

Undecidability of the Post Correspondence Problem

Second Bachelor Seminar Talk

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Post Correspondence Problem

$$A_{TM} = \{(M, w) \in TM \times \Sigma^* \mid M \text{ accepts } w\}$$

$$A_{TM} \leq_m PCP$$

Σ : discrete type

domino := $\Sigma^* \times \Sigma^*$

pcp := finite set of dominos

A list S is a solution of a PCP instance P if

$S \neq \emptyset \wedge S \subseteq P \wedge \text{match } S$

1	10111	10
111	10	0

10111	1	1	10
10	111	111	0

1	0	1	1	1	1	1	0
1	0	1	1	1	1	1	0

Post Correspondence Problem

$$A_{TM} = \{(M, w) \in TM \times \Sigma^* \mid M \text{ accepts } w\}$$

$$PCP = \{P : pcp \mid \exists S. S \neq \emptyset \wedge S \subseteq P \wedge \text{match } S\}$$

$$A_{TM} \leq_m MPCP \leq_m PCP$$

Modified Post Correspondence Problem

$$A_{TM} = \{(M, w) \in TM \times \Sigma^* \mid M \text{ accepts } w\}$$

$$PCP = \{P : pcp \mid \exists S. S \neq \emptyset \wedge S \subseteq P \wedge \text{match } S\}$$

$$A_{TM} \leq_m MPCP \leq_m PCP$$

$$\text{mpcp} := \text{domino} \times pcp$$

10111	1	10
10	111	0

10111	1	1	10
10	111	111	0

A list S is a solution of an MPCP instance (f, P) if

$$S \subseteq P \cup \{F\} \wedge \text{match}(f :: S)$$

1 0 1 1 1	1	1	1	0
1 0	1 1 1	1 1 1	0	

Modified Post Correspondence Problem Reduction

$$A_{TM} = \{(M, w) \in TM \times \Sigma^* \mid M \text{ accepts } w\}$$

$$PCP = \{P : pcp \mid \exists S. S \neq \emptyset \wedge S \subseteq P \wedge \text{match } S\}$$

$$MPCP = \{(f, P) : mpcp \mid \exists S. S \subseteq P \cup \{f\} \wedge \text{match}(f :: S)\}$$

$$A_{TM} \leq_m MPCP \leq_m PCP$$

$$f(P) = \left\{ \left[\frac{\star x_1}{y_1 \star} \right], \left[\frac{\star x_2}{y_2 \star} \right], \dots, \left[\frac{\star x_k}{y_k \star} \right], \left[\frac{\star x_1}{\star y_1 \star} \right], \left[\frac{\#\$}{\$} \right] \right\}$$

Modified Post Correspondence Problem Reduction

$$A_{TM} = \{(M, w) \in TM \times \Sigma^* \mid M \text{ accepts } w\}$$

$$PCP = \{P : pcp \mid \exists S. S \neq \emptyset \wedge S \subseteq P \wedge \text{match } S\}$$

$$MPCP = \{(f, P) : mpcp \mid \exists S. S \subseteq P \cup \{f\} \wedge \text{match}(f :: S)\}$$

$$A_{TM} \leq_m MPCP \leq_m PCP$$

Define a reduction g and prove its correctness

$$(M, w) \in A_{TM} \Leftrightarrow g(M, w) \in MPCP$$

Formalization of Turing Machines

Turing machine over finite alphabet Σ

$TM := (Q, \delta, q_0, F)$ with

Q : finite type of states

$\delta : Q \times \Sigma_{\perp} \rightarrow$
 $Q \times \Sigma_{\perp} \times \{L, N, R\}$

$q_0 : Q$ initial state

$F \subseteq Q$ set of halting states

Inductive tape : **Type** :=

| niltape : tape

| leftof : $\Sigma \rightarrow \text{list } \Sigma \rightarrow \text{tape}$

| rightof : $\Sigma \rightarrow \text{list } \Sigma \rightarrow \text{tape}$

| midtape : $\text{list } \Sigma \rightarrow \Sigma \rightarrow \text{list } \Sigma \rightarrow \text{tape}$.

niltape	leftof	midtape	rightof
$[]$	$[\text{abcd}]$	$[\text{abcd}]$	$[\text{abcd}]$
\uparrow	\uparrow	\uparrow	\uparrow
q	q	q	q

A **configuration** consists of the current state and the tape.

