Assignment 10
Introduction to Computational Logic, SS 2007

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Read in the lecture notes: Chapter 7

Exercise 10.1 (Quasi-Instances) Decide for each of the following formulas whether it is a quasi- or even a λ-instance of $fx \to \exists f$, where f, x are variables.

a) $\overline{fx} \to \exists x.\overline{fx}$

b) $fx \vee \exists x.\overline{fx}$

c) $fx \vee gx \vee \exists x.\overline{fx}$

d) $fx \vee \overline{\forall x.fx}$

e) $(\exists f \to x) \to fy \to x$

f) $(\exists f \to x \wedge y) \to f(gxy) \to (x \wedge y \to z) \to z$

g) $\exists f = \exists f \vee f(gx)$

Exercise 10.2 (Leibniz) Prove: $\text{HL} \vdash (x = y) = \forall f.fx \to fy$.

Exercise 10.3 (Dual of Skolem) Prove: $\text{HL} \vdash (\exists x \forall y.fxy) = \forall g \exists x.fx(gx)$.
Hint: Use the Skolem law shown in the lecture notes.

Exercise 10.4 (Strange Quantification) Let $f : ((S \to \mathbf{B}) \to \mathbf{B}) \to T \to \mathbf{B}$.

a) Determine the type indices for the occurrences of $\forall$ in the formula $\forall f.\forall(f\forall)$.

b) Prove: $\text{HL} \vdash \neg\forall f.\forall(f\forall)$.

Exercise 10.5 (Witnesses) Let $f : T \to T \to \mathbf{B}$. Prove: $\text{HL} \vdash \exists g \forall x \exists h.h(fx) \wedge \overline{hg}$.
Hint: Find witnesses $\theta g, \theta h_1, \theta h_2$ for g, h_1, h_2 such that x does not occur in θg and $\text{HL} \vdash \theta(h_1(fx) \wedge \overline{h_1 g} \vee h_2(fx) \wedge \overline{h_2 g})$.

Exercise 10.6 (Skolem Forms) A Skolem form is a formula of the form $\exists x_1 \ldots x_m \forall y_1 \ldots y_n.s$, where $m,n \geq 0$ and s contains neither quantifiers nor functional identities. Find for each of the following formulas s a Skolem form t and prove $\text{HL} \vdash s = t$. Try to find Skolem forms with a minimal number of quantifiers.

a) $\forall f \wedge \exists g$

b) $\forall f \to \exists f$

c) $\forall x.fx \to \exists f \wedge gx$

d) $\forall x \forall y \exists z.fxyz$

e) $\exists f \vee \exists g \vee \forall h$ where f, g have the same type