Semantics, WS 2003 – Assignment 8

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Recommended reading: Types and programming languages, chapter 15, 16, 18, 19

Exercise 8.1: Subtyping and References Consider a λ-calculus with subtypes and references:

$T ::= \mathit{Unit} \mid \mathit{Top} \mid T \to T \mid \mathit{Ref}\ T$

$t ::= \mathit{unit} \mid x \mid \lambda x : T.t \mid t\ t \mid \mathit{ref}\ t \mid !t \mid t := t$

(a) Assume that the subtyping relation is defined with the usual rules for *Top* and $T \to T$ and the rule $\dfrac{T <: T'}{\mathit{ref}\ T\ <:\ \mathit{ref}\ T'}$. Give a closed well-typed term whose normal form is stuck.

(b) Assume that the subtyping relation is defined with the usual rules for *Top* and $T \to T$ and the rule $\dfrac{T' <: T}{\mathit{ref}\ T\ <:\ \mathit{ref}\ T'}$. Give a closed well-typed term whose normal form is stuck.

Exercise 8.2: Subtyping and Preservation Consider a λ-calculus with subtypes as follows:

$T ::= \mathit{Top} \mid T \to T$

$t ::= x \mid \lambda x : T.t \mid t\ t$

Give a term t for which there exist T, T' such that $\varnothing \mapsto t : T$, $t \longrightarrow t'$, $\varnothing \mapsto t' : T'$ and $T' \neq T$. Why does the existence of t not violate the Preservation Property?

Exercise 8.3: Natural numbers in Java The following Java program models the natural numbers with addition:

```
class Nat extends Object {
  Nat() {}
  Nat add(Nat y) { return y; }
}

class Pos extends Nat {
  Nat pred;
  Pos (Nat x) { pred = x; }
  Nat add(Nat y) { return pred.add(new Pos(y)); }
}
```

(a) Model the classes *Object*, *Nat*, and *Pos* in the λ-calculus. Model each class C with two record types C and $CRep$ and two procedures

$classC : CRep \to (Unit \to C) \to (Unit \to C)$

$newC : CArg \to C$

You are allowed (and have to) define the type *Nat* recursively. The procedures *classPos* and *newPos* have to be mutually recursive. Try to express that using *fix*.

(b) Model the classes *Object*, *Nat*, and *Pos* in SJ. Do this by defining the following set-theoretic objects:

(i) The set Cla and the relation $<:_0 \subseteq Cla^2$

(ii) The function $typ \in Cla \to Lab \rightharpoonup Typ$

(iii) The function $mth \in Cla \to Lab \rightharpoonup Val$

(c) Extend the Java program with a method *sub* for subtraction.

Exercise 8.4: Buzzwords Explain each of the following buzzwords used with object-oriented programming with one or two sentences and an example:

(a) encapsulation

(b) multiple representations

(c) subtyping

(d) inheritance

(e) overriding

(f) late binding

(g) open recursion

Exercise 8.5: Exponentially larger normal forms Assume the pure untyped λ-calculus. Define recursively terms $t_0, t_1, t_2, \ldots$ such that for all i:

(a) t_i has a normal form and

(b) $size(\text{normal form of } t_i) > 2^i$

Assume that the size of terms is defined as follows:

$$\begin{aligned} size(x) &= 1 \\ size(\lambda x.t) &= 1 + size(t) \\ size(t_1\ t_2) &= 1 + size(t_1) + size(t_2) \end{aligned}$$

Define the terms t_i such that they have at most one free variable f.