TM Acceptability

Inductive predicate \rightarrow_M^i defining reachability

$$\frac{}{c \rightarrow_M^0 c} \qquad \frac{c_1 \rightarrow_M^i c_2 \quad \text{state } c_2 \notin F}{c_1 \rightarrow_M^{(Si)} c_2} \text{ (step } c_2)$$

TM M accepts configuration c_0 if $\exists i c_f. c_0 \rightarrow_M^i c_f \wedge \text{state } c_f \in F$

String representation $\langle \cdot \rangle$ of configurations

tape	niltape	leftof	midtape	rightof
c	$[]$	$[abcd]$	$[abcd]$	$[abcd]$
	\uparrow q	\uparrow q	\uparrow q	\uparrow q
$\langle c \rangle$	$[\mathbf{q} -]$	$[\mathbf{q} - abcd]$	$[ab \mathbf{q} cd]$	$[abcd \mathbf{q}]$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$C_T \quad \left[\frac{\quad}{*q_0aba} \right]$$

$$\begin{array}{ccccccc}
 [a\ b\ a] & \rightarrow & [x\ b\ a] & \rightarrow & [x\ b\ a] & \rightarrow & [x\ b\ x] \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 q_0 & & q_1 & & q_1 & & q_f
 \end{array}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right]$$

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right]$$

$\star q_0 a b a$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$[a\ ba] \rightarrow [x\ ba] \rightarrow [x\ ba] \rightarrow [x\ bx] \rightarrow [x\ bx]$$

\uparrow \uparrow \uparrow \uparrow \uparrow
 q_0 q_1 q_1 q_0 q_f

$$\left[\begin{array}{c} \overline{\quad} \\ \star q_0 a b a \end{array} \right] \left[\begin{array}{c} \star \\ - \\ \star \end{array} \right]$$

$$C_I \quad \left[\begin{array}{c} \overline{\quad} \\ \star q_0 a b a \end{array} \right]$$

$$C_C \quad \left[\begin{array}{c} \star \\ - \\ \star \end{array} \right] \text{ and } \left[\begin{array}{c} s \\ s \end{array} \right] \forall s \in \Sigma$$

$\star q_0 a b a$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & & &
 \end{array}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right] \begin{bmatrix} \star \\ - \\ \star \end{bmatrix} \left[\frac{q_0 a}{x q_1} \right]$$

$$C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right]$$

$$C_C \quad \begin{bmatrix} \star \\ - \\ \star \end{bmatrix} \text{ and } \begin{bmatrix} s \\ - \\ s \end{bmatrix} \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots$$

 $\star q_0 a b a$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right] \begin{bmatrix} \star \\ - \\ \star \end{bmatrix} \left[\frac{q_0 a}{x q_1} \right] \begin{bmatrix} b \\ \bar{b} \end{bmatrix}$$

$$C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right]$$

$$C_C \quad \begin{bmatrix} \star \\ - \\ \star \end{bmatrix} \text{ and } \begin{bmatrix} s \\ s \end{bmatrix} \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots$$

$\star q_0 a b a$

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$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right] \begin{bmatrix} \star \\ - \\ \star \end{bmatrix} \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix} \begin{bmatrix} b \\ \bar{b} \end{bmatrix} \begin{bmatrix} a \\ \bar{a} \end{bmatrix}$$

$$C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right]$$

$$C_C \quad \begin{bmatrix} \star \\ - \\ \star \end{bmatrix} \text{ and } \begin{bmatrix} s \\ s \end{bmatrix} \forall s \in \Sigma$$

$$C_T \quad \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix}, \begin{bmatrix} q_1 b \\ b q_1 \end{bmatrix}, \begin{bmatrix} q_1 a \\ x q_0 \end{bmatrix}, \begin{bmatrix} x q_0 \star \\ q_f x \star \end{bmatrix}, \dots$$

