A linear functional first-order intermediate language for verified compilers

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Introduction

Binding vs. assignment

Binding	Assignment

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let x=e in s

- *x* is bound in term *s*
- *functional*

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Binding vs. assignment

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let x=e in s	x := e; s

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- *functional*

Assignment

x := e; s

- x is a register
- *imperative*

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Assignment

x := e; s

- x is a register
- *imperative*

SSA-based register assignment
Translation from binding to assignment

Intermediate language IL

A linear first-order functional language with external calls

$s, t ::=$	term
$\mid \text{let } x = e \text{ in } s$	variable binding
$\mid \text{let } x = \alpha \text{ in } s$	external call
$\mid \text{if } e \text{ then } s \text{ else } t$	conditional
$\mid e$	value
$\mid \text{fun } f\ \overline{x} = s \text{ in } t$	function definition
$\mid f\ \overline{e}$	application

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   Functions f, g not first-class

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   Restricted sequentialization
   $\text{let } x = e \text{ in } s$ (not: $s; t$)

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4. **External calls** realistic
   $\text{let } x = \alpha \text{ in } s$

Example

A functional and an imperative interpretation

$$F(n, m) := n * (n + 1) * \ldots * m$$

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Functional IL

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let i = 1 in
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Imperative IL/I

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No closure created: **goto**

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Parameter passing in IL/I is parallel assignment:

$$j, p := k, m$$

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A functional and an imperative interpretation

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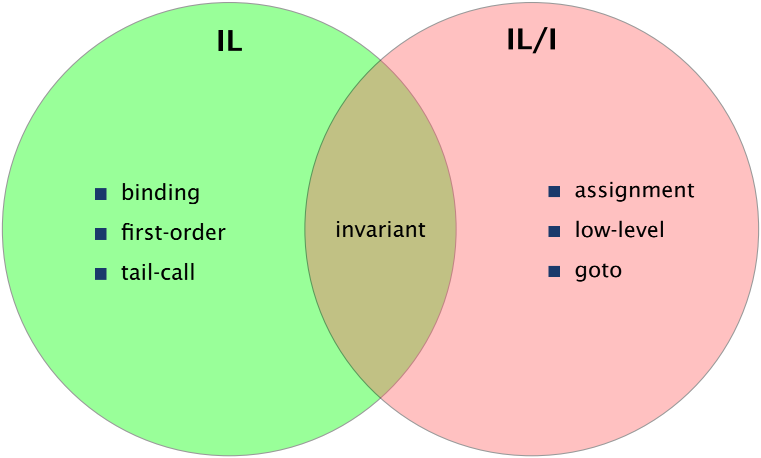
- When renamed-apart, binding and assignment interchangeable!

Overview

Translating from the functional to the imperative interpretation



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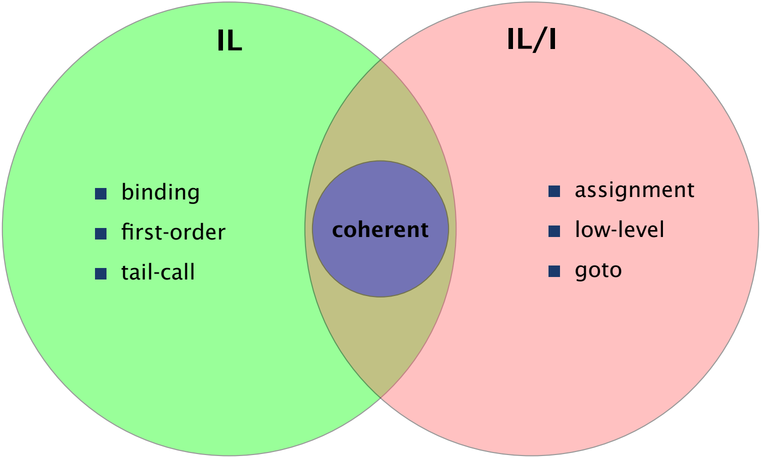

