Assignment 7
Introduction to Computational Logic, SS 2007

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Read in the lecture notes: Chapter 5

Exercise 7.1 (And) Let s, t be formulas. Find a derivation that uses no other rules but Taut and MP

a) of s from $\{s \wedge t\}$,

b) of $s \wedge t$ from $\{s, t\}$.

Exercise 7.2 (Zero-One Instances) Give all zero-one instances of the following formulas. Decide for each formula whether it is a tautology. Assume that x, y are distinct variables of type $\mathbf{B}$.

a) $x \to fx$

b) $x \to y$

c) $((x \to y) \to x) \to x$

Exercise 7.3 (Propositional Generalizations) Find most specific propositional generalizations for the following formulas. Decide for each formula whether it is tautologous. Assume that f, g have type $\mathbf{B} \to \mathbf{B}$.

a) fx

b) $fx \wedge (fx \vee fy)$

c) $(gx = gy) \to (gy = gz) \to (gx = gz)$

Exercise 7.4 (Counterexample) Let x, y be distinct variables of type $\mathbf{B}$. Give a context C such that $\mathrm{PL} \nvDash (x = y) \to (Cx = Cy)$.

Exercise 7.5 (Boolean Replacement) Let $s = (x = y) \to (fx = fy)$ and $t = (x = y) \to fx \to fy$ where f is a variable of type $\mathbf{B} \to \mathbf{B}$.

a) Give a conversion proof of $1 = s$ from $\mathrm{TL} \cup \{1 = t\}$.

b) Give a conversion proof of $1 = t$ from $\mathrm{TL} \cup \{1 = s\}$.

c) Prove $s \overset{\mathrm{PL}}{\vDash\!\dashv} t$ with (a) and (b).

Exercise 7.6 (Boolean Replacement) Find a conversion proof of

$$(x = y) \to ([z := x]s = [z := y]s) = 1$$

from TL and $(x = y) \to fx \to fy = 1$ where f is a variable of type $\mathbf{B} \to \mathbf{B}$.

Exercise 7.7 (Boolean Expansion) Let the following formulas be given where f is a variable $\mathbf{B} \to \mathbf{B}$.

$$\begin{aligned} \text{BCA} &:= f0 \to f1 \to fx \\ \text{BRep} &:= (x = y) \wedge fx = (x = y) \wedge fy \\ \text{BExp} &:= fx = \overline{x} \wedge f0 \vee x \wedge f1 \end{aligned}$$

a) Find a conversion proof of BCA = 1 from TL and BExp.

b) Find a conversion proof of BExp from TL and BRep.

c) Find a derivation of BCA from BExp. You may use Taut and Sym.

Exercise 7.8 (Tautologies) Acquaint yourself with the tautologies in the lecture notes. Make sure that you can quickly verify/falsify whether a formula is a tautology using decision trees.

Exercise 7.9 (Hypothetical Conversion Proofs) Dieter claims that the following tuple is a conversion proof of $0 = 1$ from TL with $\{x = 1\}$. Is he right?

$$\begin{aligned} 0 &= (\lambda x.x)0 && \beta \\ &= (\lambda x.1)0 && \text{Hyp } x = 1 \\ &= 1 && \beta \end{aligned}$$