$$\star q_0 a b a$$

$$\star q_0 a b a \star x q_1 b a$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right] \left[\begin{array}{c} \star \\ - \\ \star \end{array} \right] \left[\begin{array}{c} q_0 a \\ x q_1 \end{array} \right] \left[\begin{array}{c} b \\ b \end{array} \right] \left[\begin{array}{c} a \\ a \end{array} \right] \left[\begin{array}{c} \star \\ - \\ \star \end{array} \right] \left[\begin{array}{c} x \\ x \end{array} \right] \left[\begin{array}{c} q_1 b \\ b q_1 \end{array} \right] \left[\begin{array}{c} a \\ a \end{array} \right]$$

$$\star q_0 a b a \star x q_1 b a$$

$$\star q_0 a b a \star x q_1 b a \star x b q_1 a$$

$$C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right]$$

$$C_C \quad \left[\begin{array}{c} \star \\ - \\ \star \end{array} \right] \text{ and } \left[\begin{array}{c} s \\ s \end{array} \right] \forall s \in \Sigma$$

$$C_T \quad \left[\begin{array}{c} q_0 a \\ x q_1 \end{array} \right], \left[\begin{array}{c} q_1 b \\ b q_1 \end{array} \right], \left[\begin{array}{c} q_1 a \\ x q_0 \end{array} \right], \left[\begin{array}{c} x q_0 \star \\ q_f x \star \end{array} \right], \dots$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc} [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\ \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & \\ q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & & & \end{array}$$

$$\begin{aligned} C_I & \left[\frac{\quad}{\star q_0 a b a} \right] \\ C_C & \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\ C_T & \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \end{aligned}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right]$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{aligned}
 C_I & \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C & \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T & \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots
 \end{aligned}$$

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{aligned}
 C_I & \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C & \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T & \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots
 \end{aligned}$$

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{aligned}
 & \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 & \rightarrow \left[\frac{\star}{\star} \right]
 \end{aligned}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{aligned}
 C_I & \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C & \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T & \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots
 \end{aligned}$$

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{aligned}
 & \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 & \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right]
 \end{aligned}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{aligned}
 C_I & \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C & \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T & \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots
 \end{aligned}$$

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{aligned}
 & \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 & \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right]
 \end{aligned}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x}$$

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Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a\ ba] & \rightarrow & [x\ ba] & \rightarrow & [xb\ a] & \rightarrow & [xbx\] & \rightarrow & [xb\ x] \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 q_0 & & q_1 & & q_1 & & q_0 & & q_f
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 aba} \right] \\
 C_C \quad \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 aba} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_f x}{q_f} \right]
 \end{array}$$

$$\frac{\star q_0 aba \star x q_1 ba \star x b q_1 a \star x b x q_0 \star x b q_f x}{\star q_0 aba \star x q_1 ba \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a\ ba] & \rightarrow & [x\ ba] & \rightarrow & [xb\ a] & \rightarrow & [xbx\] & \rightarrow & [xb\ x] \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 q_0 & & q_1 & & q_1 & & q_0 & & q_f
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 aba} \right] \\
 C_C \quad \left[\frac{\star}{\quad} \right] \text{ and } \left[\frac{s}{\star} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 aba} \right] \left[\frac{\star}{\quad} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{\bar{b}} \right] \left[\frac{a}{\bar{a}} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\bar{x}} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{\bar{a}} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\bar{x}} \right] \left[\frac{b}{\bar{b}} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\bar{x}} \right] \left[\frac{b}{\bar{b}} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\quad} \right] \left[\frac{x}{\bar{x}} \right] \left[\frac{b}{\bar{b}} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\quad} \right]
 \end{array}$$

$$\frac{\star q_0 aba \star x q_1 ba \star x b q_1 a \star x b x q_0 \star x b q_f x}{\star q_0 aba \star x q_1 ba \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a\ ba] & \rightarrow & [x\ ba] & \rightarrow & [xb\ a] & \rightarrow & [xbx\] & \rightarrow & [xb\ x] \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 q_0 & & q_1 & & q_1 & & q_0 & & q_f
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 aba} \right] \\
 C_C \quad \left[\frac{\star}{\quad} \right] \text{ and } \left[\frac{s}{\star} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 aba} \right] \left[\frac{\star}{\quad} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{\quad} \right] \left[\frac{a}{\quad} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{\quad} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b}{\quad} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b}{\quad} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b}{\quad} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right]
 \end{array}$$

