Assignment 1
Semantics, WS 2011-2012

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Read in the lecture notes: Chapter 2

Read the web pages for the course. Install the Coq system on your computer. Read Sections 2.1-2.4 of the lecture notes for “Computational Logic”.Then do the following exercises. Also read through the chapter “Basics” of the “Software Foundations” text.

Exercise 1.1 (Disjunction) A boolean disjunction $x \lor y$ yields *false* if and only if both x and y are *false*.

a) Define disjunction as a function *orb* : *bool* → *bool* → *bool* in Coq.
b) Prove the de Morgan law $\neg(x \lor y) = \neg x \land \neg y$ in Coq.

Exercise 1.2 Define functions as follows.

a) A function *power* : *nat* → *nat* → *nat* that yields x^n for x and n.
b) A function *fac* : *nat* → *nat* that yields $n!$ for n.
c) A function *even* : *nat* → *bool* that tests whether its argument is even.
d) A function *mod3* : *nat* → *nat* that yields the remainder of x on division by 3.

Exercise 1.3 Prove the following lemmas.

```
Lemma mul_O (x : nat) : mul x O = O.

Lemma mul_S (x y : nat) : mul x (S y) = add (mul x y) x.

Lemma mul_com (x y : nat) : mul x y = mul y x.

Lemma mul_dist (x y z: nat) : mul (add x y) z = add (mul x z) (mul y z).

Lemma mul_asso (x y z: nat) : mul (mul x y) z = mul x (mul y z).
```

Exercise 1.4 Consider the following Coq proof. After each tactic in the proof, there will be a number of goals. After each tactic give the number of goals and for each of these goals give the assumptions of the goal and the claim of the goal.

```
Lemma add_S' (x y : nat) : add x (S y) = S (add x y).

Proof.
induction x.
simpl.
reflexivity.
simpl.
rewrite IHx.
reflexivity.
Qed.
```