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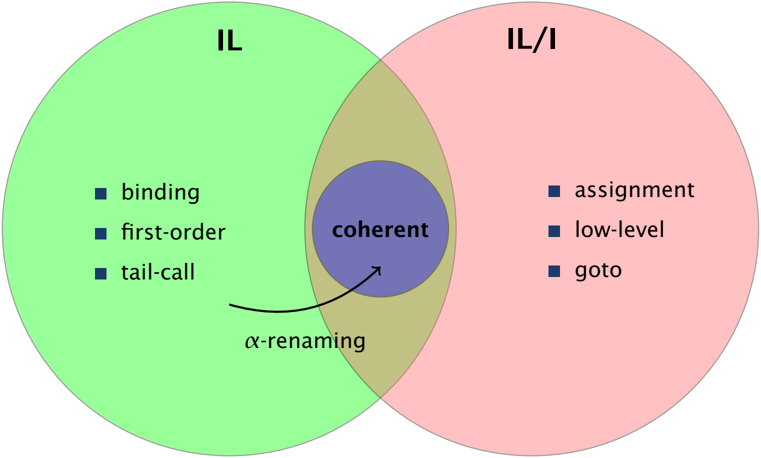

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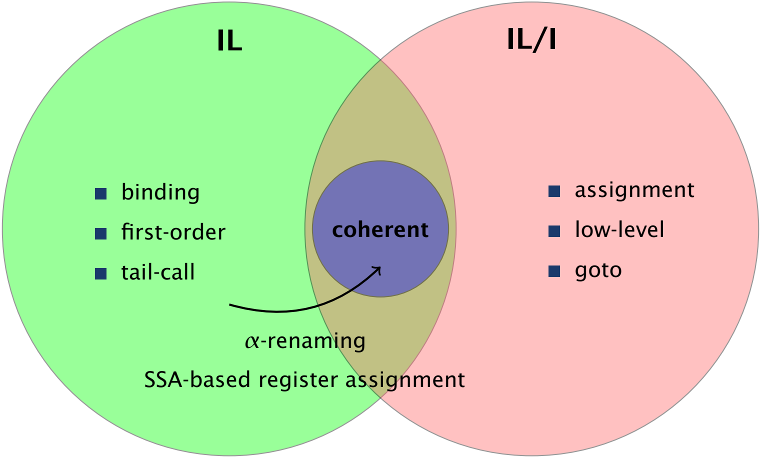

Related work

Static single assignment (SSA)

Related work

IL

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let i = 1 in
fun f (j,p) =

 let c = p <= m in
 if c then
  let k = p * j in
  let x = p + 1 in
  f (k,x)
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```

SSA

```
 i := 1;
f:
 j := φ(i,k), p := φ(n,x)
 c := n <= m;
 if c then
  k := p * j;
  x := p + 1;
  goto f
 else
  return i
```

- SSA •—• CPS due to Appel (1998) and Kelsey (1995).
- Chakravarty et al. (2003) reformulates SSA optimization on a functional language in ANF (Sabry et al. 1993).
- IL is a sub-language (up to system calls)

SSA in verified compilers

Related work

1. CompCertSSA: Barthe et al. (2012)
   - Integrates SSA-based optimization passes in CompCert (Leroy (2009))
2. VeLLVM: Zhao et al. (2012)
   - Verifies some SSA-based passes of LLVM

- SSA for optimizations
  - performance of data-flow analyses
- ϕ-functions
  - no functional language
  - underlying semantics uses imperative variables

SSA-based register allocation

Related work

- SSA-based register allocation (Hack et al. (2006))
  - allows phase separation of spilling and register assignment
  - IL version similar to Appel (1992)
  - not considered in verified setting so far:
    out of SSA + non-SSA register allocation
- Blazy et al. (2010) verify non-SSA register allocation algorithm (which must include spilling)
- We only considering register assignment, because SSA-based algorithm allows spilling to be separate phase

Functional and imperative semantics

Related work

- Beringer et al. (2003) use a language with a functional and imperative interpretation for proof-carrying code.
- Grail normal form (GNF) sufficient for functional + imperative semantics to coincide
- Main difference: GNF requires functions to be closure converted, i.e. all variables a function body depends on must be parameters

Contributions

- Coherence
  - relates binding and assignment directly
  - another perspective on SSA and functional programming
- SSA-based register assignment on IL
  - formal correctness proof (using coherence)
  - key property from SSA holds on IL:
    spilling can be considered separately (not possible without SSA)
- Coq development available online:
  `www.ps.uni-saarland.de/~sdschn/publications/lvc15`

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Semantics and program equivalence