$$\frac{\star q_0 aba \star x q_1 ba \star x b q_1 a \star x b x q_0 \star x b q_f x}{\star q_0 aba \star x q_1 ba \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C \quad \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b q_f}{q_f} \right]
 \end{array}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C \quad \left[\frac{\star}{\quad} \right] \text{ and } \left[\frac{s}{\star} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\quad} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{\quad} \right] \left[\frac{a}{\quad} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{\quad} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b}{\quad} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b}{\quad} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b}{\quad} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\quad} \right] \left[\frac{x}{\quad} \right] \left[\frac{b q_f}{q_f} \right] \left[\frac{\star}{\quad} \right]
 \end{array}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C \quad \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b q_f}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x q_f}{q_f} \right]
 \end{array}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f \star q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & &
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 a b a} \right] \\
 C_C \quad \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 a b a} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b q_f}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x q_f}{q_f} \right] \left[\frac{\star}{\star} \right]
 \end{array}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f \star q_f}$$

Constructing an MPCP Match

Example: Turing machine T accepts all inputs with an even number of a -symbols, replacing them with x .

$$\begin{array}{ccccccccc}
 [a & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & a] & \rightarrow & [x & b & x] & \rightarrow & [x & b & x] \\
 \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & & \uparrow & & & \\
 q_0 & & & & q_1 & & & & q_1 & & & & q_0 & & & & q_f & & &
 \end{array}$$

$$\begin{array}{l}
 C_I \quad \left[\frac{\quad}{\star q_0 a b a \star} \right] \\
 C_C \quad \left[\frac{\star}{\star} \right] \text{ and } \left[\frac{s}{s} \right] \forall s \in \Sigma \\
 C_T \quad \left[\frac{q_0 a}{x q_1} \right], \left[\frac{q_1 b}{b q_1} \right], \left[\frac{q_1 a}{x q_0} \right], \left[\frac{x q_0 \star}{q_f x \star} \right], \dots \\
 C_D \quad \left[\frac{s q_f}{q_f} \right], \left[\frac{q_f s}{q_f} \right] \forall s \in \Sigma \\
 C_F \quad \left[\frac{q_f \star}{\quad} \right]
 \end{array}$$

$$\begin{array}{l}
 \left[\frac{\quad}{\star q_0 a b a \star} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{x q_1} \right] \left[\frac{b}{b} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{q_1 b}{b q_1} \right] \left[\frac{a}{a} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_1 a}{x q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{x q_0 \star}{q_f x \star} \right] \rightarrow \\
 \rightarrow \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b}{b} \right] \left[\frac{q_f x}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x}{x} \right] \left[\frac{b q_f}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{x q_f}{q_f} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_f \star}{\quad} \right]
 \end{array}$$

$$\frac{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f \star q_f \star}{\star q_0 a b a \star x q_1 b a \star x b q_1 a \star x b x q_0 \star x b q_f x \star x b q_f \star x q_f \star q_f \star}$$

Transforming a TM into an MPCP Instance

TM M accepts configuration $c_0 \Leftrightarrow g(M, c_0) \in MPCP$

Definition of MPCP dominos

C_I fixed initial card $\left[\frac{\star}{\star \langle c_0 \rangle} \right]$

C_C copy cards $\left[\frac{\star}{\star} \right]$ and $\left[\frac{s}{s} \right] \forall s \in \Sigma$

C_T transition cards e.g. $\left[\frac{q_1 a}{x q_2} \right]$ if $q_1 \in Q \setminus F \wedge \delta(q_1, a) = (q_2, x, R)$

C_D deletion cards $\left[\frac{s q}{q} \right], \left[\frac{q s}{q} \right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$

