Using Oz for College Timetabling

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Abstract. In this paper, we concentrate on a typical scheduling problem: the computation of a timetable for a German college. Like many other scheduling problems, this problem contains a variety of complex constraints and necessitates special-purpose search strategies. Techniques from Operations Research and traditional constraint logic programming are not able to express these constraints and search strategies on a sufficiently high level of abstraction. We show that the higher-order concurrent constraint language Oz provides this high-level expressivity, and can serve as a useful programming tool for college timetabling.

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1 Introduction

Constraint Logic Programming over finite domains is a rapidly growing research area aiming at the solution of large combinatorial problems. For many real-world problems the constraint logic programming approach (extended to the concurrent constraint approach) is competitive or better than traditional Operations Research (OR) algorithms. OR techniques lack flexibility and the effort to achieve customized solutions is often unaffordable.

The power of constraint logic programming has been proven by languages such as `CHIP` [DVS$^+$88], Prolog III [Col90] or CLP(R) [JM87]. To solve real-world problems, several new constraints were added as primitives (like *atmost* or the *cumulative* constraint [AB93] for scheduling or placement problems importing experience from OR). This approach might be viable for well known problems but is not going to foster the exploration of new areas of applications. More clarity and flexibility for the programmer was achieved by `clp(FD)` [DC93] (inspired by [VSD91]). This approach is based on a single primitive constraint (called indexical) with which more complicated constraints may be defined. In terms of efficiency, `clp(FD)` is competitive with `CHIP` for certain benchmarks, and adds some flexibility. But still missing is one characteristic that we consider essential for solving certain constraint problems: *the flexibility to exploratively invent new constraints and search strategies.* While experimental languages such as `cc(FD)` [VSD95] and AKL(FD) [CCD94] provide flexibility in formulating constraints, their search strategies are still fixed.

Oz [Smo95b, SS94, SSW94, ST95] is a concurrent constraint language providing for functional, object-oriented and constraint programming. It is based on a simple yet powerful computation model [Smo95a], which can be seen as an extension of the concurrent constraint model [SR90].

In this paper, we describe how the unique features of Oz contribute to computing the timetable of a German college that we describe in Section 2. The problem contains a combination of complex constraints preventing the application of more standard timetabling techniques.

What can Oz offer to solve this problem?

Constraint Programming. The concurrent constraint language Oz allows to restrict the possible values of variables to finite sets of integers. In Section 3, we introduce some basics about concurrent constraint programming. Crucial for constraint programming is the ability to add constraints on variables concurrently and incrementally. In Oz, this is done by *propagators*, which can express several constraints of our timetabling problem as shown in Section 3.

Reified Constraints. A general scheme, called *reified constraints*, allows to express the remaining constraints of our timetable problem, as discussed in Section 4. Reified constraints allow to reflect the fact that a constraint holds into a 0/1-valued variable.

Constructive Disjunction. Disjunctive constraints (like resource or capacity constraints), can be used constructively in Oz, i.e. information common to different branches can be lifted out for active pruning. We describe constructive disjunction [VSD95] in Section 5 and show how it can be used for ovelap constraints in our timetabling problem.

Flexible Enumeration. In Oz, the programmer can invent customized search strategies for solving the timetabling problem and optimizing the solutions found. In Section 6, we develop a search strategy especially adapted to our problem, combining the first-fail heuristic for variable selection with a priority scheme for value selection. This strategy results in an efficiently computed first solution approximating an optimality criteria for the distribution of the college courses.

Optimization. This first solution can be further optimized using a branch-and-bound technique. Optimization can be achieved through an incremental process, allowing the user to inspect the current solution *any time* and to interrupt and resume the optimization at will, as described in Section 7.

Interoperability. The interoperability libraries of Oz allow convenient programming of graphical user interfaces, including the visualization of the computed timetables, as presented in Section 8.

In Section 9, we compare the described techniques with other approaches in constraint logic programming.

Due to space limitations we cannot further detail other aspects of Oz like conditionals and disjunctions, which have shown to be useful for other constraint problems. The interested reader is referred to the documentation of the Oz system (see concluding remark of the paper).

