Quantification in Dynamic Lambda Calculus

(joint work with Michael Kohlhase)

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Motivation

⇒ A text creates a kind of dynamic potential.

U
$man(U)$
$walk(U)$

He whistles.

Make use of this property!?!

⇒ (Compositional) semantic construction: using the structural layer of type theory (λ-calculus)

⇒ Combine these two goals!

⇒ Advantages:

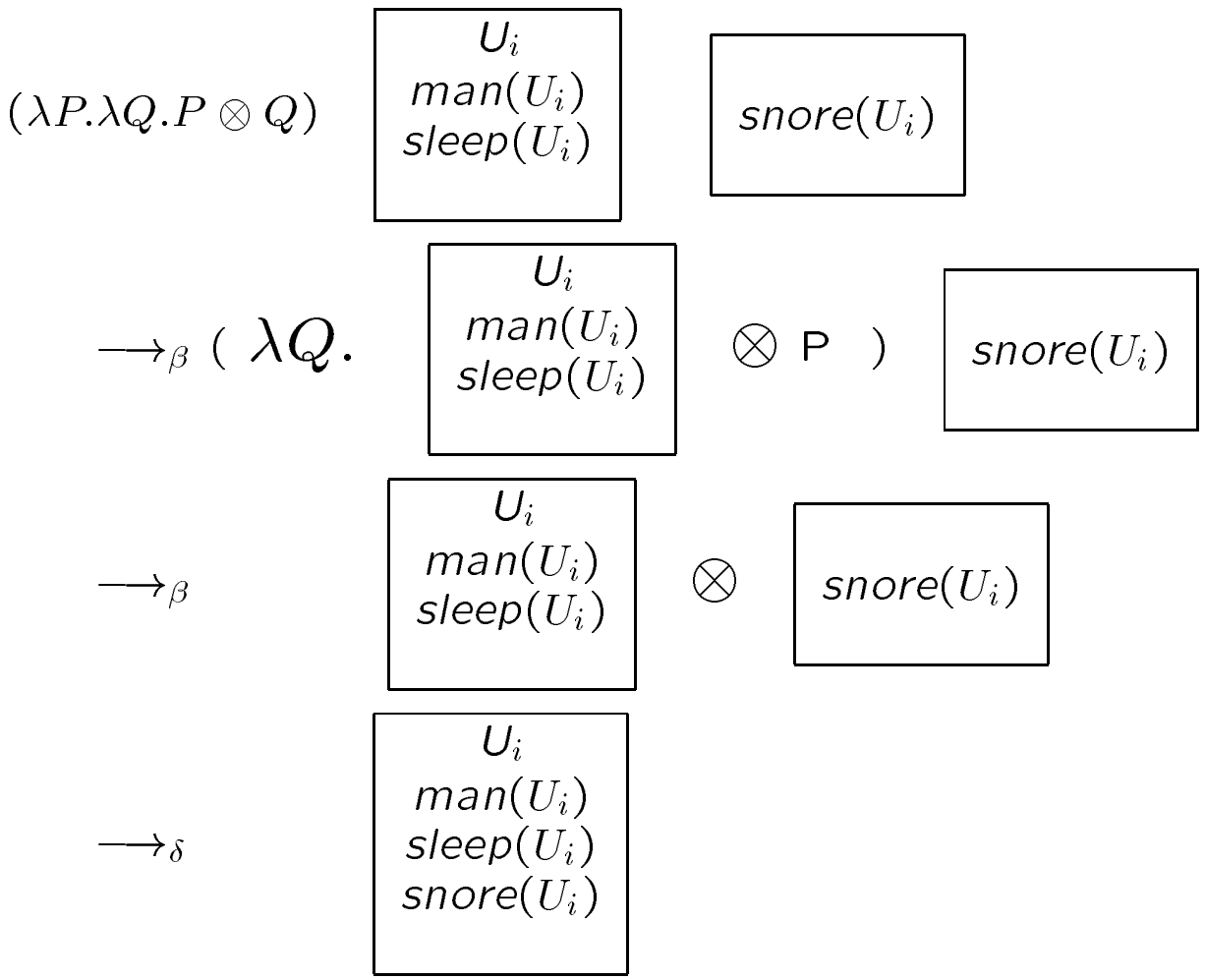
→ extend known techniques to include dynamic properties

→ rigid modelling of the interaction of dynamic and functional parts

→ utilisation of binding information for control of structural processes

The Interaction of Two Concepts

$\Rightarrow$ dynamic binding extends over β-reduction



$\Rightarrow$ variable capture !!

