Assignment 11
Introduction to Computational Logic, SS 2012

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Read in the lecture notes: Chapters 10-11

Exercise 11.1 Prove that weakening is admissible for the tableau system. Rewrite with the list equivalence *rotate*.

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Lemma tabW C s :
tab C -> tab (s :: C).
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Exercise 11.2 Give signed tableau rules for conjunction and disjunction.

Exercise 11.3 Give complete signed tableaux for the following clauses. A tableau is complete if every branch is either closed or solved.

a) $\{\neg\neg x \to \neg y \to \neg(x \to y)^-\}$

b) $\{\neg x \to \neg y \to \neg(y \to x)^-\}$

Exercise 11.4 Make sure you understand every detail of the decision procedure *dec*. You should be able to write the code of *dec* given your understanding of the signed tableau rules. Don't worry about the first line and the proof obligations.

Exercise 11.5 Prove the lemmas *sat_dec*, *tab_dec*, *nd_dec*, *tab_unsat*, *valid_dec*, and *valid_refut*.

Exercise 11.6 Let $(W, \leq, \alpha)$ be a Kripke model. Argue the following facts where (w, w' and w'' are in W).

a) $w \vDash \neg s$ if and only if $w' \nvDash s$ for all $w' \geq w$.

b) $w \nvDash \neg s$ if and only if $w' \vDash s$ for some $w' \geq w$.

c) $w \vDash \neg\neg s$ if and only if for every $w' \geq w$ there is some $w'' \geq w'$ such that $w'' \vDash s$.

Exercise 11.7 There is no such soundness result for classical provability. Which rule of the classical ND calculus causes a problem?

Exercise 11.8 Let $(W, \leq, \alpha)$ be a Kripke model. Argue the following facts (where $w \in W$ and s, t, u are formulas).

a) $w \vDash s \to \neg\neg s$

b) $w \vDash s \to t \to s$

c) $w \vDash (s \to t \to u) \to (s \to t) \to s \to u$

Exercise 11.9 Suppose $\emptyset$ and s are such that $\emptyset \vdash_{\mathcal{NC}} s$. Argue that $\emptyset \nvdash_{\mathcal{N}} \neg s$.

Exercise 11.10 Which of the following formulas are independent? Justify your answer either by giving appropriate proofs in the intuitionistic ND system or by giving appropriate Kripke models.

a) $\neg(\neg\neg x \to x)$

b) $(x \to y) \to (\neg x \to y) \to y$

c) $((x \to y) \to x) \to x$

Exercise 11.11 Prove the following in Coq.

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Lemma unprovable_PWM : ~nd nil (Imp (Imp (Not x) y) (Imp (Imp (Not (Not x)) y) y)).

Lemma unprovable_nPWM : ~nd nil (Not (Imp (Imp (Not x) y) (Imp (Imp (Not (Not x)) y) y))).

Lemma indep_PWM : indep (Imp (Imp (Not x) y) (Imp (Imp (Not (Not x)) y) y)).
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