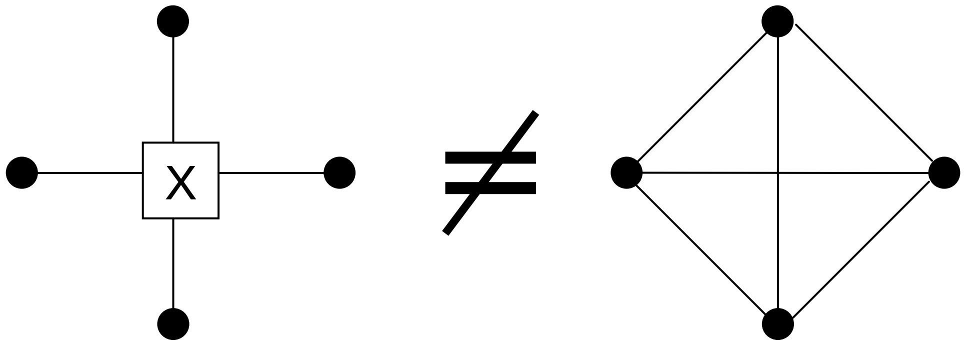
Context-Free Graph Grammars

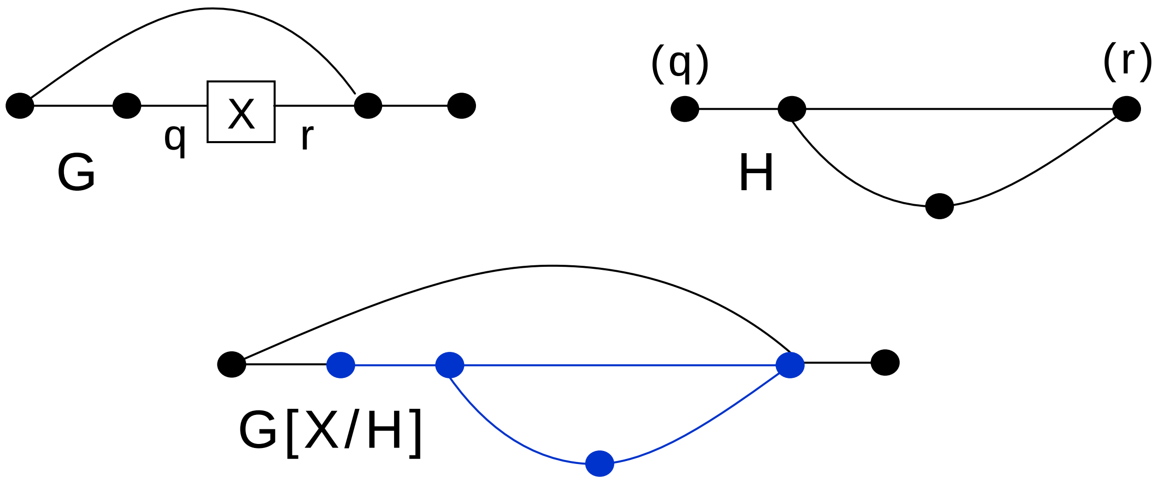
Kai Mittermüller

Übersicht

- Hypergraphen und Graphgrammatiken
- Kontextfreiheit von Graphgrammatiken
- Hierarchie von String-Graphgrammatiken

