Assignment 2
Introduction to Computational Logic, SS 2007

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Read in the lecture notes: Chapter 2

Exercise 2.1 (Intensions and Extensions) Give the intension (tree representation) and the extension for each of the following notations.

a) $2 + 3$

b) $2 < 3 - 2$

c) $(\lambda x{\in}\mathbb{N}.\, x{\cdot}x)\, 4$

d) $(\lambda f{\in}\mathbb{N} \to \mathbb{N} \to \mathbb{N}.\, \lambda x{\in}\mathbb{N}.\, \lambda y{\in}\mathbb{N}.\, fyx)\,(-)\,2$

Exercise 2.2 (Substitution) Apply the following substitutions.

a) $[x := y](fx(\lambda x.y))$

b) $[y := x](\lambda x.\, fxy)$

c) $[f := gxy](\lambda x.\, \lambda y.\, fxy)$

Exercise 2.3 (Term Equality) The following notations describe terms. Draw for each notation the tree representation of the term described. Which of the notations describe the same term? (Two terms are equal if they have the same tree representation.)

a) $\lambda x.\, x$

b) $\lambda y.\, y$

c) $\lambda y.\, x$

d) $[x := y](\lambda x.\, x)$

e) $[y := x](\lambda x.\, y)$

f) $\lambda x.\, \lambda y.\, fxy$

g) $\lambda y.\, \lambda x.\, fyx$

Exercise 2.4 (Counterexamples) Find counterexamples for the following claims.

a) $t = [x' := x]t' \wedge x' \notin \mathcal{N}(\lambda x.t) \Longrightarrow \lambda x.t = \lambda x'.t'$

b) $t' = [x := x']t \Longrightarrow \lambda x.t = \lambda x'.t'$

Exercise 2.5 (Normal Forms) Derive the $\beta\eta$-normal forms of the following terms.

a) $\lambda xy.fx$

b) $\lambda xy.fy$

c) $\lambda xy.fxy$

d) $(\lambda xy.fyx)zxy$

e) $(\lambda xy.fxy)ab$

f) $(\lambda z.a)(\lambda xy.fxy)$

g) $(\lambda xu.x)(\lambda vy.y)$

h) $(\lambda f.fa)(\lambda xy.fxy)b$

i) $(\lambda z.(\lambda xy.fxy)((\lambda z.z)z))ab$

j) $(\lambda fgx.fx(gx))(\lambda xy.x)(\lambda xz.x)$

Exercise 2.6 (Explicit Formulations) The formulation of the axioms for untyped terms leave implicit:

- The universal quantification of meta-variables.
- The syntactic operations A, L, and **S**.

For instance, an explicit formulation of Axiom CL would look as follows:

$$\forall x \in Nam\ \forall t \in Ter\colon |Lxt| = 1 + |t|$$

Give explicit formulations of the axioms SA, IL, and SL.

Exercise 2.7 Make sure that you can name all the sets and all the functions of the axiomatization of terms. Moreover, given the name of an axiom, you should be able to reproduce the axiom.

Exercise 2.8 (Identity Substitution) Let $\theta = (\lambda x{\in}Nam.x)$. Prove

$$\forall t \in Ter\colon \theta t = t$$

by induction on $|t|$. Make sure you mention all the axioms used. Take the proof of Proposition 2.1 in the lecture notes as template.

Exercise 2.9 (Coincidence) Prove the following statement by induction on $|t|$.

$$\forall t \in Ter\ \forall \theta, \theta' \in Sub\colon (\forall x \in \mathcal{N}t\colon \theta x = \theta' x) \implies \theta t = \theta' t$$

Make sure you mention all the axioms used.