2 The Problem

Our goal was to find a weekly timetable for the Catholic College for Social Work in Saarbrücken, Germany, in the spring semester 95. The school offers a four year program for a degree in social work. Some courses are mandatory for and dedicated to students of a certain year while others are optional and open to all students. There are 91 courses, 34 instructors and 7 rooms of varying size. The assignment of instructors to courses is fixed. Each course needs to take place in a room of sufficient size. There are five school days, and the courses are being held between 8:15 am and 5 pm. They may start every quarter of an hour. There are short courses of 3 quarters and long courses of 6 quarters. There must be a break of at least one quarter after a short course and of at least two quarters after a long course.

In the following we state the additional constraints that a schedule must fulfill.

C1) Some courses are limited to certain time slots.
C2) Some instructors have times of unavailability.
C3) There are different lunch breaks for the different years in the program.
C4) Some courses must be held after others.
C5) There are sets of courses whose members must be held in parallel.
C6) To every course, a room of sufficient size must be assigned.
C7) An instructor can only teach one course at a time.
C8) Two instructors want to take turns in caring for an infant child and therefore cannot teach at the same time.
C9) The mandatory courses of each year must not overlap.
C10) Some optional courses must not overlap with courses of the first two years, and others not with courses of any year.
C11) Two mandatory courses of a year may overlap if they are split in groups.
C12) The instructors do not want to teach on more than 3 days a week.
C13) All members of some sets of courses must be held on different days.

The schedule should obey the following criteria as well as possible: The courses of the first two years should be grouped around Monday, Tuesday and Wednesday. If this is not possible they should be scheduled on Thursday or Friday morning. The third year should be placed on Wednesday and the fourth preferably on Tuesday and Thursday. The number of courses after the lunch break and on Thursday[1] and Friday should be minimized.

3 Constraints and Propagators in Oz

Our goal is to assign to every course a starting time and a room. Note that the assignment of instructors to courses is fixed in our college. Here, we concentrate on the starting time and show in Section 4 how the room assignment is handled.

The courses are held between 8:15 am and 5 pm on five school days and may start every quarter of an hour. Thus, there are $36 * 5 = 180$ possible starting times for each course. Since there are 91 courses, the overall search space contains 180^{91} elements. Instead of enumerating the whole search space and testing whether a valuation satisfies all the constraints (generate & test), the idea of constraint programming is to restrict the search space a priori through constraints. While the search space is being explored, more information on the starting times becomes known, i.e., the search space can be further pruned, while it is being explored. A programming language for constraint programming needs to provide flexible means to express pruning operators. In this and the following section, we concentrate on pruning operators, while the exploration of the search space is described in Section 6 and Section 7.

We represent a course, say "Psychology 101", by grouping together its start time, duration, instructor, room size, and its name in a record of the form

[1] In Germany, several holidays in the spring semester fall on a Thursday.

```
Psych101 = course(start     : _
                  dur       : 6
                  instructor: ´Smith´
                  roomsize  : big
                  name      : ´Psychology 101´)
```

The underscore _ indicates that no information on the `start` value is known. The starting time of a course is represented by an integer denoting the corresponding quarter of the school week. Because initially it is known that the course must start between quarter 1 and quarter 180 we can add the constraint

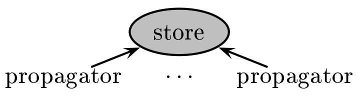
```
Psych101.start :: 1#180
```

expressing that this course can take values between 1 and 180, i.e., `Psych101.start` $\in \{1, \ldots, 180\}$ in a more mathematical notation. We say that the starting time is constrained to a *finite domain*.

Such constraints are stored in a *constraint store*. For the constraints residing in the constraint store Oz provides efficient algorithms to decide satisfiability. The largest set of integers satisfying all the constraints for a variable in the store, is called the *domain* of that variable. To distinguish the constraints in the store from more complex constraints, we often call them *basic constraints*.

The idea of constraint programming is to install constraints that further limit the possible `start` value for every course. For example, a constraint of type C1 may say that the course "Psychology 101" must be held on Monday morning or on Tuesday morning. This constraint is *imposed* on the constraint store by the expression `Psych101.start ::[1#18 37#54]`[2] limiting the start value in the constraint store to satisfy `Psych101.start` $\in \{1, \ldots, 18, 37, \ldots, 54\}$.

Besides C1, the constraints C2 and C3 can be expressed by such constraints.



For more complex arithmetic constraints it is known that deciding their satisfiability is not computationally tractable (there are several NP-complete problems on finite domains, e.g. graph coloring). Thus, such constraints are not contained in the constraint store but are modeled by *installing* so called *propagators* inspecting the constraint store, as depicted above.