= establishment of anaphoric binding

$\mathcal{DLC}$'s Idea: Extending the Type System

$\Rightarrow$ every expression has a mode which contains the binding information

- $\rightarrow$ U^+: all variables called U will be captured
- $\rightarrow$ A^-: variable A is liable for capturing (effectively: free!)

$\Rightarrow$ add mode information in type: $\Gamma\#\alpha$ where $\Gamma = U^+, A^-$ is the mode, α the type in the usual sense

$\Rightarrow$ positive variables overwrite negative ones $U^+, U^- = U^+$

$\Rightarrow$ have dynamic and functional variables

$\Rightarrow$ type function for variables $\tau()$ and for the usual Σ for constants

$\Rightarrow$ definition of well-formed formulae:

$$\frac{\Sigma(c) = \alpha}{c\colon \alpha}\ \mathit{wff}\!:\!\mathit{const}$$

$$\frac{A \notin \mathbf{DP}(\tau(A))}{A\colon A^{-}\#\overline{\overline{\tau(A)}}}\ \mathit{wff}\!:\!\mathit{var}$$

$$\frac{\mathbf{A}\colon \alpha \quad \tau(U) = \beta}{(\delta U_{\beta}\mathbf{.A})\colon U^{+}\#\alpha}\ \mathit{wff}\!:\!\mathit{dyn}$$

$$\frac{\mathbf{A}\colon \alpha \quad X : X^{-}\#\overline{\overline{\beta}} \quad X \notin \mathbf{Dom}(\Gamma^{-})}{(\lambda X_{\beta}\mathbf{.A})\colon (\Gamma^{-}\#\overline{\overline{\beta}}) \to [\Gamma^{-}/X]\alpha}\ \mathit{wff}\!:\!\mathit{abs}$$

$$\frac{\mathbf{A}\colon \Gamma^{-}\#\alpha \to \beta \quad \mathbf{B}\colon \alpha}{\mathbf{AB}\colon \Gamma^{-}\#\beta}\ \mathit{wff}\!:\!\mathit{app}$$

Example

$$\dfrac{\dfrac{\dfrac{\tau(P) = \gamma}{P\colon P^- \# \gamma} \qquad U\colon U^- \# \theta}{\delta U.P\colon U^+, P^- \# \gamma \qquad f(U)\colon U^- \# o}}{\dfrac{\lambda P_\gamma.\delta U.P\colon (\Gamma^- \# \gamma) \to (U^+, \Gamma^- \# \gamma)}{(\lambda P_\gamma.\delta U.P) f(U)\colon U^+, U^- \# \alpha}}$$

$\Gamma^- \# \gamma = U^- \# \alpha$

Thus, $(\lambda P_\gamma.\delta U.P) f(U)\colon U^+ \# \alpha$

and $(\lambda P_\gamma.\delta U.P) f(U) \longrightarrow_\beta \delta U.f(U)$.

What about Other Formalisms?

⇒ e.g. λ-DRT:

→ set of base types = $\{e, o\}$

→ specify logical constants, e.g. dynamic negation, implication $\underline{\neg}, \underline{\Rightarrow}$

→ example:

$$\dfrac{\dfrac{\dfrac{\tau(U) = e}{U\colon U^- \# e} \quad \dfrac{\tau(Q) = \Delta^+ \# o}{Q\colon Q^-, \Delta^+ \# o}}{\delta U.\mathsf{man}(U)\colon U^+ \# o} \quad \dfrac{\tau(Q) = \Delta^+ \# o}{Q\colon Q^-, \Delta^+ \# o}}{\delta U.\mathsf{man}(U) \underline{\Rightarrow} Q\colon (Q^-/U) \# o}$$

$$\dfrac{\dfrac{\tau(U) = e}{U\colon U^- \# e}}{\mathsf{sleep}(U)\colon U^- \# o}$$

⇒ likewise for other formalisms (Zeevat, Muskens, Groenendijk and Stockhof)

Further Features of $\mathcal{DLC}$

$\Rightarrow$ Structural:

- $\rightarrow$ (type-preserving) notion of substitution
- $\rightarrow$ alphabetical variants, dynamic variants
- $\rightarrow$ $\beta\eta$-reduction, confluent and terminating
- $\rightarrow$ head normal form

$\Rightarrow$ Semantics:

- $\rightarrow$ extension of Henkin models by a domain for (potentially) dynamic structures.
- $\rightarrow$ reduction system correct wrt. to this semantics
- $\rightarrow$ dynamic operators and expressions can be expressed as extensions of their static counterparts.