Semantics of IL and IL/I

Reduction, events, configurations

- Small-step relation $\xrightarrow{\phi}$
- Decorated with events ϕ

$$\phi ::= \tau \qquad \text{silent event}$$
$$\mid v = \alpha \qquad \text{external event}$$

- Configurations

$$\text{IL: } (F, V, s) \qquad \text{IL/I: } (L, V, s)$$

  - F function env. (with closures)
  - L block env. (no closures)
  - V variable env.
  - s program

Program equivalence

Non-determinism and equivalence

$$\textsc{Extern}\quad \frac{v \in \mathbb{V}}{\begin{array}{ll} & F \mid V \qquad\quad \mid \mathrm{let}\ x = \alpha\ \mathrm{in}\ s \\ \xrightarrow{v=\alpha} & F \mid V[x \mapsto v] \mid s \end{array}}$$

- $\xrightarrow{\phi}$ forms a LTS
- Internally deterministic reduction systems (IDRS)
  - $\sigma \xrightarrow{\phi} \sigma_1 \wedge \sigma \xrightarrow{\tau} \sigma_2 \Rightarrow \phi = \tau$ — τ-deterministic
  - $\sigma \xrightarrow{\phi} \sigma_1 \wedge \sigma \xrightarrow{\phi} \sigma_2 \Rightarrow \sigma_1 = \sigma_2$ — action-deterministic
- Configurations are equivalent ($\simeq$), if they allow the same partial traces

$$\pi ::= \epsilon \mid v \mid \bot \mid v = \alpha, \pi$$

- Sound and complete characterization via (stutter) bisimulation

Liveness

Liveness

Judgment

$$(L, V, s) \stackrel{?}{\simeq} (L, W, s)$$

Liveness

Judgment

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$$V =_X W \Rightarrow (L, V, s) \stackrel{?}{\simeq} (L, W, s)$$

Liveness

Judgment

$$V =_X W \Rightarrow (L, V, s) \overset{?}{\simeq} (L, W, s)$$

$$\boxed{\Lambda \vdash \textbf{live}\; s : X}$$

Λ	live variables of functions
s	program
X	set of live variables

- embedded liveness analysis results as annotations in syntax:

$$\text{fun}\; f\; \overline{x} : X_1 = s_1 \;\text{in}\; s_2$$

- syntactic structure allows for inductive specification
- useful for imperative IL/I
- judgment monotonic in X (larger sets are sound)

Liveness

Properties for IL/I

Theorem (Decidability)

$\Lambda \vdash \textbf{live}\, s : X$ *decidable.*

Theorem (Soundness)

If

1. $\Lambda \vdash \textbf{live}\, s : X$ — *liveness information sound*
2. $L \vDash \Lambda$ — *Λ sound for blocks L*
3. $V =_X W$ — *V, W agree on live set X*

then

$$(L, V, s) \simeq (L, W, s)$$

Coherence

Coherence

Intuition

$$F, f : (W, \overline{x}, s) \mid V \mid f\,\overline{e} \quad\longrightarrow\quad F, f : (W, \overline{x}, s) \mid W[\overline{x} \mapsto \overline{v}] \mid s$$

$$\overset{?}{\simeq}$$

$$F, f : (W, \overline{x}, s) \mid V[\overline{x} \mapsto \overline{v}] \mid s$$

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$$\overset{?}{\simeq}$$

$$F, f : (W, \overline{x}, s) \mid V[\overline{x} \mapsto \overline{v}] \mid s$$

1. If $\boxed{\Lambda \vdash \textbf{live}\; s : X}$ then it suffices if W and V agree on $X \setminus \overline{x}$
2. Call $X \setminus \overline{x}$ **globals of function** f
3. Liveness definition is arranged such that context Λ records globals
4. Define coherence to ensure environments agree on globals at every application

Coherence

Inductive definition

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f *available* as long as no global rebound

Coherence

Inductive definition

f available as long as no global rebound

not invariant

```
let x = 7 in
fun f () : {x} = x in
let x = 5 in
f ()
```

Coherence

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1 let x = 7 in
2 fun f () : {x} = x in
3 let x = 5 in
4 f ()
```

f *unavailable* after line 3

Coherence

Inductive definition

f available as long as no global rebound

not invariant	coherent

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let x = 7 in
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f *unavailable* after line 3

