Assignment 8
Introduction to Computational Logic, SS 2011

Prof. Dr. Gert Smolka, Dr. Chad Brown
www.ps.uni-saarland.de/courses/cl-ss11/

Read in the lecture notes: Chapters 4-5

Note: It is very important to do all the examples in the lecture notes and the exercises below in the system Coq.

Exercise 8.1 Prove $\forall X{:}\mathit{Type}\ \forall p{:}X{\to}X{\to}\mathit{Prop}\ \forall x{:}X\ \exists y{:}X.\ \neg(pxy \leftrightarrow \neg pyy)$.

Exercise 8.2 Prove the following lemma, which says that a barber who shaves exactly those males who don't shave themseves cannot be male.

```
Lemma Barber (X : Type) (m : X -> Prop) (s : X -> X -> Prop) (b :X) :
(forall x,  m x -> (s b x <-> ~ s x x)) -> ~ m b.
```

Exercise 8.3 Prove the following generalized form of Cantor's Theorem.

```
Lemma GCantor (X Y : Type) (n : Y -> Y) :
(forall y, n y <> y) -> forall f : X -> X -> Y, exists g : X -> Y, forall x:X, f x <> g.
```

Exercise 8.4 Here are some challenging lemmas. Try to prove the lemmas with *tauto* and *generalize*. Once you have found a proof with *tauto*, you may fill in by hand the parts found by *tauto*.

```
Lemma DN_Contra : DN <-> forall X Y: Prop, (~Y -> ~X) -> X -> Y.
Lemma XM_Contra : XM <-> forall X Y: Prop, (~Y -> ~X) -> X -> Y.
Lemma DN_Peirce : DN <-> forall X Y: Prop, ((X -> Y) -> X) -> X.
```

Exercise 8.5 Give the typing rules for dependent matches discriminating on options and list. Use the *Check* command to find out whether your rules are correct.

Exercise 8.6 Prove the following lemmas.

```
Definition boolp (x : bool) : Prop := if x then True else False.
Lemma boolp_injective (x y : bool) : boolp x = boolp y -> x = y.
Lemma boolp_True (x : bool) : boolp x = True -> x = true.
```

Exercise 8.7 Prove the following lemmas.

```
Lemma negb_neq (x : bool) : negb x <> x.
Lemma true_or_false (x : bool) : x=true \/ x=false.
Lemma xor_com (x y : bool) : (if x then negb y else y) = (if y then negb x else x).
```

Exercise 8.8 Prove the following version of Cantor's Theorem. (Hint: Try to use the generalized Cantor Theorem from Exercise 8.3 above.)

```
Lemma BCantor (X : Type) :
forall f : X -> X -> bool, exists g : X -> bool, forall x:X, f x <> g
```

Exercise 8.9 Find ground terms that have the following types.

a) $\forall f : bool \to Type.\ f\ true \to f\ false \to \forall x : bool.\ f\ x$

b) $\forall p : bool \to Prop.\ p\ true \to p\ false \to \forall x : bool.\ p\ x$

c) $\forall p : bool \to Prop\ \forall x : bool.\ (x = true \to p\ true) \to (x = false \to p\ false) \to p\ x$