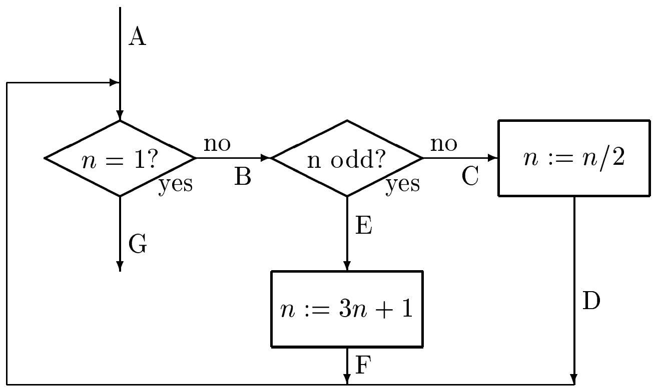
collecting/accumulating semantics — Jan 14 2002

$acc : p \mapsto \{s : Var \to Val \mid s\ visible\ at\ p\}$
given: $p, S_0 = \{all\ input\ values\ for\ n\} = \mathbb{N}$
lattice $L = \langle 2^{Store},\ \subseteq \rangle$

$L_{A,\ldots,G} = \langle \{\langle S_A, \ldots, S_G\rangle \mid S_A, \ldots, S_G \in 2^{Store}\},\ \dot{\subseteq} \rangle$
$acc \in L_{A,\ldots,G},\ acc = F(acc)$ or: $acc = lfp(F) = \bigcup_n F^n(\perp)$
$F(\langle S_A, \ldots, S_G\rangle) = \langle S'_A, \ldots, S'_G\rangle$
$S'_D = \{s' \in Store \mid \exists s \in S_C : \langle C, s\rangle \to \langle D, s'\rangle\}$

set of program points PP.

$$\begin{array}{rcl} F : (2^{Store})^{PP} & \to & (2^{Store})^{PP} \\ d & \mapsto & d' : PP \mapsto 2^{Store} \\ & & d'(p') = \{s' \in Store \mid \exists p \in PP, s \in d(p) :\ \langle p, s\rangle \to \langle p', s'\rangle\} \end{array}$$



$$\begin{array}{rcl} acc(A) & = & S_0 \\ acc(B) & = & (acc(A) \cup acc(D) \cup acc(F)) \cap \text{“} \neq 1'' \\ acc(C) & = & acc(B) \cap even \\ acc(D) & = & \{s' \mid s \in acc(C),\ s'(n) = s(n)/2\} \\ & = & f_{n/2}(acc(C)) \\ \vdots & & \end{array}$$

$$\begin{array}{rcl} acc^\#(A) & = & \top \\ acc^\#(B) & = & (acc^\#(A) \sqcup acc^\#(D) \sqcup acc^\#(F)) \sqcap \top \\ \ldots & & \\ acc^\#(D) & = & f^\#_{n/2}(acc^\#(C)) \\ \vdots & & \\ acc^\#(G) & = & \ldots \end{array}$$

$$\begin{array}{llcll} \alpha : & 2^{Store} & \longrightarrow & \{even,\ odd,\ \perp,\ \top\} = L^\# & \text{monotone} \\ \gamma : & \{even,\ odd,\ \perp,\ \top\} & \longrightarrow & 2^{Store} & \text{monotone} \end{array}$$

$\top$
/ \
$even$ odd
\ /
$\perp$

$acc^\#(G) \sqsupseteq \overbrace{\alpha(acc(G))}^{\perp}$ — Safety
$(\alpha(S) = \perp\ \Rightarrow\ S = \emptyset\ (by\ def.\ of\ \alpha))$
$acc(G) \subseteq \underbrace{\gamma(\underbrace{acc^\#(G)}_{\perp})}_{\emptyset}$

$\alpha(acc(B)) = acc^\#(B)$ — completeness of abstraction

$acc^\# :\ p \mapsto \{a \in \{e, o, \perp, \top\} :\ a\ is\ 'visible'\ at\ p\}$
$L^\#_{A,\ldots,G} = \langle \{\langle a_A, \ldots, a_G\rangle \mid a_A, \ldots, a_G \in \{e, o, \perp, \top\}, \dot{\sqsubseteq} \rangle$

$$
\begin{array}{rcl}
F^{\#} : L^{\#^{PP}} & \to & L^{\#^{PP}} \\
e & \mapsto & e' : PP \mapsto L^{\#} \\
 & & e'(p') = \{ s' \in AbsStore \mid \exists p \in PP, s \in e(p) : \ \langle p, s \rangle \mapsto \langle p', s' \rangle \}
\end{array}
$$

$AbsStore = L^{\#^{PP}} = \{ e : \ PP \mapsto \{e, o, \bot, \top\}\}$

$\{A, \ldots, G\} \to \{e, o, \bot, \top\}$

$\alpha(\{1\}) \qquad \alpha(\{s : \underbrace{Var}_{\{n\}} \to \underbrace{Val}_{\mathbb{N}} \mid s(n) = 1\})$

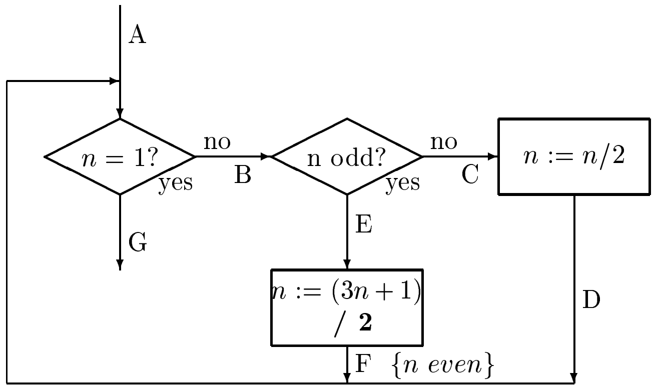
$\alpha(s) = e$

$\gamma(e) = even\ numbers$

$\alpha(\{2\}) = e$

$f^{\#}_{3n+1} : o \to e$

	A	B	C	D	E	F	G
$\dot{\bot}$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
$F^{\#}(\dot{\bot})$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
$F^{\#}(F^{\#}(\dot{\bot})) = F^{\#^2}(\dot{\bot})$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	o
$F^{\#^3}(\dot{\bot})$	$\top$	$\top$	e	$\bot$	o	$\bot$	o
$F^{\#^4}(\dot{\bot})$	$\top$	$\top$	e	$\top$	o	e	o
$F^{\#^5}(\dot{\bot})$	$\top$	$\top$	e	$\top$	o	e	o



<u>Constant propagation</u>

$$
\alpha : S \mapsto \begin{array}{ll} \bot & \text{if } S = \emptyset \\ k & \text{if } S = \{k\} \\ \top & \text{otherwise} \end{array}
$$

$acc^{\#}(p) = k$

$acc(p) = \{k\}$

<u>Dead code elimination</u>

$$
\alpha : S \mapsto \begin{array}{ll} \bot & \text{if } S = \emptyset \\ \top & \text{if } S \neq \emptyset \end{array}
$$

$acc^{\#}(p) = \bot \ \Rightarrow \ p\ not\ reachable$

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