```
let x = 7 in
fun f () : {x} = x in
let y = 5 in
f ()
```

f *available* in line 4

Coherence

Inductive definition

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Coherence judgment $\Lambda \vdash \mathbf{coh}\ s$

- ensures s only applies available functions
- defined relative to liveness information

Coherence

Rules

$\Lambda - \{x\}$ removes definitions from Λ that require x as global

$$\textsc{Coh-Op}\ \frac{\Lambda - \{x\} \vdash \textbf{coh}\ s}{\Lambda \vdash \textbf{coh}\ \text{let}\ x = e\ \text{in}\ s}$$

$$\textsc{Coh-App}\ \frac{\Lambda f \neq \bot}{\Lambda \vdash \textbf{coh}\ f\ \overline{y}}$$

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$\lfloor \Lambda \rfloor_X$ removes definitions from Λ that require more globals than X

$$\textsc{Coh-Fun}\ \frac{\Lambda; f : X \vdash \textbf{coh}\ t \qquad \lfloor \Lambda; f : X \rfloor_X \vdash \textbf{coh}\ s}{\Lambda \vdash \textbf{coh}\ \text{fun}\, f\,\overline{x} : X = s\, \text{in}\, t}$$

Coherence

Results

We define $strip(V, \overline{x}, s) \;=\; (\overline{x}, s)$ and lift *strip* pointwise to contexts.

Theorem (Coherence implies invariance)

If

1. $\Lambda \vdash \textbf{coh}\ s$ — *s is coherent*
