



## Seminar Advanced Topics in Semantics, Summer 2008 Session 2

Dr. Jan Schwinghammer, Prof. Dr. Gert Smolka  
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Logical Relations and a Case Study in Equivalence Checking

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### Small-group work

Discuss some or all of the following questions about Karl Cray's article "Logical Relations ..."

- a) Solve Exercise 6.3.1: Recall that  $\Gamma \vdash x \equiv y : \text{Unit}$  is an example showing that equivalence is type-dependent (and goes beyond merely checking for equal normal forms). Give two *closed* terms that are equivalent but have distinct normal forms.
- b) Can you think of types other than `Unit` that require a type-dependent notion of equivalence?
- c) Solve Exercise 6.6.4: Suppose that the language contains exactly one constant `k`. Produce a counterexample for monotonicity, i.e., find terms `s`, `t`, such that  $\Gamma \vdash s \text{ is } t : T$  but not  $\Gamma, x : S \vdash s \text{ is } t : T$ .
- d) Solve Exercise 6.9.13: Argue that the equivalence algorithm terminates.
- e) Solve Exercise 6.9.14: Extend the language with pairs, and sketch how the statements and proofs have to be adapted. Challenge: What about sum types?
- f) Solve Exercise 6.9.15: Where does the proof break down when considering polymorphic types? What about recursive types?

Discuss any further problems, or questions you find interesting about the chapter.