



Assignment 3 Semantics, WS 2011-2012

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Exercise 3.1 Prove the following.

Goal $\text{forall } X:\text{Prop}, \sim X \leftrightarrow \sim\sim X$.

Exercise 3.2 Prove the following.

a)

Goal $\text{False} \leftrightarrow \text{forall } Z:\text{Prop}, Z$.

b)

Goal $\text{forall } X:\text{Prop}, \sim X \leftrightarrow \text{forall } Z:\text{Prop}, X \rightarrow Z$.

c)

Goal $\text{forall } X:\text{Type}, \text{forall } x\ y:X, x = y \leftrightarrow \text{forall } p:X \rightarrow \text{Prop}, p\ x \rightarrow p\ y$.

d)

Goal $\text{forall } X\ Y:\text{Prop}, X \wedge Y \leftrightarrow \text{forall } Z:\text{Prop}, (X \rightarrow Y \rightarrow Z) \rightarrow Z$.

e)

Goal $\text{forall } X\ Y:\text{Prop}, X \vee Y \leftrightarrow \text{forall } Z:\text{Prop}, (X \rightarrow Z) \rightarrow (Y \rightarrow Z) \rightarrow Z$.

Exercise 3.3 Prove the following.

a)

Goal $\text{forall } X\ Y:\text{Prop}, \sim(X \vee Y) \leftrightarrow \sim X \wedge \sim Y$.

b)

Goal $\text{forall } X\ Y\ Z:\text{Prop}, (X \vee (Y \wedge Z)) \leftrightarrow (X \vee Y) \wedge (X \vee Z)$.

Exercise 3.4 Prove the following. (This exercise may be tough.)

Goal $(\text{forall } X:\text{Prop}, \sim\sim X \rightarrow X) \rightarrow (\text{forall } X:\text{Prop}, X \vee \sim X)$.

Exercise 3.5 Prove the following.

a)

```
Goal forall p:nat -> Prop, forall x:nat, p 0 -> (forall x:nat, p x -> p (S x)) -> p x.
```

b)

```
Goal forall X:Type, forall p:list X -> Prop, forall xs:list X, p nil ->
(forall x:X, forall xs:list X, p xs -> p (x :: xs)) -> p xs.
```

Exercise 3.6 Extend the compiler correctness development with an operator for subtraction.

Exercise 3.7 (Challenge) Write a decompilation function that recovers an expression from the program it compiles to and prove the correctness of your function.

Exercise 3.8 Consider the following alternative definition of a compiler.

```
Fixpoint compile' (e : exp) : prog :=
  match e with
  | Const n => iConst n :: nil
  | Binop b e1 e2 => compile' e1 ++ compile' e2 ++ iBinop b :: nil
  end.
```

Consider the binary operators for addition (+), multiplication (*) and subtraction (-). What is the maximum set of these three operators for which this compiler is correct?