Assignment 12
Introduction to Computational Logic, SS 2011

Prof. Dr. Gert Smolka, Dr. Chad Brown

`www.ps.uni-saarland.de/courses/cl-ss11/`

Read in the lecture notes: Chapter 7

Note: It is very important to read the lecture notes, do all the examples in the lecture notes and do the exercises below in the system Coq.

Exercise 12.1 Study the proof of *Ap_append*. Make sure you can state all the subgoals after each tactic.

Exercise 12.2 Give the normal forms of the following terms: *FinVal F01*, *FinVal F02*, *FinVal F12*, *FinVal F03*, *FinVal F13* and *FinVal F23*.

Exercise 12.3 Give the normal forms of the following terms: *@P bool 3 F03*, *@P bool 3 F13*, *P F03 0 1 2* and *P F23 0 1 2*.

Exercise 12.4 Prove the following lemmas.

```
Lemma Pex1 : comp negb 1 (P F01) == negb.

Lemma Pex2 :
comp2 implb 2 (P F02) (P F12) == (fun x y => implb x y)
\/
comp2 implb 2 (P F02) (P F12) == (fun x y => implb y x).

Lemma Pex3 :
comp2 implb 2 (P F12) (P F02) == (fun x y => implb x y)
\/
comp2 implb 2 (P F12) (P F02) == (fun x y => implb y x).
```

Exercise 12.5 Define a function *get* of type

$$\forall A : \mathsf{T}.\ \forall n : nat.\ Fin\, n \to ilist\, A\, n \to A$$

such that $get\, A\, n\, k\, l$ returns the k^{th} element of the length-indexed list l.

Exercise 12.6 Use *Fin_Inv* and scripts to define a predecessor function with the following type and prove the function correct.

```
Definition predFin {n : nat} (x : Fin n) : option (Fin (pred n)).

Lemma predFin_correct {n:nat} :
predFin (@FinO n) = None /\ forall (k:Fin (S n)), predFin (FinS k) = Some k.
```

Exercise 12.7 Use *Fin_Inv* to prove the following.

```
Lemma Fin1 (k:Fin 1) : k = FinO 0.
```

Exercise 12.8 Prove the following lemmas.

```
Lemma B_NotTru_Fal_ex : (B_Not B_Tru) = B_Fal \/ (B_Not B_Tru) <> B_Fal.

Lemma B_NotTru_Fal_ex2 : [[B_Not B_Tru]] == [[B_Fal]] \/ ~ [[B_Not B_Tru]] == [[B_Fal]].
```

Exercise 12.9 Define *B_Or* and prove it is interpreted using *orb*.

```
Definition B_Or (s t : B) : B :=
...

Lemma B_Or_orb (s t : B) : [[B_Or s t]] == (comp2 orb n [[s]] [[t]]).
...
```

Exercise 12.10 Give the type and normal form of the following terms.

a) *[[F01 ==> #]] true*

b) *[[F01 ==> #]] false*

c) *[[(F02 ==> #) ==> F12]] false false*

d) *[[(F02 ==> #) ==> F12]] false true*

e) *[[(F02 ==> #) ==> F12]] false*

Exercise 12.11 Let $g : bool^3 \to bool$ be the boolean function such that $g\,a\,b\,c$ is *false* iff a and b are *true* and c is *false*. Find a formula $s \in B_3$ such that $[\![s]\!] \equiv g$. Prove your solution is correct by filling in the missing boolean formula in the following Coq proof.

```
Lemma exg :
let g := fun a b c => match a,b,c with true,true,false => false | _,_,_ => true end
in
{s:B 3|[[s]] == g}.
exists ...
intros a.
simpl.
destruct (hd a) ; destruct (hd (tl a)) ; destruct (hd (tl (tl a))) ; reflexivity.
Qed.
```