C_F final card $\left[\frac{q \star}{\star} \right] \forall q \in F$

$g(M, c_0) := (C_I, C_C \cup C_T \cup C_D \cup C_F)$

Transition Dominos

$\forall q_1 \notin F, q_2 \in Q, a b z \in \Sigma$

$$\delta(q_1, a) = (q_2, b, L) \quad \left[\begin{array}{c} \star q_1 a \\ \star q_2 b \end{array} \right] \text{ and } \left[\begin{array}{c} z q_1 a \\ q_2 z b \end{array} \right]$$

$$\delta(q_1, -) = (q_2, b, R) \quad \left[\begin{array}{c} \star q_1 - \\ \star b q_2 \end{array} \right] \text{ and } \left[\begin{array}{c} q_1 \star \\ b q_2 \star \end{array} \right]$$

$$\delta(q_1, -) = (q_2, -, N) \quad \dots$$

Correctness Proof

$$\forall M c_0. \exists c_f i. c_0 \xrightarrow{i}_M c_f \wedge (\text{state } c_f) \in F \Leftrightarrow \\ \exists P \subseteq TM_{\text{cards}}. \text{match} \left(\left[\frac{}{\star\langle c_0 \rangle} \right] :: P \right)$$

Correctness Proof

$$\forall M c_0. \exists c_f i. c_0 \xrightarrow{i}_M c_f \wedge (state\ c_f) \in F \Leftrightarrow \\ \exists P \subseteq TM_{cards}. match \left(\left[\frac{\quad}{\star \langle c_0 \rangle} \right] :: P \right)$$

Proof direction \Rightarrow with induction on i :

$$i = 0 \wedge (state\ c_0) \in F$$

We remove all symbols to the left and to the right of the state using **deletion cards**.

Example: $\langle c_0 \rangle = q_0 a b$

$$\left[\frac{\quad}{\star q_0 a b} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 a}{q_0} \right] \left[\frac{b}{b} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 b}{q_0} \right] \left[\frac{\star}{\star} \right] \left[\frac{q_0 \star}{\quad} \right]$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } \text{match} \left(\left[\frac{\quad}{\star\langle \text{step } c_0 \rangle} \right] :: A \right)$$

WANT

$$\exists P, \text{match} \left(\left[\frac{\quad}{\star\langle c_0 \rangle} \right] :: P \right)$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } \text{match} \left(\left[\frac{\quad}{\star\langle \text{step } c_0 \rangle} \right] :: A \right)$$

WANT

$$\exists P, \text{match} \left(\left[\frac{\quad}{\star\langle c_0 \rangle} \right] :: P \right)$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } \text{match} \left(\left[\frac{\quad}{\star\langle \text{step } c_0 \rangle} \right] :: A \right)$$

WANT

$$\exists P, \text{match} \left(\left[\frac{\quad}{\star\langle c_0 \rangle} \right] :: P \right)$$

$$P := (\text{step_cards } c_0) ++ A$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } \text{match} \left(\left[\frac{\quad}{\star \langle \text{step } c_0 \rangle} \right] :: A \right)$$

WANT

$$\exists P, \text{match} \left(\left[\frac{\quad}{\star \langle c_0 \rangle} \right] :: P \right)$$

$$P := (\text{step_cards } c_0) ++ A$$

step_cards : *conf* \rightarrow *list domino*

Example: $\langle c_0 \rangle = q_0 a b c$ and $\delta(q_0, a) = (q_1, x, R)$

$$\text{step_cards } c_0 = \begin{bmatrix} \star \\ \star \end{bmatrix} \left[\begin{array}{c} q_0 a \\ x q_1 \end{array} \right] \left[\begin{array}{c} b \\ b \end{array} \right] \left[\begin{array}{c} c \\ c \end{array} \right]$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } match \left(\left[\frac{\quad}{\star \langle step\ c_0 \rangle} \right] :: A \right)$$

$$\begin{aligned} \#_1(step_cards\ c_0) &= \star \langle c_0 \rangle \\ \#_2(step_cards\ c_0) &= \star \langle step\ c_0 \rangle \end{aligned}$$