2. $\Lambda \vdash \textbf{coh}\ F$ — *definitions in F are coherent*
3. $\Lambda' \vdash \textbf{live}\ s : X$ *for* $\Lambda \preceq \Lambda'$ — *liveness information is sound*
4. $V =_X W$ — *V, W agree on X*
5. $F, V \vDash \Lambda$ — *closures in F agree with V on globals*

then

$$(F, V, s)_F \simeq (strip\, F, W, s)_I$$

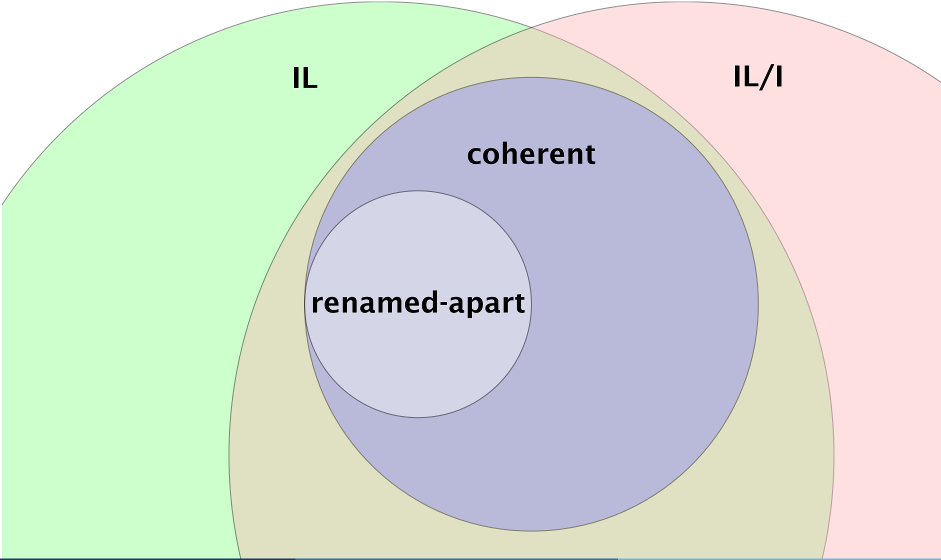
Register assignment

Register assignment

- State-of-the-art SSA-based register assignment algorithm
  - decouples spilling from assignment:
    number of registers bounded by largest live set
  - polynomial-time (coalescing is NP-hard)
  - critically depends on dominance ordering
- Register assignment for functional language IL
  - same properties: register bound, polynomial time
  - straight-forward recursion on syntax
- Correctness argument of assignment phase
  - does not involve dominance
  - via coherence and α-equivalence

Register assignment

Proof overview



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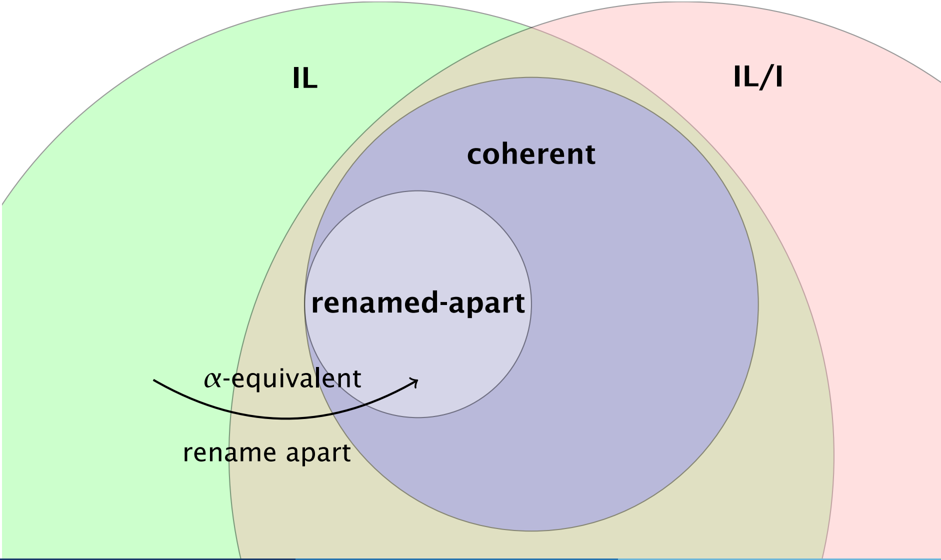

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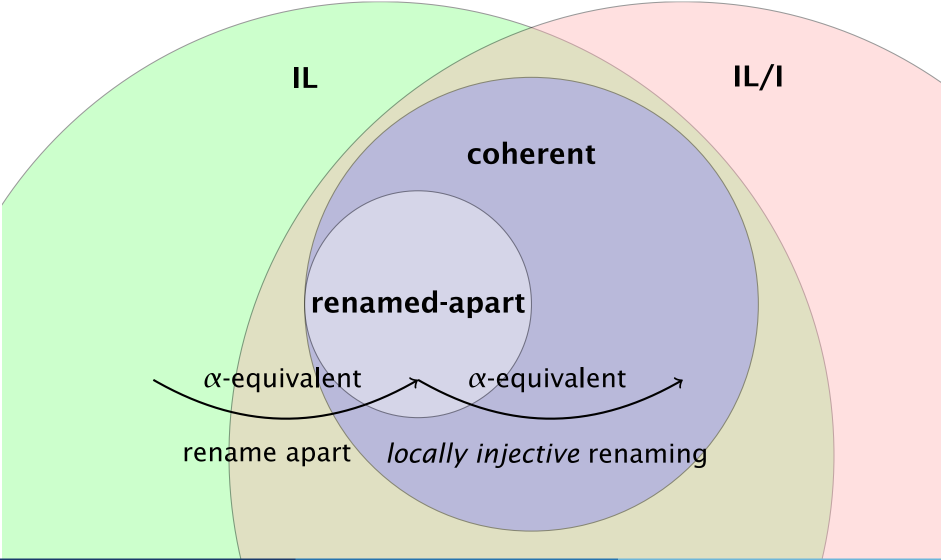

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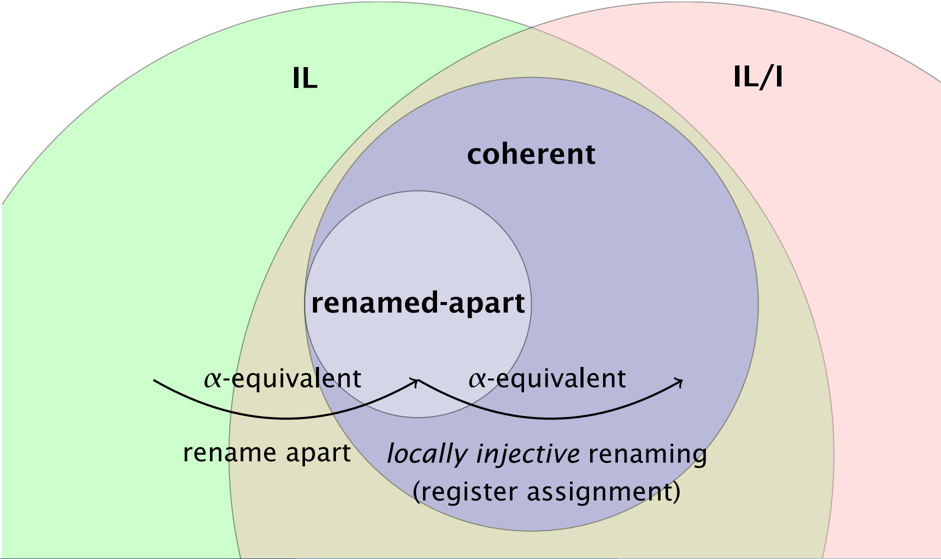

