}{lll} & \forall f & \\ = & \overline{\exists x.\, \overline{fx}} & d\Pi, BA \\ = & \overline{(\lambda x.\overline{fx})(C(\lambda x.\overline{fx}))} & \text{Choi} \\ = & f(C(\lambda x.\overline{fx})) & \beta, BA \end{array}$$

□

Duality can be preserved by introduction of dual choice operator:

**ĈD** $\quad \hat{C}f = C(\lambda x.\overline{fx})$