Tableaux and Higher Order Logics

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Outline

- What is a higher-order logic?
- Complete cut-free tableaux for classical extensional higher-order logic with equality (Gert Smolka)
- Translating from classical extensional tableau refutations to intuitionistic intensional ND derivations (Christine Rizkallah)
- Adding if-then-else, description and choice (Julian Backes)
- An implementation in GWT (Matthias Höschele)

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$$\neg : oo$$
$$\rightarrow : ooo$$
$$=_\sigma : \sigma\sigma o$$
$$\forall_\sigma : (\sigma o)o$$

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$$\begin{aligned} \neg &: oo \\ \rightarrow &: ooo \\ =_\sigma &: \sigma\sigma o \\ \forall_\sigma &: (\sigma o)o \end{aligned}$$

- $[s]$ is normal form of s (OK to think β-normal).

Appetizer: Mensa Example

Let $a, b, c : o$.

Let $f_1, f_2, f_3, g_1, g_2, g_3 : o\iota$.

Is the following set satisfiable?

$$f_1 a =_\iota g_1 b$$
$$f_1 b \neq_\iota g_1 a$$

$$f_2 b =_\iota g_2 c$$
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No. There are only two Booleans, so either $a = b$, $b = c$ or $a = c$. Each case contradicts two of the formulas.

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ND and Higher Order Logics

Restrict to logical constants $\longrightarrow$ and $\forall_\sigma$. Easy ND calculus:

$$\rightarrow I \; \frac{\begin{matrix} s \\ \vdots \\ t \end{matrix}}{s \longrightarrow t} \qquad \rightarrow E \; \frac{s \longrightarrow t \qquad s}{t}$$

$$\forall I_y \; \frac{[sy]}{\forall_\sigma s} \; y : \sigma \text{ fresh} \qquad \forall E \; \frac{\forall_\sigma s}{[st]}$$

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- Corresponds to proof terms (Curry-Howard-deBruijn)

Intuitionistic and Intensional Higher Order

- Peirce's Law: $((p \to q) \to p) \to p$
- Boolean Extensionality:

$$(p \to q) \to (q \to p) \to \forall u.up \to uq$$

- Functional Extensionality:

$$(\forall x.\forall w.w(fx) \to w(gx)) \to \forall u.uf \to ug$$

- ξ Extensionality:

$$(\forall x.\forall w.w(fx) \to w(gx)) \to \forall u.u(\lambda x.fx) \to u\lambda x.gx$$

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- Every object of function type is an abstraction.

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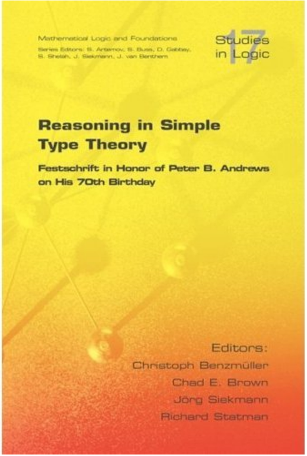
Peter B. Andrews



1971. Peter B. Andrews.
Resolution in Type Theory.
Journal of Symbolic Logic.
Reprinted in the recent Festschrift.

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- All logical constants definable from $=$ (Henkin, Andrews)
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- Consider the following example:
- Let $p : (\iota o)o$, $f : \iota o$ and $x : \iota$.

- Can

$$pf \qquad \text{and} \qquad \neg p(\lambda x.\neg\neg fx)$$

  both be true?

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- Higher-order variables, λ-abstractions, embedded formulas
- No. Unsatisfiable since f and $\lambda x.\neg\neg fx$ are the same (classically, extensionally).

Example (Mating)

$$p\, f \qquad \neg p(\lambda x.\neg\neg f x)$$

Unsatisfiable. Need extra rules to refute this branch.

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Add disequation $f \neq (\lambda x.\neg\neg f x)$ to the branch.

Example (Functional Extensionality)

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Add to branch:

$$fx \neq \neg\neg fx$$

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Two branches, both easily closed:

fx	$\neg fx$
$\neg\neg\neg fx$	$\neg\neg fx$

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$$\mathcal{T}_{\mathrm{MAT}}\ \frac{xs_1 \ldots s_n\,,\ \neg xt_1 \ldots t_n}{s_1 \neq t_1 \mid \cdots \mid s_n \neq t_n}$$

Equality on Individuals

How do we refute the following?

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Decomposition Rule:

$$\mathcal{T}_{\text{DEC}} \quad \frac{x s_1 \ldots s_n \neq_\iota x t_1 \ldots t_n}{s_1 \neq t_1 \mid \cdots \mid s_n \neq t_n}$$

Now $x \neq_\iota x$ is refuted using $n = 0$ case.

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$$\mathcal{T}_{\text{CON}} \quad \frac{s =_\iota t \, , \, u \neq_\iota v}{s \neq u \, , \, t \neq u \mid s \neq v \, , \, t \neq v}$$

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Transitivity example:

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$$\begin{array}{c|c} \multicolumn{2}{c}{x =_\iota y} \\ \multicolumn{2}{c}{y =_\iota z} \\ \multicolumn{2}{c}{x \neq_\iota z} \\ \hline & x \neq z \\ x \neq x & y \neq z \\ y \neq x & \end{array}$$

Equality on Individuals

Confrontation Rule:

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y \neq x & \begin{array}{c|c} y \neq y & y \neq z \\ z \neq y & z \neq z \end{array}
\end{array}$$

