



Assignment 13 Semantics, WS 2007/08

Prof. Dr. Gert Smolka, Dr. Jan Schwinghammer
www.ps.uni-sb.de/courses/sem-ws07/

Read Chapters 23 and 29 of Pierce's TAPL.

Exercise 13.1 (Booleans) Express the following values in F:

$false : bool$

$true : bool$

$if : \forall X Y. bool \rightarrow (1 \rightarrow X) \rightarrow (1 \rightarrow X) \rightarrow X$

Exercise 13.2 (Natural Numbers) Express the following values in F:

$0 : nat$

$succ : nat \rightarrow nat$

$iter : \forall X. nat \rightarrow (X \rightarrow X) \rightarrow X \rightarrow X$

$plus : nat \rightarrow nat \rightarrow nat$

$power : nat \rightarrow nat \rightarrow nat$

Exercise 13.3 (Lists) Express lists in F. First find a type construction $list\ S$ and then express the following operations:

$nil : \forall X. list\ X$

$cons : \forall X. X \rightarrow list\ X \rightarrow list\ X$

$foldl : \forall X Y. (X \rightarrow Y \rightarrow Y) \rightarrow Y \rightarrow list\ X \rightarrow Y$

Exercise 13.4 (Products) Express the following values in F:

$pair : \forall X Y. X \rightarrow Y \rightarrow X \times Y$

$fst : \forall X Y. X \times Y \rightarrow X$

$snd : \forall X Y. X \times Y \rightarrow Y$

Exercise 13.5 (Sums) Express the following values in F:

$inl : \forall X Y. X \rightarrow X + Y$

$inr : \forall X Y. Y \rightarrow X + Y$

$case : \forall X Y Z. X + Y \rightarrow (X \rightarrow Z) \rightarrow (Y \rightarrow Z) \rightarrow Z$

Exercise 13.6 (Side Condition) Consider the closed term

$$t = \lambda X. \lambda x:X. \lambda X. \lambda f:X \rightarrow X. fx$$

- a) Convince yourself that t is ill-typed.
- b) Convince yourself that the term becomes well-typed if the side condition “ Y not free in Γ or t ” in the typing rule for polymorphic procedures is dropped.
- c) Convince yourself that $t \text{ bool true nat succ} \rightarrow^* \text{succ true}$. Why does this reduction show that t must be ill-typed?
- d) Explain why the side condition doesn't prevent that the following term is well-typed: $\lambda X. \lambda x:X. \lambda X. \lambda f:X \rightarrow X. \lambda x:X. fx$.