Assignment 6
Semantics, WS 2007/08

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Read Chapter 3 of the lecture notes and Chapters 6, 8 and 9 of Pierce's TAPL

Exercise 6.1 (Contextual equivalence)

a) Show that $s \sim t \implies \theta s \sim \theta t$ doesn't hold in the pure calculus.

b) Show that $x \sim (\lambda y.x)y$ doesn't hold if contextual equivalence is defined without the closedness condition.

c) Show that $x \approx \lambda y.xy$ doesn't hold in the extended calculus.

Exercise 6.2 (De Bruijn terms) We implement ordinary and de Bruijn terms as follows:

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type var = int
datatype ter = V of var | A of ter * ter | L of var * ter
datatype dbt = DV of var | DA of dbt * dbt | DL of dbt
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a) Write a procedure $\mathit{free} : \mathit{var} \to \mathit{dbt} \to \mathit{bool}$ that tests whether a variable occurs free in a de Bruijn term.

b) Write a procedure $\mathit{subst} : (\mathit{var} \to \mathit{dbt}) \to \mathit{dbt} \to \mathit{dbt}$ that applies a de Bruijn substitution to a de Bruijn term.

c) Write a procedure $\mathit{db} : \mathit{ter} \to \mathit{dbt}$ that translates a term into a de Bruijn term.

Exercise 6.3 (Simple types)

a) Show that for any T, the n-th Church numeral $c_n := \lambda f{:}T \to T.\lambda x{:}T.f^n\, x$ has type $(T \to T) \to T \to T$.

b) Give a type-annotated version of *succ* (see Sect. 3.4 of the notes), and verify that it has type $((T \to T) \to T \to T) \to (T \to T) \to T \to T$. Can you do the same for $\mathit{power} = \lambda mn.n\,m$, taking 2 arguments of type $(T \to T) \to T \to T$?

c) Show that $\omega = \lambda x{:}T.x\,x$ is not well-typed, for any choice of T. (By Exercise 6.6 below, you may therefore conclude that also $\Omega = \omega\,\omega$ is not typable.) **Hint:** use the Inversion Lemma.

Exercise 6.4 (Expansion) Show that *expansion* does not hold: find Γ, t, t' and T such that $\Gamma \vdash t' : T$ and $t \to t'$, but $\Gamma \vdash t : T$ does not hold.

Exercise 6.5 (Relevance) Prove that $\Gamma \vdash t : T$ and $x \in \mathit{FV}\,t$ implies that $x \in \mathit{Dom}\,\Gamma$.