Hypergraphen

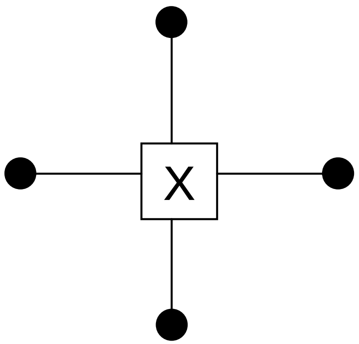




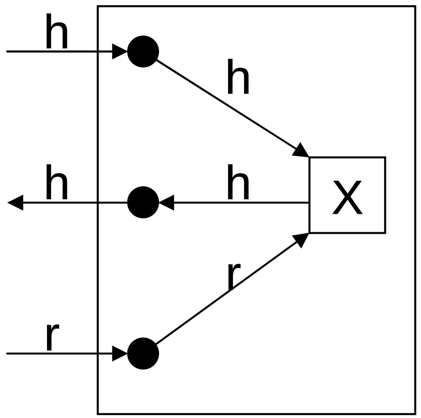
Hypergraphen

Zwei Arten von Hypergraphen

Hyperkanten



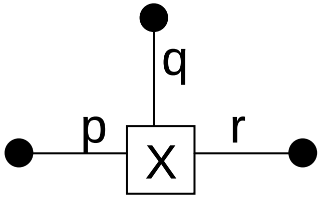
Hyperknoten



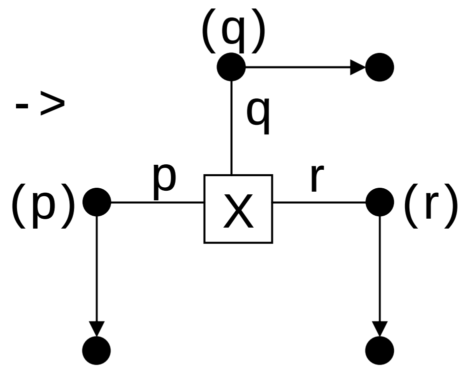
Hyperkanten-Graphen sind äquivalent zu Hyperknoten-Graphen

Ableitungen

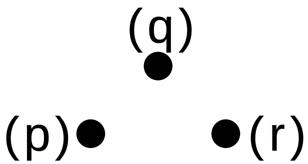
S ->



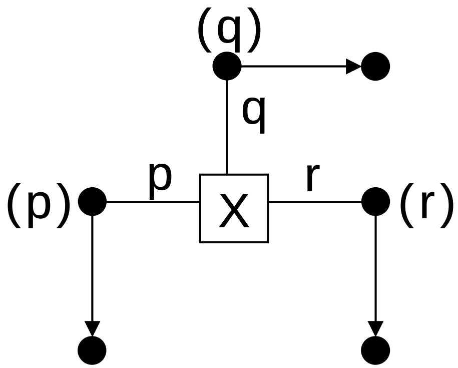
X ->



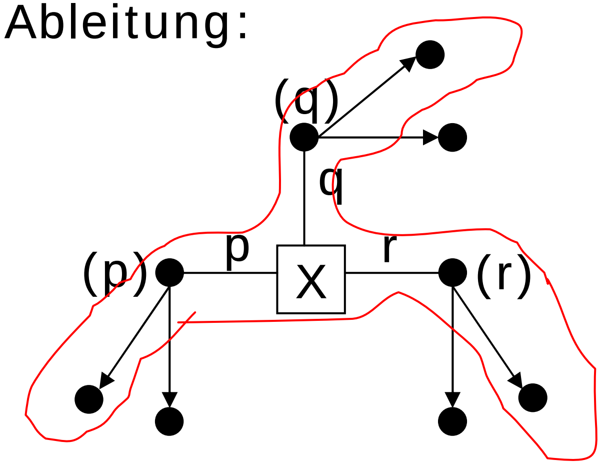
X ->



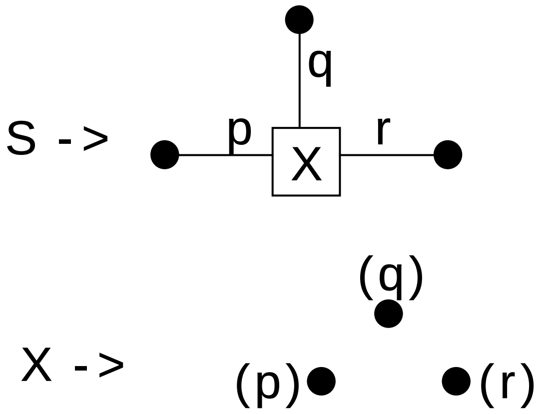
1. Ableitung:

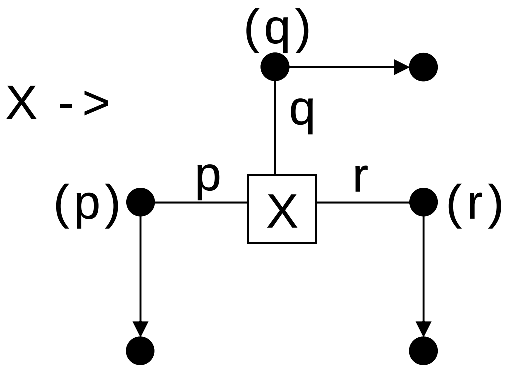


2. Ableitung:

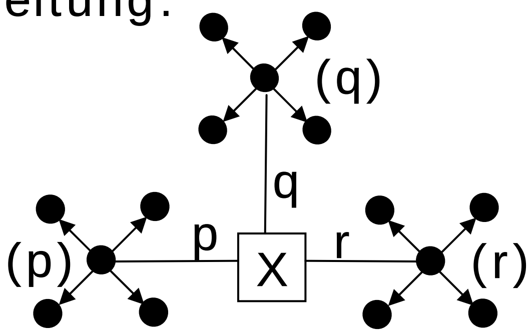


Ableitungen

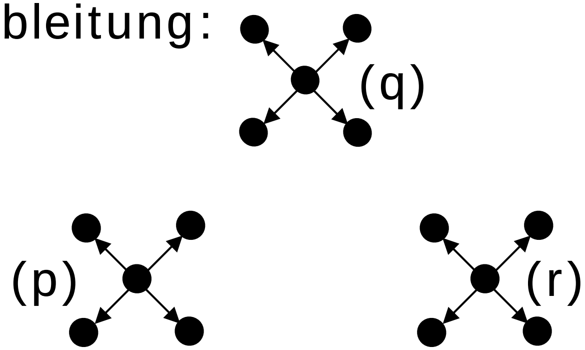




4. Ableitung:



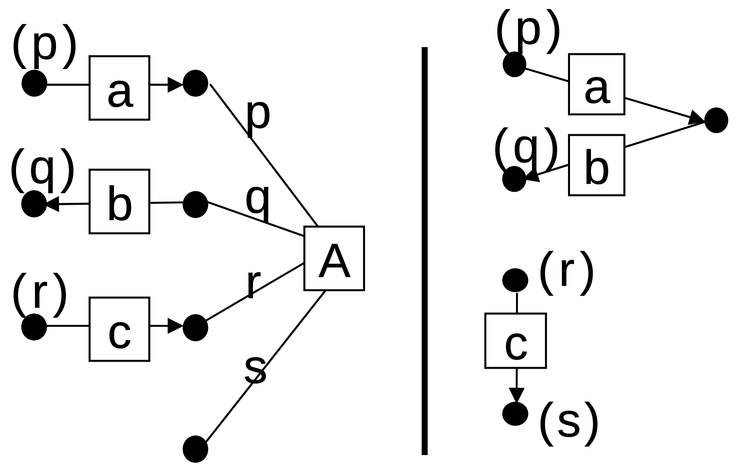
5. Ableitung:



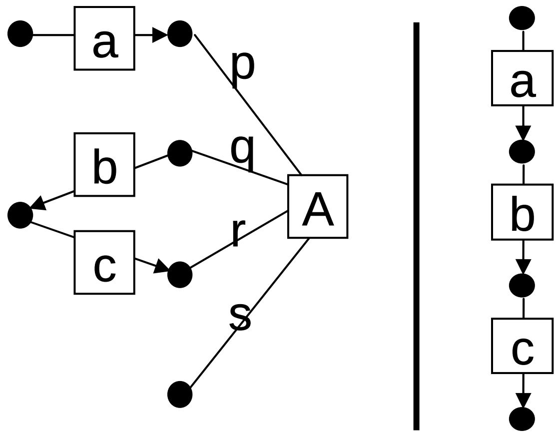
Stringsprachen

$$L = \{a^n b^n c^n\}$$

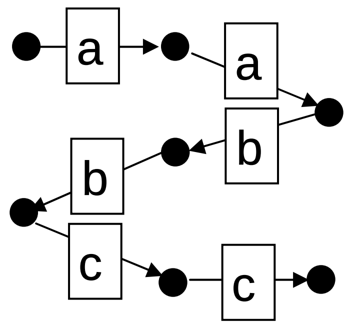
A ->



S ->



Beispiel
Ableitung



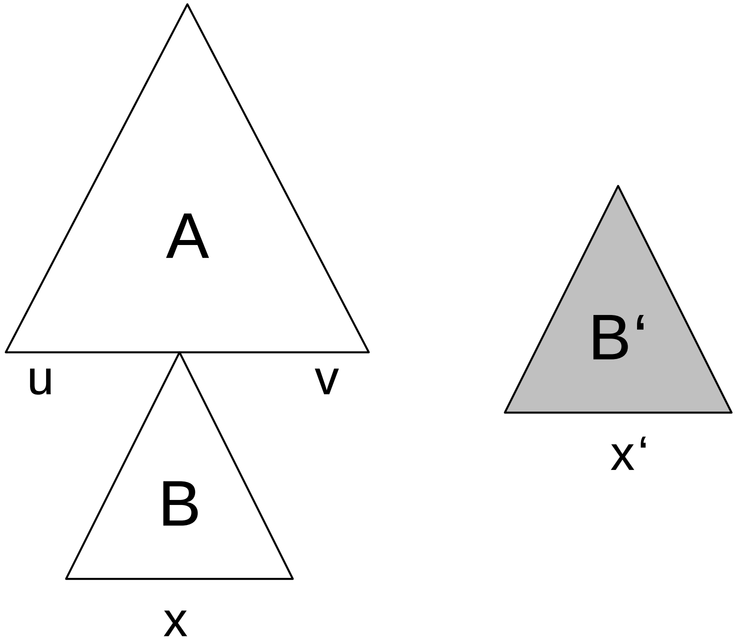
Kontextfreiheit

$$A \Rightarrow^* uBv$$

$$\Rightarrow^* uxv$$

$$B' \Rightarrow^* x'$$

$$\Rightarrow A \Rightarrow^* ux'v$$



Kontextfreiheit

Lemma:

$Seien\ H\ und\ K\ Hypergraphen.$

$Sei\ e_1 \ldots e_n\ alle\ Nichtterminal-Kanten \in H.$

$Gdw.\ H \Rightarrow {}^*K\ existieren\ Hypergraphen\ K_1 \ldots K_n,$

$so\ dass\ K = H\left[e_1 \backslash K_1\right] \ldots \left[e_n \backslash K_n\right]\ mit$

$e_i \Rightarrow {}^*K_i\ \text{für}\ alle\ 1 \le i \le n.$

Kontextfreiheit

Beweisskizze:

$$Induktion\ \ über\ die\ Länge\ \ der\ Ableitung:$$

$$I.A: Länge = 0 \quad \Rightarrow \quad H = K$$

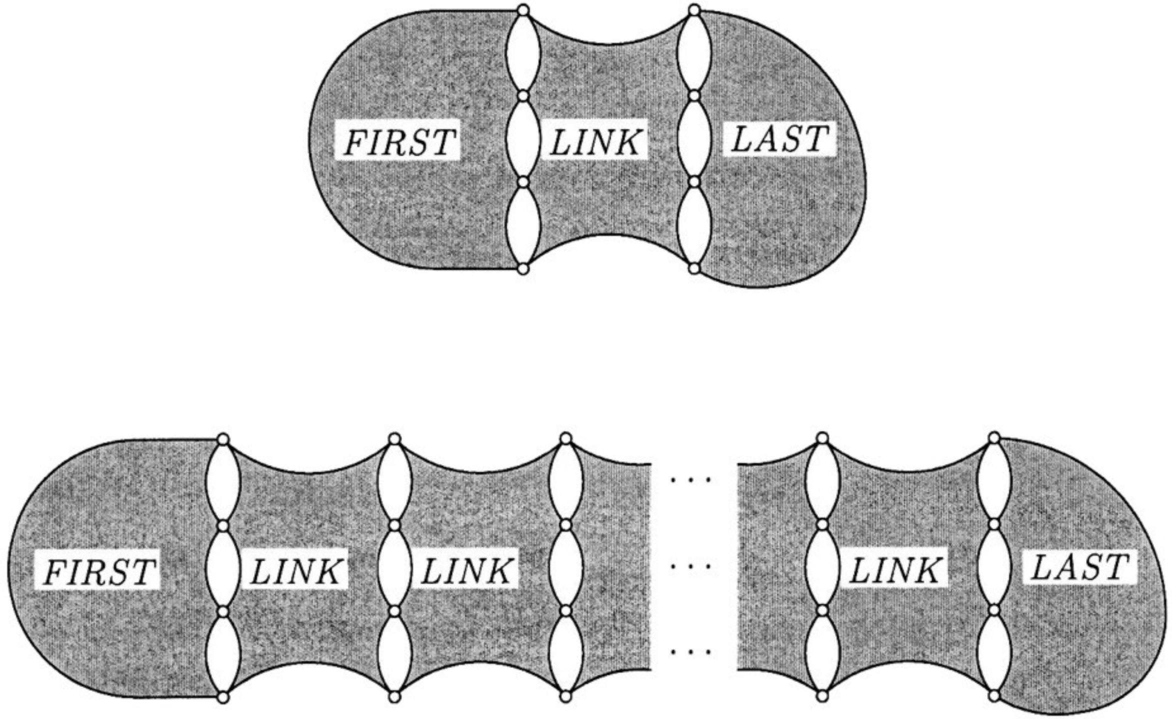
$$I.S: Sei\ K' = H[e_1 \setminus K_1]\ldots[e_{n-1} \setminus K_{n-1}]$$