WANT

$$\exists P, match \left(\left[\frac{\quad}{\star \langle c_0 \rangle} \right] :: P \right)$$

$$P := (step_cards\ c_0) ++ A$$

$step_cards : conf \rightarrow list\ domino$

Example: $\langle c_0 \rangle = q_0\ a\ b\ c$ and $\delta(q_0, a) = (q_1, x, R)$

$$step_cards\ c_0 = \begin{bmatrix} \star \\ \star \end{bmatrix} \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix} \begin{bmatrix} c \\ c \end{bmatrix}$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } \text{match} \left(\left[\frac{\quad}{\star \langle \text{step } c_0 \rangle} \right] :: A \right)$$

$$\begin{aligned} \#_1(\text{step_cards } c_0) &= \star \langle c_0 \rangle \\ \#_2(\text{step_cards } c_0) &= \star \langle \text{step } c_0 \rangle \end{aligned}$$

WANT

$$\exists P, \text{match} \left(\left[\frac{\quad}{\star \langle c_0 \rangle} \right] :: P \right)$$

$$P := (\text{step_cards } c_0) ++ A$$

$$\frac{\quad}{\star \langle c_0 \rangle} \frac{\#_1(\text{step_cards } c_0)}{\#_2(\text{step_cards } c_0)} \frac{\#_1 A}{\#_2 A}$$

$\text{step_cards} : \text{conf} \rightarrow \text{list domino}$

Example: $\langle c_0 \rangle = q_0 a b c$ and $\delta(q_0, a) = (q_1, x, R)$

$$\text{step_cards } c_0 = \begin{bmatrix} \star \\ \star \end{bmatrix} \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix} \begin{bmatrix} c \\ c \end{bmatrix}$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } match \left(\left[\frac{\quad}{\star \langle step\ c_0 \rangle} \right] :: A \right)$$

$$\begin{aligned} \#_1(step_cards\ c_0) &= \star \langle c_0 \rangle \\ \#_2(step_cards\ c_0) &= \star \langle step\ c_0 \rangle \end{aligned}$$

WANT

$$\exists P, match \left(\left[\frac{\quad}{\star \langle c_0 \rangle} \right] :: P \right)$$

$$P := (step_cards\ c_0) ++ A$$

$$\frac{\quad}{\star \langle c_0 \rangle} \frac{\#_1(step_cards\ c_0)}{\#_2(step_cards\ c_0)} \frac{\#_1 A}{\#_2 A}$$

$$\frac{\star \langle c_0 \rangle}{\star \langle step\ c_0 \rangle} \frac{\#_1 A}{\#_2 A}$$

$step_cards : conf \rightarrow list\ domino$

Example: $\langle c_0 \rangle = q_0\ a\ b\ c$ and $\delta(q_0, a) = (q_1, x, R)$

$$step_cards\ c_0 = \begin{bmatrix} \star \\ \star \end{bmatrix} \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix} \begin{bmatrix} c \\ c \end{bmatrix}$$

Correctness \Rightarrow cont.

HAVE

$$\text{IH: } \text{match} \left(\left[\frac{\quad}{\star \langle \text{step } c_0 \rangle} \right] :: A \right)$$

$$\text{IH': } \#_1 A = \star \langle \text{step } c_0 \rangle :: \#_2 A$$

$$\#_1(\text{step_cards } c_0) = \star \langle c_0 \rangle$$

$$\#_2(\text{step_cards } c_0) = \star \langle \text{step } c_0 \rangle$$

WANT

$$\exists P, \text{match} \left(\left[\frac{\quad}{\star \langle c_0 \rangle} \right] :: P \right)$$

$$P := (\text{step_cards } c_0) ++ A$$

$$\frac{\quad}{\star \langle c_0 \rangle} \frac{\#_1(\text{step_cards } c_0)}{\#_2(\text{step_cards } c_0)} \frac{\#_1 A}{\#_2 A}$$

$$\frac{\quad}{\star \langle c_0 \rangle} \frac{\star \langle \text{step } c_0 \rangle}{\star \langle \text{step } c_0 \rangle} \frac{\#_1 A}{\#_2 A}$$

$$\frac{\quad}{\star \langle c_0 \rangle} \frac{\star \langle \text{step } c_0 \rangle}{\star \langle \text{step } c_0 \rangle} \frac{\star \langle \text{step } c_0 \rangle \#_2 A}{\#_2 A}$$