A propagator inspects the store and when values are ruled out from the domain of a variable, it may add more information to the store, i.e., it may amplify the store by adding more basic constraints. Thus, we are replacing *global consistency*, which is assured for the constraints in the store, by *local consistency* where unsatisfiability may not be detected. It is important to allow propagators to act concurrently since it is not statically known when they will be able to perform

[2] The term `[1#18 37#54]` denotes a list consisting of the two pairs `1#18` and `37#54`.

their computation. This is one major motivation behind *concurrent* constraint programming.

As an example consider a constraint of type C4, which states that the course "Sociology 101" must be held after our course "Psychology 101". It can be expressed by installing the propagator

```
Psych101.start + Psych101.dur =<: Socio101.start
```

For example, if `Psych101.start` is constrained as above to Monday morning or Tuesday morning, and `Psych101.dur` to 6, then the propagator will add the basic constraint

```
Socio101.start :: 7#180
```

to the constraint store. Vice versa, if later on it becomes known that `Socio101` starts the latest at 10:30 am on Monday (`Socio101.start` $\in \{7, \ldots, 10\}$), then `Psych101.start` will be constrained to start the latest at 9:00 am (`Psych101.start` $\in \{1, \ldots, 4\}$). Note that the propagator remains active, waiting for more information on either `Psych101.start` or `Socio101.start` to come.

Observe that in the implementation we have to add the necessary break after `Psych101` (depending on `Psych101.dur`) but we omit the breaks in this presentation for simplicity.

The constraint C5 is modeled by using the propagator `=:` expressing equality. Note that `S=:T` is modeled by `S=<:T S>=:T`. Thus holes in the domains of `S` and `T` as in `Psych101.start` are not considered for `=:`. It implements only partial arc-consistency instead of full arc-consistency in the terminology of [Mac77]. Interval-consistency can be implemented efficiently since the propagator only needs to watch the currently smallest and biggest possible values for the involved variables.

4 Reified Constraints

We have seen in the previous section that propagators are crucial components in a constraint programming system, since they allow to prune the search space. A constraint programming language therefore must strive to easily express many kinds of propagators. In this section, we introduce *reified constraints* as a generic tool to express new propagators and show how the remaining constraints C6 - C13 of our college problem can be expressed with them.

The constraint C6 says that for every point in time the number of courses of a certain size must not exceed the number of rooms of that size. Assume that there are `NumberOfRooms` different rooms available of a given size. If we are able to compute for every quarter of an hour `Q` of the teaching week the number of courses `CoursesAtQ` of the given size being held in this quarter, then we only need to install the propagator

```
CoursesAtQ =<: NumberOfRooms
```

to express C6 for every quarter `Q` and every room size. To compute `CoursesAtQ` it would be convenient to be able to constrain a boolean variable `CAtQ` to 1 if a given course overlaps `Q` and to 0 if it does not. Then `CoursesAtQ` can be obtained simply by computing the sum of all `CAtQ` over all courses of the given size. To compute `CAtQ`, we use *reified constraints*, i.e. propagators that reflect the validity of a constraint into a variable. Reified constraints are also known in the literature as *nested constraints* [BO92, Sid93].

The constraint whose validity we want to reflect, has the form

```
Course.start :: Q-Course.dur+1 # Q
```

expressing that `Course` has started before or at quarter `Q`, but not finished at `Q`.

Now, we reflect the validity of this constraint into the variable `CAtQ`:

```
CAtQ = Course.start :: Q-Course.dur+1 # Q
```

First of all, every reified constraint always constrains the first variable to be either 0 or 1, i.e. `CAtQ` $\in \{0, 1\}$. As in previous propagators, information flows either way. If the store logically implies `Course.start` $\in$ {`Q-Course.dur+1`, ..., `Q`}, `CAtQ` is constrained to 1. If the store implies `Course.start` $\notin$ {`Q-Course.dur+1`, ..., `Q`}, `CAtQ` is constrained to 0. Vice versa, if `CAtQ` is constrained to 1 or 0, the basic constraint `Course.start :: Q-Course.dur+1 # Q` or its negation `Course.start \:: Q-Course.dur+1 # Q` is imposed on the store. It is essential that while `CAtQ` is not determined to 0 or 1, the constraint on the right-hand side is used only for checking but not for pruning. If already `NumberOfRooms` courses are scheduled at `Q`, the remaining boolean variables `CAtQ` will be constrained to 0 by the propagator `CoursesAtQ =<:NumberOfRooms`.

