



Constraint Programming: Assignment no. 3

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In the following, let P be some fixed constraint satisfaction problem.

Recall that an *assignment* for P is a function $\alpha : \text{var}(P) \rightarrow \text{dom}(D)$, and that a *constraint store* for P is a function $s : \text{var}(P) \rightarrow \mathfrak{P}(\text{dom}(D))$. We write $\text{ass}(P)$ for the set of all assignments for P , and $\text{stores}(P)$ for the set of all constraint stores for P .

Exercise 3.1 (Assignments and stores)

Recall that the set of all assignments licensed by a constraint store s is defined as

$$\text{ass}(s) := \{ \alpha \in \text{ass}(P) \mid \forall x \in \text{var}(P). \alpha(x) \in s(x) \}.$$

For a given assignment α , we define the special store $\text{store}(\alpha)$ that licenses exactly the assignment α by putting $\text{store}(\alpha) := \{ x \mapsto \{ \alpha(x) \} \mid x \in \text{var}(P) \}$. Show that for all assignments $\alpha \in \text{ass}(P)$ and for all stores $s \in \text{stores}(P)$,

$$\alpha \in \text{ass}(s) \iff \text{store}(\alpha) \leq s \tag{1}$$

Exercise 3.2 (Implementation)

Intuitively, propagators should ‘implement constraints’. What we want this to mean is that a propagator should never remove a solution of a constraint from a store. In this exercise, we will see that it is sufficient to require a weaker condition. Let us say that a propagator p *implements* a constraint C , if for all assignments $\alpha \in \text{ass}(P)$,

$$\alpha \in C \iff p(\text{store}(\alpha)) = \text{store}(\alpha).$$

Show that the following properties hold for all stores $s \in \text{stores}(P)$, all constraints $C \in \text{con}(P)$, and all propagators p that implement C :

$$\alpha \in \text{ass}(s) \wedge \alpha \in C \implies \alpha \in \text{ass}(p(s)) \tag{2}$$

$$\alpha \notin C \implies |\text{ass}(p(\text{store}(\alpha)))| = 0 \tag{3}$$