$$und\ es\ exitiert\ Ableitung\ e_n \Rightarrow {}^*K_n$$

$$\Rightarrow K = K'[e_n \setminus K_n]$$

Zusätzlich muss gezeigt werden, das die Reihenfolge und der Zeitpunkt der Ersetzung keinen Unterschied machen.

Pumping-Lemma



Pumping-Lemma

$$\forall L \in hyperedge\ replacement\ language\ with\ order(L) = r$$

$$\exists\, p, q \in N$$

$$\forall H \in L,\ |H| > p$$

$$\exists\, First, Link, Last : First \otimes Link \otimes Last = H,\ |Link \otimes Last| \leq q$$

$$und\ type(Link) \leq r$$

$$\forall\, k \in N : First \otimes Link^{k} \otimes Last \in L$$

Pumping-Lemma (regulär)

$$\forall L \in regular\ language$$

$$\exists\ p \in N$$

$$\forall h \in L : |h| > p$$

$$\exists\ u, v, w : uvw = h,\ |v| > 0, |uv| < p$$

$$\forall\ k \in N : uv^k w \in L$$

Hierarchie von Stringgrammatiken

$$1)\, STR(HR_2) <=> CFL$$

$$2)\, \forall k > 1: STR(HR_{2k}) = STR(HR_{2k+1}) \subset STR(HR_{2k+2})$$

$$Anmerkung:$$

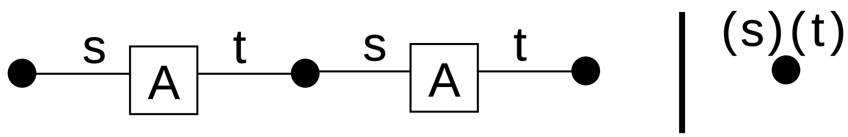
$$STR(HR_{2k}) <=> L_{2k} = \{a_1^n \ldots a_{2k}^n\}$$