$\text{step_cards} : \text{conf} \rightarrow \text{list domino}$

Example: $\langle c_0 \rangle = q_0 a b c$ and $\delta(q_0, a) = (q_1, x, R)$

$$\text{step_cards } c_0 = \begin{bmatrix} \star \\ \star \end{bmatrix} \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix} \begin{bmatrix} c \\ c \end{bmatrix}$$

Correctness \Leftarrow

$$\forall P c_0. P \subseteq TM_{cards} \wedge match\left(\left[\frac{\cdot}{\star(c_0)}\right] :: P\right) \rightarrow \exists c_f i. c_0 \rightarrow_M^i c_f \wedge (state\ c_f) \in F$$

Proof:

1. Size induction on $|P|$
2. Case analysis on $state\ c_0 \in F$

Correctness \Leftarrow

$$\forall P c_0. P \subseteq TM_{cards} \wedge match\left(\left[\frac{}{\star(c_0)}\right] :: P\right) \rightarrow \exists c_f i. c_0 \rightarrow_M^i c_f \wedge (state\ c_f) \in F$$

Proof:

1. Size induction on $|P|$
2. Case analysis on $state\ c_0 \in F$

$$state\ c_0 \in F : c_f := c_0$$

$state\ c_0 \notin F$: prove that P can be split into $(step_cards\ c_0) ++ P'$

use IH with P' and $step\ c_0$ to get i and c_f with

$$step\ c_0 \rightarrow_M^i c_f \wedge (state\ c_f) \in F$$

$$c_0 \xrightarrow{1}_M step\ c_0 \rightarrow_M^i c_f$$

$$c_0 \xrightarrow{(Si)}_M c_f$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star\langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] :: P$$

$$\frac{\quad}{\star q_0 aba}$$

$$C_I \quad \left[\frac{\quad}{\star\langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star\langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{\$}{\$ \star q_0 aba}\right] :: P'$$

$$\frac{\$}{\star q_0 aba \$ \star q_0 aba}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = \begin{array}{c} [a\ ba] \\ \uparrow \\ q_0 \end{array}$ and $step\ c_0 = \begin{array}{c} [x\ b\ a] \\ \uparrow \\ q_1 \end{array}$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{q_0 a}{x q_1}\right] :: P'$$

$$\frac{q_0 a}{\star q_0 abaxq_1}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{sq}{q}\right], \left[\frac{qs}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q\star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{q_f a}{q_f}\right] :: P'$$

$$\frac{q_f a}{\star q_0 aba q_f}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star\langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{q_f \star}{_}\right] :: P'$$

$$\frac{q_f \star}{\star q_0 aba}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star\langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{a}{a}\right] :: P'$$

$$\frac{a}{\star q_0 abaa}$$

$$C_I \quad \left[\frac{\$}{\$\star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star\langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] :: P'$$

$$\frac{\star}{\star q_0 aba \star}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{\$}{\$ \star q_0 aba}\right] :: P'$$

$$\frac{\star \$}{\star q_0 aba \star \$ \star q_0 aba}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{a}{a}\right] :: P'$$

$$\frac{\star a}{\star q_0 aba \star a}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{\star}{\star}\right] :: P'$$

$$\frac{\star\star}{\star q_0 aba\ \star\star}$$

$$C_I \quad \left[\frac{\$}{\$\star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q\star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{q_f a}{q_f}\right] :: P'$$

$$\frac{\star q_f a}{\star q_0 aba \star q_f}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star\langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{q_f \star}{_}\right] :: P'$$

$$\frac{\star q_f \star}{\star q_0 aba \star}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = [a\ ba]$ and $step\ c_0 = [x\ b\ a]$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{q_0 a}{x q_1}\right] :: P'$$

$$\frac{\star q_0 a}{\star q_0 aba\ \star x q_1}$$

$$C_I \quad \left[\frac{\$}{\$\star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = \begin{bmatrix} a & ba \end{bmatrix}$ and $step\ c_0 = \begin{bmatrix} x & b & a \end{bmatrix}$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \begin{bmatrix} \star \\ \star \end{bmatrix} \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix} :: P'$$

