Semantics of Programming Languages
Assignment 10

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`http://www.ps.uni-sb.de/courses/sem-ws01/`

INFORMATIK

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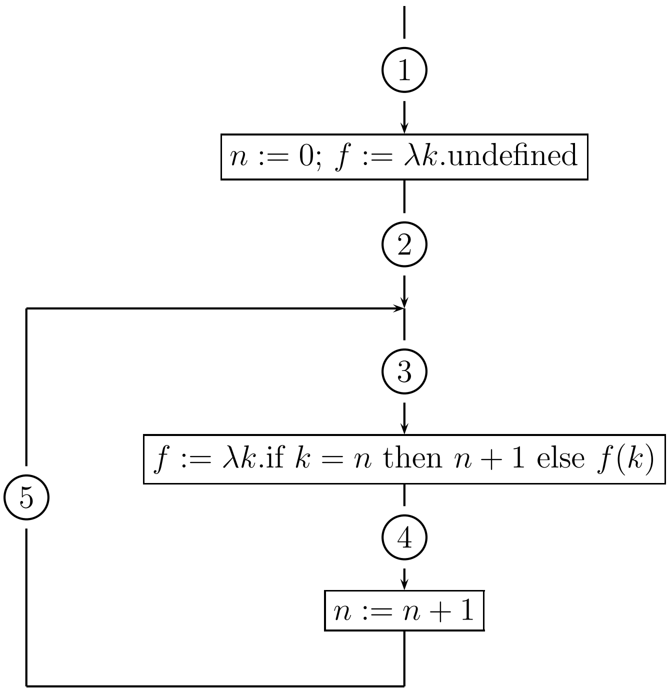
Exercise 10.1 (4) Let $\langle L, \sqsubseteq, \top \rangle$ a poset with greatest element $\top$, let S and T be sets such that $T \subseteq S$, and let $\sqsubseteq_S$ resp. $\sqsubseteq_T$ be the pointwise ordering on L^S resp. L^T induced by $\sqsubseteq$. Let $\gamma : L^T \to L^S : g \mapsto f_g$ with

$$f_g : S \to L : x \mapsto \begin{cases} g(x) & \text{if } x \in T \\ \top & \text{otherwise} \end{cases}$$

Is there an $\alpha : L^S \to L^T$ such that $\langle L^S, \sqsubseteq_S \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle L^T, \sqsubseteq_T \rangle$? Prove your answer.

For the following exercises, consider the program below with program points $PP = \{1, \ldots, 5\}$ and variables $Var = \{n, f\}$. The store maps n to natural numbers and f to partial functions over the naturals, i. e., $Store = \{s \mid s(n) \in \mathbb{N}, s(f) \in \mathbb{N} \xrightarrow{p} \mathbb{N}\}$.



Exercise 10.2 (1) Guess what the program computes.

Exercise 10.3 (3) Specify the collecting semantics as a system of equations.

Exercise 10.4 (5) Specify the collecting semantics as a least fixpoint: Give an operator $F : (2^{Store})^{PP} \to (2^{Store})^{PP}$ such that $acc = \operatorname{lfp} F$. Compare acc with your guess from exercise 10.2.
Hint: Think of F as function over quintuples of sets of stores and decompose F via $F(d) = \langle f_1(d), \dots, f_5(d)\rangle$; the functions $f_i : (2^{Store})^{PP} \to 2^{Store}$, which map quintuples of sets of stores to sets of stores, should closely resemble your equations from exercise 10.3.

We do a definedness analysis to find out for which natural numbers the program defines the partial function f. Our concrete domain $\langle D, \subseteq\rangle$ is $D = 2^{Store}$ ordered by inclusion. As abstract domain $\langle D^{\#}, \sqsubseteq\rangle$, we take $D^{\#} = 2^{\mathbb{N}}$ ordered by inverse inclusion, i. e., $M \sqsubseteq M'$ iff $M' \subseteq M$. The abstraction α maps $S \in D$ to $\alpha(S) = \bigcap\{dom(s(f)) \mid s \in S\} \in D^{\#}$, where $dom(h) \subseteq \mathbb{N}$ denotes the domain of a partial function $h \in \mathbb{N} \xrightarrow{p} \mathbb{N}$.

Exercise 10.5 (4) Find the corresponding concretization $\gamma : D^{\#} \to D$ and prove that $\langle D, \subseteq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle D^{\#}, \sqsubseteq\rangle$.

Exercise 10.6 (5) Give the best approximations $f_i^{\#}$ of f_i (see exercise 10.4) and the corresponding operator $F^{\#} : {D^{\#}}^{PP} \to {D^{\#}}^{PP}$. Compute $\operatorname{lfp} F^{\#}$. For which natural numbers does the program define f?

Exercise 10.7 (3) Prove that the above analysis can be refined to arbitrary precision.
Hint: Unfold the loop once, twice, ...