Semantics of Programming Languages
Assignment 12

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INFORMATIK

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Consider the following program P; the integer variables x and y both are initialized to 0.

```
l1: while (*) {
l2:   x++;
l3:   y++;
    }
l4: while (*) {
l5:   x--;
l6:   y--;
    }
l7: if (x == y) {
l8:   assert(0);
    }
```

Exercise 12.1 (10) Let the set of predicates be $\{p_1\}$ where $p_1 \equiv x = y$.

1. For each of the assignments in P, compute the corresponding abstract statement.
2. Construct the abstract program $P^{\#}$ such that *post* of $P^{\#}$ equals $post^{\#}_{cart}$ of P.
3. Compute the collecting semantics of $P^{\#}$. Is the label $l8$ reachable in $P^{\#}$?

Exercise 12.2 (5) Refine the abstraction (i. e., specify additional predicates) from exercise 12.1 such that the label $l8$ is not reachable in the abstract program. Give the new abstract program (without proof).

Exercise 12.3 (5) Formalize constant propagation (see exercise 9.3) as predicate abstraction, i. e., give the predicates and the boolean abstraction.

Why is this analysis not implementable? Suggest a way to make it implementable, at least for some programs.

Discuss the effect of a cartesian abstraction (see exercises 8.2 and 8.3) on top of the boolean abstraction.

Exercise 12.4 (5) Let $\mathbf{D} = \langle D, \sqsubseteq \rangle$ and $\mathbf{D}^\# = \langle D^\#, \sqsubseteq^\# \rangle$ be posets. A function $f : D \to D^\#$ is called *additive* if for all $a, b \in D$, the existence of $a \sqcup b$ implies that $f(a) \sqcup^\# f(b)$ exists and $f(a \sqcup b) = f(a) \sqcup^\# f(b)$.

Let $\mathbf{D} \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \mathbf{D}^\#$ be a Galois connection. Prove that α is additive. Give a counterexample showing that γ need not be additive even if γ is the identity, i. e., $\gamma(y) = y$ for all $y \in D^\#$.

Generalize the concept of additive functions to *completely additive* functions and prove that α is completely additive. What related property holds for γ?

Just for fun: Considering the posets $\mathbf{D}$ and $\mathbf{D}^\#$ as CPOs, what are the relations between additive, completely additive and continuous functions?