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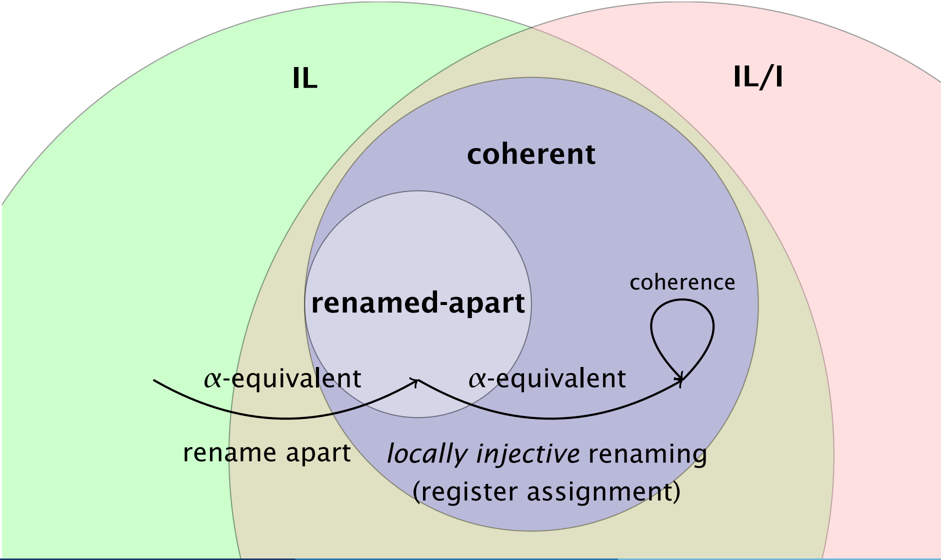

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   - Every assignment can be represented as $\rho : \mathcal{V} \to \mathcal{V}$

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1. Rename apart
   - Every assignment can be represented as $\rho : \mathcal{V} \to \mathcal{V}$
2. Rename with *locally injective* ρ
   - A ρs is α-equivalent and coherent
   - B register assignment algorithm yields locally injective renaming

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2. Rename with *locally injective* ρ

   A ρs is α-equivalent and coherent

   B register assignment algorithm yields locally injective renaming
3. Reinterpret binding as assignment: IL/I

Side Conditions: Liveness and Renamed-Apart

- Call $\boxed{\Lambda \text{ and } s \textit{ suitable}}$ if
  1. s renamed-apart
  2. $\Lambda \vdash \textbf{live}\, s : X$ — liveness sound
     - ⋆ write $[s]$ for X

Side Conditions: Liveness and Renamed-Apart

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     - ⋆ write $[s]$ for X
  3. $s \subseteq \text{fv}(s)$ — no variable occurs in annotation before it is bound

