Assignment 5
Introduction to Computational Logic, SS 2011

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Read in the lecture notes: Chapter 4

Note: It is very important to do all the examples in the lecture notes and the exercises below in the system Coq.

Exercise 5.1 Find proofs for the following propositions. Check your answers with Coq.

a) $\forall X\,Y\!:\!Prop.\;\; X \to (X \to Y) \to Y$
b) $\forall X\,Y\!:\!Prop.\;\; (X \to X \to Y) \to X \to Y$
c) $\forall X\,Y\,Z\!:\!Prop.\;\; (X \to Y \to Z) \to (X \to Y) \to X \to Z$
d) $\forall X\,Y\!:\!Prop.\;\; X \to Y \to \forall Z\!:\!Prop.\; (X \to Y \to Z) \to Z$
e) $\forall X\,Y\!:\!Prop.\;\; (\forall Z\!:\!Prop.\; (X \to Y \to Z) \to Z) \to X$
f) $\forall X\,Y\!:\!Prop.\;\; X \to \forall Z\!:\!Prop.\; (X \to Z) \to (Y \to Z) \to Z$

Exercise 5.2 Find proofs for the following propositions. Check your answers with Coq.

a) $\neg\neg\neg\bot$
b) $\forall X\!:\!Prop.\; X \to \neg\neg X$
c) $\forall X\!:\!Prop.\; \neg\neg\neg X \to \neg X$
d) $\forall X\!:\!Prop.\; \neg X \to (\neg X \to X) \to \bot$

Exercise 5.3 Prove that the proposition $\neg\neg\bot$ is not provable.

Exercise 5.4 Suppose the proofs of the lemmas *R* and *circuit2* would end with *Defined*. What term would you get with *Compute circuit2*? Check your answer with Coq.

Exercise 5.5 Establish the lemmas *circuit*, *R*, and *circuit2* with transparent proofs (i.e., *Defined* in place of *Qed*) and prove the following lemma.

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Lemma test : circuit = circuit2.
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Exercise 5.6 Prove $\forall X\,Y\!:\!Prop.\; \neg X \to ((X \to Y) \to X) \to \bot$. (Hint: You may find it useful to use the *assert* tactic with *False*.)

Exercise 5.7 Find proofs for the following propositions. Check your answers with Coq.

a) $\forall A : \mathsf{T}. \forall z : A. \forall r : A \to A \to \mathit{Prop}. (\forall x\,y : A. r\,x\,y) \to r\,z\,z.$

b) $\forall A : \mathsf{T}. \forall x\,y : A. \forall q : A \to \mathit{Prop}. q\,x \to (\forall p : A \to \mathit{Prop}. p\,x \to p\,y) \to q\,y.$

c) $\forall A : \mathsf{T}. \forall x\,y : A. \forall q : A \to \mathit{Prop}. q\,y \to (\forall p : A \to \mathit{Prop}. p\,x \to p\,y) \to q\,x.$