Because we guarantee that at each time there are sufficiently many rooms available, it is straightforward to assign appropriate rooms to the courses. In particular, the room assignment can be performed after the timetable computation, which reduces the complexity of the problem considerably.

The constraints C7 - C11 express overlapping conditions on courses. Assume that `Psych101` and `Socio101` are mandatory courses for first year students. Then C9 says that they may not overlap. This constraint can be expressed with a disjunction of the following form

$$\begin{array}{ll} & \texttt{Psych101.start} + \texttt{Psych101.dur} \leq \texttt{Socio101.start} \\ \vee & \texttt{Socio101.start} + \texttt{Socio101.dur} \leq \texttt{Psych101.start} \end{array}$$

If we are able to install a propagator stating that at least one of a given set of complex constraints is valid, we can express this disjunction. Thus, our problem is solved by reifying complex constraints:

```
B1 = Psych101.start + Psych101.dur =<: Socio101.start
B2 = Socio101.start + Socio101.dur =<: Psych101.start      (1)
B1 + B2 >=: 1
```

Let us consider the first reified constraint. If `Psych101.start`+`Psych101.dur` $\leq$ `Socio101.start` is logically implied by the constraint store, the basic constraint `B1::1` is imposed on the store. If `Psych101.start` + `Psych101.dur` $>$ `Socio101.start` is logically implied by the constraint store, the basic constraint `B1::0` is imposed on the store. Vice versa, if `B1` is constrained to 1, the propagator `Psych101.start + Psych101.dur =<: Socio101.start` is installed and if `B1` is constrained to 0, the propagator `Psych101.start + Psych101.dur >: Socio101.start` is installed.

As an example consider now `Psych101.start` $\in \{8,\ldots,12\}$, `Psych101.dur` $= 6$, `Socio101.start` $\in \{10,\ldots,14\}$ and `Socio101.dur` $= 6$. The second reified constraint constrains `B2` to 0 because the constraint store implies `Socio101.start`+ `Socio101.dur` $>$ `Psych101.start`. The constraint `B2 ::0` in the store allows the propagator `B1 + B2 >=:1` to constrain `B1` to 1. This allows the first reified propagator to install the propagator `Psych101.start + 6 =<:Socio101.start`, constraining `Psych101.start` to 8 and `Socio101.start` to 14, the only possible values, if the two courses do not overlap.

Encoding the constraints C7 - C10 now becomes straightforward. For example, C7 says that no two courses of an instructor must overlap. Thus, for every pair of courses of an instructor, a propagator of the above form must be installed.

The constraint C11 boils down to the constraint that a certain course, say `SplitCourse`, may overlap with at most one of a list `OtherSplitCourses` of other courses. If we are able to install a propagator that constrains a variable `Overlap` to 1, if a course overlaps with the course `SplitCourse` and to 0, if it does not, we can build the sum `Sum` of these variables over `OtherSplitCourses`. Then, we only need to impose the constraint `Sum =<:1`, stating that at most one of `OtherSplitCourses` overlaps with `SplitCourse`.

So how can we constrain the `Overlap` variables? Two courses `SplitCourse` and `OtherSplit` overlap, if `OtherSplit` starts before `SplitCourse` is finished and vice versa, i.e. if both the constraints

```
SplitCourse.start+SplitCourse.dur >: OtherSplit.start
```

and

```
OtherSplit.start+OtherSplit.dur >: SplitCourse.start
```

hold. The variable `Overlap` must be constrained to 1, if these two constraints hold and to 0 otherwise:

```
B1 = SplitCourse.start+SplitCourse.dur >:
OtherSplit.start
B2 = OtherSplit.start+OtherSplit.dur >: SplitCourse.start        (2)
Overlap = B1 + B2 =: 2
```

As usual, this constraint works also the other way around, e.g. if `Overlap` is known to be 1, then `B1 + B2 =:2` is installed. Thus, `B1` and `B2` are constrained to 1 and, hence, both propagators are installed. If `Overlap` is known to be 0, for example because another `Overlap` variable in `Sum` is already 1, the propagator `B1+B2 \=:2` is installed stating that only one of `B1` and `B2` may be constrained to 1. Thus, if, for example, `B2` is constrained to 1, then `B1` is constrained to 0 and the reified constraint for `B1` installs the non-reified version of its negation: `SplitCourse.start + SplitCourse.dur =<: OtherSplit.start`.