```
fun f () : {y} = 7 in
let y = 5 in
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- Call $\boxed{\Lambda \text{ and } s \textit{ suitable}}$ if
  1. s renamed-apart
  2. $\Lambda \vdash \textbf{live}\, s : X$ — liveness sound
     - write $\lceil s \rceil$ for X
  3. $\lceil s \rceil \subseteq \text{fv}(s)$ — no variable occurs in annotation before it is bound

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  4. $\Lambda \subseteq \text{fv}(s)$ — no global from Λ bound in s

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Local Injectivity

Local Injectivity $\boxed{\rho \vdash \textbf{\textit{inj}}\ s}$ requires $\rho : \mathcal{V} \to \mathcal{V}$ to be injective on every live set X that appears in the liveness derivation.

ρ^{-X}: inverse of ρ on X,

Local Injectivity

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Theorem (A)

If

1. Λ *and* s *suitable*
2. s *without unreachable code*
3. $\rho \vdash \textbf{\textit{inj}}\ s$ — ρ *locally injective*

then

1. $\rho\ (\lfloor \Lambda \rfloor_{[s]}) \vdash \textbf{coh}\ (\rho\ s)$ — ρs *coherent*
2. ρ *injective on* $\text{fv}(s) \Longrightarrow \rho, \rho^{-\text{fv}(s)} \vdash \rho\ s \sim_\alpha s$ — ρs *α-equivalent to* s

---

ρ^{-X}: inverse of ρ on X,

Register assignment algorithm

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Definition

- Assume $\mathrm{fresh} : \mathit{set}\,\mathcal{V} \to \mathcal{V}$ such that $\mathrm{fresh}\,X \notin X$ for all finite X.
  - separation of concerns: correctness and code quality

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$$
\begin{array}{lcl}
\text{rassign}\,\rho\,(\text{let}\,x = e\,\text{in}\,s) & = & \text{rassign}\,(\rho[x \mapsto y])\,s \\
\quad \text{where } y = \text{fresh}\,(\rho([s] \setminus \{x\})) & & \\
\text{rassign}\,\rho\,(\text{if}\,e\,\text{then}\,s\,\text{else}\,t) & = & \text{rassign}\,(\text{rassign}\,\rho\,s)\,t \\
\text{rassign}\,\rho\,e & = & \rho \\
\text{rassign}\,\rho\,(f\,\overline{e}) & = & \rho \\
\text{rassign}\,\rho\,(\text{fun}\,f\,\overline{x}\,:\,X' = s\,\text{in}\,t) & = & \text{rassign}\,(\text{rassign}\,(\rho[\overline{x} \mapsto \overline{y}])\,s)\,t \\
\quad \text{where } \overline{y} = \text{freshlist}\,(\rho([s] \setminus \overline{x}))\,|\overline{x}| & &
\end{array}
$$

rassign recurses on program structure,
while SSA algorithm must process statements in dominance order

Register assignment algorithm

Correctness and bound on registers

Theorem (B)

Let Λ *and* s *suitable and* ρ *injective on* $[s]$*. Then:* $\mathrm{rassign}\,\rho\, s \vdash$ ***inj*** s.

Assume variables totally ordered: $x_0 < x_1 < x_2 \ldots$

Theorem (Register Bound)

If

1. Λ *and* s *suitable*
2. $\forall$ *finite sets of variables* Y*: fresh* $Y \in \{x_0, \ldots, x_{|Y|}\}$
3. k *is size of largest set of live variables in* s
4. $\rho(\mathrm{fv}(s)) \subseteq \{x_0, \ldots, x_n\}$.
5. $\rho' = \mathrm{rassign}\,\rho\, s$

Then $\rho'\,(\mathcal{V}_O(s)) \subseteq \{x_0, \ldots, x_{max\{n,k\}}\}$

Coq Development

- This work is part of a very simple verified compiler
- Extraction yields binary that handles the running example
  - Efficient finite set library with type classes (Lescuyer 2012)
  - Cannot assume set extensionality
  - Decision procedures for equivalence on many types
- Development almost completely constructive
  - UIP required for Paco Library (Hur et al. (2013))
- Formal development contains proofs of
  - Backwards translation: IL/I to IL (SSA-construction)
  - Dead code elimination
  - Sparse conditional constant propagation
  - Translation validation for analysis results

Conclusion

- Coherence relates binding to assignment
- Correctness proof of register assignment on IL
  - same advantages as SSA (register bound)
  - correctness via coherence and α-equivalence
  - structural recursion instead of dominance ordering
- Coq development is available online[1]

[1] www.ps.uni-saarland.de/~sdschn/publications/lvc15

Conclusion

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Thanks! Questions?

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Thank you for your attention! Questions? I

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Semantics of IL and IL/I

Common rules

$\phi ::= \tau \mid v = \alpha$ events

$\xrightarrow{\phi}$ small step relation

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Semantics of IL and IL/I

Common rules

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$\xrightarrow{\phi}$ small step relation

(F, V, s)
- F function env.
- V variable env.
- s program

Op

$$\frac{[\![e]\!]\,V = v}{\begin{array}{ll} & F \mid V \mid \text{let } x = e \text{ in } s \\ \xrightarrow{\tau} & F \mid V[x \mapsto v] \mid s \end{array}}$$

Extern

$$\frac{v \in \mathbb{V}}{\begin{array}{ll} & F \mid V \mid \text{let } x = \alpha \text{ in } s \\ \xrightarrow{v=\alpha} & F \mid V[x \mapsto v] \mid s \end{array}}$$