STR(HR_2) <=> CFL

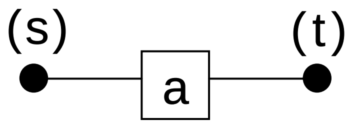
S -> AA | e

A -> a

S ->

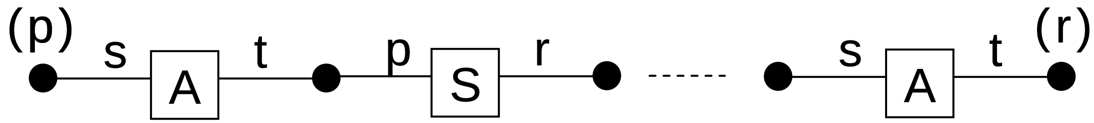


A ->



STR(HR$_2$) <=> CFL

S ->



A ->

S -> AS...A

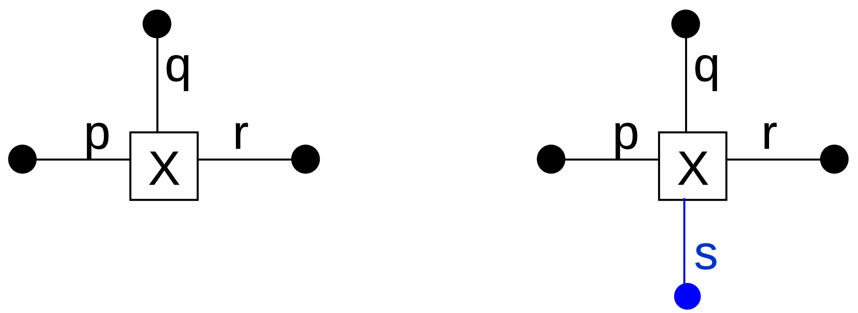
A -> a...c

$STR(HR_{2k+1}) \subset STR(HR_{2k+2})$

- Es ist offensichtlich das

$$HR_l \subseteq HR_{l+1}$$

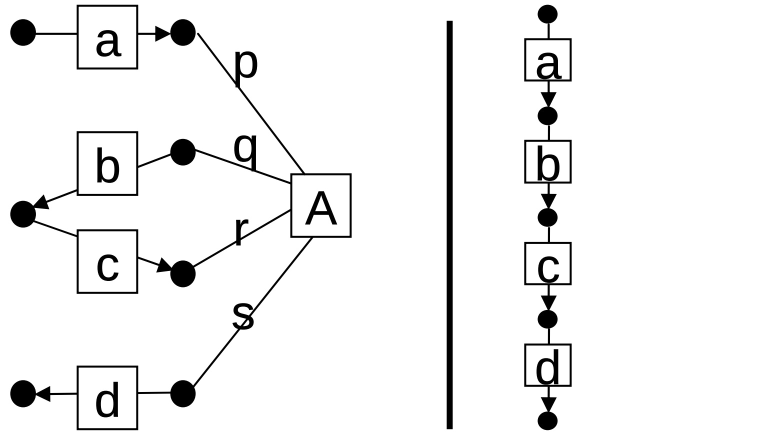
da jede Hyperkante einfach um einen zusätzlichen Knoten erweitert werden kann:



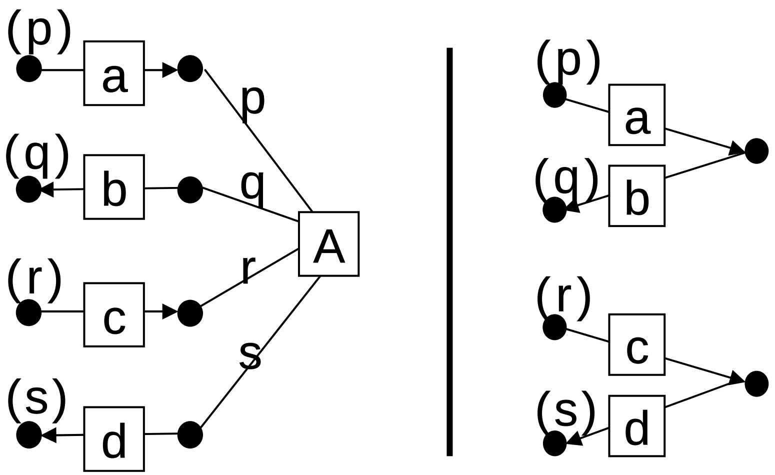
STR(HR$_{2k+1}$) ⊂ STR(HR$_{2k+2}$)

$$L_4 = \{a^n b^n c^n d^n\}$$

S ->



A ->



$$\Rightarrow L_4 \in HR_4$$

STR(HR$_{2k+1}$) ⊂ STR(HR$_{2k+2}$)

$z.z.: L_4 \notin HR_3$

Annahme : $L_4 \in HR_3$

dann gilt nach dem Pumpinglemma :

$L_4 \; mit \; order(L_4) = 3$

$\exists \; p, q \in N$

$\forall H \in L, \; |H| > p$

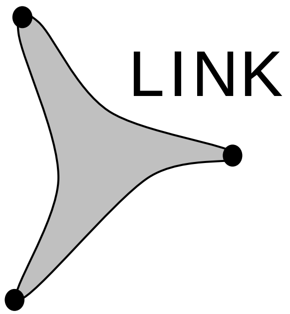
$\exists First, Link, Last : First \otimes Link \otimes Last = H, \; |Link \otimes Last| \leq q$

$und \; type(Link) \leq 3$

$\forall \; k \in N : First \otimes Link^{k} \otimes Last \in L$

$STR(HR_{2k+1}) \subset STR(HR_{2k+2})$

$type(Link) \leq 3$



da L_4 eine Stringgrammatik ist, muss LINK Kanten der Form

enthalten.

Damit fehlt aber eine Verbindungsstelle für eine Kante.

Zusammenfassung

- Hypergraphen
- Kontextfreie Graphgrammatiken
- Pumpinglemma
- Stringgrammatiken

Literatur

- Jost Engelfriet: Context-Free Graph Grammars, Handbook of Formal Languages, 1997
- F. Drewes, H.-J. Kreowski, A. Habel: Hyperedge Replacement Graph Grammars, Handbook of Graph Grammars and Computing by Graph, 1997
- A.Habel, H.-J. Kreowski: Some structural aspects of hypergraph languages generated by hyperedge replacement, LNCS 247, 1987