Complete Cut-Free Extensional Tableau Tableau

$$\mathcal{T}_{\neg\neg} \quad \frac{\neg\neg s}{s} \qquad \mathcal{T}_{\mathrm{BQ}} \quad \frac{s =_o t}{s, t \mid \neg s, \neg t} \qquad \mathcal{T}_{\mathrm{BE}} \quad \frac{s \neq_o t}{s, \neg t \mid \neg s, t}$$

$$\mathcal{T}_{\mathrm{FQ}} \quad \frac{s =_{\sigma\tau} t}{[su] = [tu]} \quad u : \sigma \qquad \mathcal{T}_{\mathrm{FE}} \quad \frac{s \neq_{\sigma\tau} t}{[sx] \neq [tx]} \quad x : \sigma \text{ fresh}$$

$$\mathcal{T}_{\mathrm{MAT}} \quad \frac{xs_1 \dots s_n \, , \, \neg xt_1 \dots t_n}{s_1 \neq t_1 \mid \cdots \mid s_n \neq t_n} \qquad \mathcal{T}_{\mathrm{DEC}} \quad \frac{xs_1 \dots s_n \neq_\iota xt_1 \dots t_n}{s_1 \neq t_1 \mid \cdots \mid s_n \neq t_n}$$

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Completeness: Building a Model

Discriminants + Possible Values + Logical Relations
$\Rightarrow$ Henkin Model

Discriminant: Maximum set of compatible discriminating terms.

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Let E be a set of normal formulas.

A term s_ι is discriminating if $s \neq t$ or $t \neq s$ is in E (for some t).

Δ is a discriminant if it is a set of discriminating terms such that there are no $s, t \in \Delta$ such that $s \neq t \in E$ and for any discriminating term $u \notin \Delta$ there is some $s \in \Delta$ such that $s \neq u \in E$ or $u \neq s \in E$.

Completeness: Possible Values Model

- Given: Irrefutable branch A.
- Extend to E satisfying certain properties. ($A \subseteq E$)
- Define $\mathcal{I}\sigma$ and $\triangleright_\sigma \subseteq \Lambda_\sigma \times \mathcal{I}\sigma$ by induction on types.

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- For $f : \mathcal{I}\sigma \to \mathcal{I}\tau$ let $s \triangleright_{\sigma\tau} f$ hold if

$$t \triangleright a \Rightarrow st \triangleright fa \text{ (logical relations)}$$

Let $\mathcal{I}(\sigma\tau) = \{f : \mathcal{I}\sigma \to \mathcal{I}\tau | \exists s \in \Lambda_\sigma . s \triangleright f\}$.

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- Choose $\mathcal{I}x$ such that $x \triangleright \mathcal{I}x$. This gives a model of E.

Two Interesting Fragments

- EFO: Essentially First Order
  (Brown, Smolka – TPHOLs 2009)

- Basic
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  - λ-free EFO
  - Decidable. (Tableau Calculus decides unsatisfiability.)
  - Finite model property

Translating to ND

- Can we translate tableau refutations into pure ND proofs?

Christine Rizkallah

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- ND: Intuitional, Intensional
- Transformation: Φ, if A is closed and tableau refutable, then $\Phi(A)$ is ND refutable.
- Mostly using Girard definitions in terms of $\rightarrow$ and $\forall$.
- $\Phi(=_\iota)$ as Leibniz
- $\Phi(=_o)$ as equivalence
- $\Phi(=_{\sigma\tau})fg$ as $\forall xy : \sigma.\Phi(=_\sigma)xy \rightarrow \neg\neg\Phi(=_\tau)(fx)(gy)$

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If-Then-Else, Description, Choice

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Julian Backes

If-Then-Else, Description, Choice

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  - If-Then-Else: $\text{if}_\sigma : o\sigma\sigma\sigma$ where $\text{if}\top xy = x$ and $\text{if}\bot xy = y$

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  - Choice: $\epsilon_\sigma : (\sigma o)\sigma$ where $p(\epsilon p)$ unless p is contantly false.
- Yes! We have nice cut-free rules for each.

Julian Backes

If-Then-Else

$$\mathcal{T}_{\mathsf{if}_o} \quad \frac{\mathsf{if}\; s\; t\; u\; v_1 \cdots v_n}{s, t\; v_1 \cdots v_n \mid \neg s, u\; v_1 \cdots v_n}$$

If-Then-Else

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If-Then-Else

$$\mathcal{T}_{\text{if}_o} \quad \frac{\text{if } s\, t\, u\, v_1 \cdots v_n}{s, t\, v_1 \cdots v_n \mid \neg s, u\, v_1 \cdots v_n}$$

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- Henkin Completeness
- Standard Completeness for EFO + if_σ at all types.

Choice

$$\mathcal{T}_\epsilon \quad \frac{}{\neg[st] \mid [s(\epsilon_\sigma\, s)]}\; t : \sigma$$

Choice

$$\mathcal{T}_\epsilon \quad \frac{}{\neg[st] \mid [s(\epsilon_\sigma\, s)]}\ t : \sigma,\ \epsilon_\sigma\, s \text{ accessible}$$

- $\epsilon_\sigma s$ is accessible if one of the following forms is on the branch:

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- Standard Completeness of EFO $+\ \epsilon_\iota$

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- Henkin Completeness (using ideas from Mints 1996)
- Standard Completeness of EFO + ϵ_ι
- (Description is similar)
- Conjecture: Standard Completeness of EFO + ι_σ at all types is possible.

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- Includes the tableau decision procedure for the Basic Fragment

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Thanks ...

Hopefully a demo of Jitpro in GWT follows...