Cond

$$\frac{[\![e]\!]\,V = v \qquad \beta(v) = i}{\begin{array}{ll} & F \mid V \mid \text{if } e \text{ then } s_0 \text{ else } s_1 \\ \xrightarrow{\tau} & F \mid V \mid s_i \end{array}}$$

Semantics IL and IL/I

Differences

IL

FUN

$$\frac{}{F \mid V \mid \mathrm{fun}\, f\, \overline{x} = s\, \mathrm{in}\, t \xrightarrow{\tau} F; f : (V, \overline{x}, s) \mid V \mid t}$$

APP

$$\frac{[\![\overline{e}]\!]\, V = \overline{v} \qquad Ff = (W, \overline{x}, s)}{F \mid V \mid f\, \overline{e} \xrightarrow{\tau} F^f \mid W[\overline{x} \mapsto \overline{v}] \mid s}$$

Semantics IL and IL/I

Differences

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$$\frac{[\![\overline{e}]\!]\, V = \overline{v} \qquad Ff = (\mathrm{W}, \overline{x}, s)}{F \mid V \mid f\, \overline{e} \;\xrightarrow{\tau}\; F^f \mid \mathrm{W}[\overline{x} \mapsto \overline{v}] \mid s}$$

IL/I

I-FUN

$$\frac{}{L \mid V \mid \mathrm{fun}\, f\, \overline{x} = s\, \mathrm{in}\, t \;\xrightarrow{\tau}_I\; L; f : (\overline{x}, s) \mid V \mid t}$$

I-APP

$$\frac{[\![\overline{e}]\!]\, V = \overline{v} \qquad Lf = (\overline{x}, s)}{L \mid \mathrm{V} \mid f\, \overline{e} \;\xrightarrow{\tau}_I\; L^f \mid \mathrm{V}[\overline{x} \mapsto \overline{v}] \mid s}$$

Program equivalence

Internally deterministic reduction systems

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Definition

A *reduction system* (RS) is a tuple $(\Sigma, \mathcal{E}, \longrightarrow, \tau, res)$, s.t.

1. $(\Sigma, \mathcal{E}, \longrightarrow)$ is a LTS
2. $\tau \in \mathcal{E}$
3. $res : \Sigma \to \mathbb{V}_\perp$
4. $res\, \sigma = v \Rightarrow \sigma$ $\longrightarrow$-terminal

An *internally deterministic* reduction system (IDRS) additionally satisfies

5. $\sigma \xrightarrow{\phi} \sigma_1 \wedge \sigma \xrightarrow{\phi} \sigma_2 \Rightarrow \sigma_1 = \sigma_2$ action-deterministic
6. $\sigma \xrightarrow{\phi} \sigma_1 \wedge \sigma \xrightarrow{\tau} \sigma_2 \Rightarrow \phi = \tau$ τ-deterministic

Program equivalence

Trace equivalence

$\Pi \ni \pi ::= \epsilon \mid v \mid \perp \mid \phi\pi$	$\phi \neq \tau$	partial trace
$\sigma \triangleright \pi$		σ poduces π

Program equivalence

Trace equivalence

$$\Pi \ni \pi ::= \epsilon \mid v \mid \bot \mid \phi\pi \qquad \phi \neq \tau \qquad \text{partial trace}$$

$$\sigma \rhd \pi \qquad \sigma \text{ poduces } \pi$$

Definition (Trace equivalence)

$\sigma \simeq \sigma' :\Longleftrightarrow \forall \pi, \sigma \rhd \pi \iff \sigma' \rhd \pi$

Program equivalence

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$\sigma \sim \sigma'$ bisimilarity

Program equivalence

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Definition (Trace equivalence)

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$\sigma \sim \sigma'$ bisimilarity

Theorem (Soundness and completeness)

Let $(S, \mathcal{E}, \longrightarrow, res, \tau)$ be an IDRS and $\sigma, \sigma' \in S$. Then:

$$\sigma \sim \sigma' \iff \sigma \simeq \sigma'$$

Local Injectivity

The problem with unreachable code

```
fun f () = x in
y
```

```
fun f () = y in
y
```

- $\{x \mapsto y, y \mapsto y\}$ locally injective
- Programs not α-equivalent