$$\frac{\star q_0 ab}{\star q_0 aba \star x q_1 b}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \begin{bmatrix} \star \\ \star \end{bmatrix} \text{ and } \begin{bmatrix} s \\ s \end{bmatrix} \forall s \in \Sigma$$

$$C_T \quad \begin{bmatrix} q_0 a \\ x q_1 \end{bmatrix}$$

$$C_D \quad \begin{bmatrix} s q \\ q \end{bmatrix}, \begin{bmatrix} q s \\ q \end{bmatrix} \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{_}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = \begin{bmatrix} a & ba \end{bmatrix}$ and $step\ c_0 = \begin{bmatrix} x & b & a \end{bmatrix}$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{_}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{q_0 a}{x q_1}\right] \left[\frac{b}{b}\right] \left[\frac{a}{a}\right] :: P'$$

$$\frac{\star q_0 aba}{\star q_0 aba \star x q_1 ba}$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{_}\right] \forall q \in F$$

Structure of the Solution List

Assumptions: $P \subseteq TM_{cards}$, state $c_0 \notin F$, match $\left(\left[\frac{\quad}{\star \langle c_0 \rangle}\right] ++ P\right)$

Goal: $P = (step_cards\ c_0) :: P'$

Example: $c_0 = \begin{matrix} [a\ ba] \\ \uparrow \\ q_0 \end{matrix}$ and $step\ c_0 = \begin{matrix} [x\ b\ a] \\ \uparrow \\ q_1 \end{matrix}$ with $\delta(q_0, a) = (q_1, x, R)$

$$\left[\frac{\quad}{\star q_0 aba}\right] \left[\frac{\star}{\star}\right] \left[\frac{q_0 a}{x q_1}\right] \left[\frac{b}{b}\right] \left[\frac{a}{a}\right] :: P'$$

$$\frac{\star q_0 aba}{\star q_0 aba \star x q_1 ba}$$

$$P = \left[\frac{\star}{\star}\right] \left[\frac{q_0 a}{x q_1}\right] \left[\frac{b}{b}\right] \left[\frac{a}{a}\right] :: P'$$

$$P = (step_cards\ c_0) ++ P'$$

$$C_I \quad \left[\frac{\$}{\$ \star \langle c_{start} \rangle}\right]$$

$$C_C \quad \left[\frac{\star}{\star}\right] \text{ and } \left[\frac{s}{s}\right] \forall s \in \Sigma$$

$$C_T \quad \left[\frac{q_0 a}{x q_1}\right]$$

$$C_D \quad \left[\frac{s q}{q}\right], \left[\frac{q s}{q}\right] \forall s \in \Sigma \cup \{-\}, \forall q \in F$$

$$C_F \quad \left[\frac{q \star}{\quad}\right] \forall q \in F$$

Conclusion

What we have

- Formal definitions of single-tape Turing machines, MPCP and PCP
- Verified reductions from A_{TM} to $MPCP$ and $MPCP$ to PCP

Future work

- Undecidability of $\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset$ for CFG's G_1, G_2
- Reduction of A_{TM} to the word problem for string rewriting systems (SRS)
- Reduction of SRS to PCP

$$A_{TM} \leq SRS \leq PCP^1$$

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Course Technology, second edition, 2006.

	Spec	Proof	Σ
MPCP \leq PCP	130	180	310
$A_{TM} \leq$ MPCP	370	540	910
TM	170	110	280
Σ	670	830	1500

Formalization of TM Configurations

Variable Σ : finType.

Variable Q : finType.

Inductive tape : Type :=

| niltape : tape

| leftof : $\Sigma \rightarrow \text{list } \Sigma \rightarrow \text{tape}$

| rightof : $\Sigma \rightarrow \text{list } \Sigma \rightarrow \text{tape}$

| midtape : $\text{list } \Sigma \rightarrow \Sigma \rightarrow \text{list } \Sigma \rightarrow \text{tape}$.

Definition tape' : Type := $\Sigma^* \times Q \times \Sigma^*$

Additional blank symbol needed to express $[\underset{q}{\uparrow} abcd]$ vs. $(q, \text{leftof } a [b; c; d])$