Assignment 7, Semantics of Programming Languages

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The purpose of this assignment is to prepare you for the exam on Wednesday, December 19, 2001, 8am-11am. We start at 8.00am sharp. The exam will be strictly closed book (i.e., you must not use any notes and books, and you may only use paper that we provide).

Exercise 7.1 Make sure that you can solve the following exercises from Mitchell's book:

- Exercise 2.2.8, page 57
- Exercise 2.2.11, page 59
- Exercise 2.4.9, page 84
- Exercise 2.4.18, page 91
- Exercise 2.5.18, page 113
- Exercise 2.7.24, page 222
- Exercise 3.7.25, page 222

Exercise 7.2 Make sure that you can solve the exercises 6.4, 6.5 and 6.6 from Assignment 6.

Exercise 7.3 Perform the following substitution (types are omitted):

$$((\lambda f.\lambda y.f\ (x+y))\ (\lambda x.x+y)\ [y+3/x]$$

Do not reduce the expression.

Exercise 7.4 Give a term of PCF that has a normal form but is not strongly normalizing.

Exercise 7.5 Give an equation that defines a fixed point combinator for eager PCF.

Exercise 7.6 Define the following types declared in Standard ML in PCF with unit, sums and recursive types:

```
datatype Bool = F | T
datatype Nat  = O | S of Nat
datatype List = N | C of Nat * List
```

Exercise 7.7 Find rewrite systems as follows:

(a) A terminating but nonconfluent rewrite system with exactly one rule.

(b) A nonconfluent rewrite system that is locally confluent and has nontrivial critical pairs.

(c) A nonconfluent rewrite system that has no critical pairs.

(d) A canonical rewrite system that has nontrivial critical pairs.

(e) A terminating but nonconfluent rewrite system that rewrites formulas with $\neg, \wedge, \vee$ into disjunctive normal form.

Prove that the rewrite system you give for (e) is nonconfluent.

Exercise 7.8 Prove the following equivalence in $\lambda^{\rightarrow}$.

$$\Gamma, x{:}\sigma \triangleright f\ x = M{:}\tau \quad \Leftrightarrow \quad \Gamma \triangleright f = \lambda x{:}\sigma.\, M{:}\sigma \rightarrow \tau$$

Exercise 7.9 Assume a signature that specifies the following term constants:

$$\begin{aligned} \text{fix} &: (\sigma \rightarrow \sigma) \rightarrow \sigma \\ \text{K} &: \sigma \rightarrow \tau \rightarrow \sigma \\ \text{S} &: (\rho \rightarrow \sigma \rightarrow \tau) \rightarrow (\rho \rightarrow \sigma) \rightarrow \rho \rightarrow \tau \end{aligned}$$

Give closed equations

$$\begin{aligned} \text{fix} &= \dots \\ \text{K} &= \dots \\ \text{S} &= \dots \end{aligned}$$

that specify the semantics of the respective combinators. Omit the types in lambda abstractions.

Exercise 7.10 Suppose that the combinators K and S are given for all types. Give the rules that rewrite terms with abstractions into equivalent combinatory terms. Omit the types.

Exercise 7.11 Translate the following terms into combinatory terms with K and S (types are omitted):

(a) $\lambda x.\, \lambda y.\, y$

(b) $\lambda x.\, \lambda y.\, \lambda z.\, y$

(c) $\lambda x.\, \lambda y.\, \lambda z.\, x$