Assignment 8
Introduction to Computational Logic, SS 2007

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Read in the lecture notes: Chapter 5,6

Exercise 8.1 (Type Representation) Prove that for every type T there exist types $T_1, \dots, T_n$ and a sort S such that $T = T_1 \to \dots \to T_n \to S$.
Hint: Use induction on T.

Exercise 8.2 (Notation) Prove that the terms $\uparrow_T v$, $\forall_T$, and $\approx_T$ are closed for every type T and every $v \in \hat{\mathcal{T}}\,T$.
Hint: Use mutual induction on T.

Exercise 8.3 (Deductive Completeness) Let $\Delta \subseteq \Gamma$. Prove:
A is Γ-complete $\implies$ A is Δ-complete.

Exercise 8.4 (Deductive Completeness) Prove:
A is Γ-complete $\iff$ A is $\{s \mid \exists t \in \Gamma\colon A \vdash s = t\}$-complete.
Hint: To prove one of the directions, Exercise 8.3 may be useful.

Exercise 8.5 (PL without 1) Let $\mathrm{PL}_0 := \{\mathrm{E1}, \mathrm{E0}, \mathrm{I1}, \mathrm{BCA}\}$.

a) Give a stable, idempotent substitution θ such that:
   (i) $\Delta_{\theta(\mathrm{PL}_0)} \subseteq \Delta_{\mathrm{PL}_0}$,
   (ii) $\theta(\mathrm{PL}_0)$ does not contain the parameter 1,
   (iii) $\mathrm{PL}_0 \vDash \theta(\mathrm{PL}_0)$.
b) Which parameters occur in $\Delta_{\theta(\mathrm{PL}_0)}$?
c) Prove: PL_0 is Δ_{PL_0}-complete $\iff$ $\theta(\mathrm{PL}_0)$ is $\Delta_{\theta(\mathrm{PL}_0)}$-complete.
   Hint: Look at the proof of Proposition 6.14.
d) Prove: PL is Δ_{PL}-complete $\iff$ $\theta(\mathrm{PL}_0)$ is $\Delta_{\theta(\mathrm{PL}_0)}$-complete.
   Use (c) and Proposition 6.14.

Exercise 8.6 (Propositional Logic) Take a good look at the chapter on propositional logic (Chapter 5). It will be relevant for a good portion of next week's test.