In a similar way, the constraint C12 can be expressed. Assume that all courses that a given instructor teaches are contained in the list `Courses`. For every instructor and every day, we compute the boolean value `TeachesOnDay` with

```
TeachesOnDay  =  1 =<: SumOfCoursesOnDay
```

where `SumOfCoursesOnDay` is the sum of the boolean variables obtained by reifying for every element of `Courses` a constraint that states that the course is taught on that day. For every instructor, we can express that she only teaches on three days with the propagator

```
TeachesOnDays =<: 3
```

where `TeachesOnDays` is the sum of all `TeachesOnDay` variables over the week. For example, if three courses have already been placed on different days, say Monday through Wednesday, then all the remaining courses are constrained to be held on Monday through Wednesday, thus, reducing the search space considerably.

The same technique can be applied for our last constraint C13. Assume that all elements of the list `DifferentDayCourses` with length `NumberOfDifferentDayCourses` must be held on a different day. Then the propagator

```
TeachesOnDays =: NumberOfDifferentDayCourses
```

does the job, where `TeachesOnDays` is defined as for C12.

5 Constructive Disjunction

In this section, we reconsider how to model the overlapping of two courses. Let us assume that the starting time of our two courses `Psych101` and `Socio101` is between (and including) 8:15 am (quarter 1) and 10:30 am (quarter 10), i.e., `Psych101.start`, `Socio101.start` $\in \{1, \ldots, 10\}$, and both durations are 6 quarters. Then the non-overlapping constraint of the courses expresses the disjunction

$$\texttt{Psych101.start}+6 \leq \texttt{Socio101.start} \vee \texttt{Socio101.start}+6 \leq \texttt{Psych101.start}$$

The left alternative of the disjunction constrains `Psych101.start` to $\{1,\ldots,4\}$, i.e., `Psych101` must start before or at 9 am. Analogously, the right alternative constrains `Psych101.start` to $\{7,\ldots,10\}$, i.e., it must start after or at 9:45 pm. Thus, independent of which alternative will succeed, we know that `Psych101` cannot start at 9:15 (quarter 5) am or at 9:30 am (quarter 6), i.e., `Psych101` $\in \{1,\ldots,4,7,\ldots,10\}$.

The propagators in Section 4, however, do not extract this valuable information on `Psych101`. To obtain more pruning, there is a more active form of disjunction available in Oz, called *constructive disjunction* [VSD95]. It extracts the common information from the alternatives of a disjunction. We replace program (1) by constructive disjunction, supported in the following syntax:

```
dis Psych101.start + Psych101.dur =<: Socio101.start
[] Socio101.start + Socio101.dur =<: Psych101.start
end
```

As in (1), if one alternative is unsatisfiable, the propagator corresponding to the other alternative is installed. Additionally, common information is extracted as described above. While the pruning is enhanced by constructive disjunction, it is also potentially more expensive, since extraction of common information may be attempted relatively often. Thus it takes some experimentation to find out which form of disjunction is most appropriate for a given application. It is essential that propagators also take holes in the domains into account, because in our problem constructive disjunction cuts holes in domains of variables. This is the case for reified basic constraints like `B = X::9#10`. If, for example, the basic constraint `X::[1#8 11#15]` is added, `B` is constrained to 0. The use of constructive disjunction for all non-overlap constraints (C7 - C11) in our college problem resulted in a speed up of more than one order of magnitude.

For modeling constraint C12, we use a ternary constructive disjunction instead of program (2):

```
dis B=:1 SplitCourse.start+SplitCourse.dur >: OtherSplit.start
        OtherSplit.start+OtherSplit.dur >: SplitCourse.start
[] B=:0 SplitCourse.start+SplitCourse.dur =<: OtherSplit.start
[] B=:0 OtherSplit.start+OtherSplit.dur =<: SplitCourse.start
end
```

The common information is in this case extracted from all three alternatives (or two if one alternative is known to be unsatisfiable). If all but one alternative are known to be unsatisfiable, the propagators of the remaining alternative are installed (since the disjunction must be true).

6 Enumeration

To achieve maximal pruning of the search space, we allow the propagators to *exhaustively* amplify the constraint store. We call a store, together with all the propagators, *stable* if none of the propagators can add any more information to it. Typically many variables still have more than one possible values after stability of the store. Thus, we want to explore the remaining search space. We proceed in two steps. First, we compute a fairly good first solution as described in this section and then we optimize starting from this solution as described in Section 7.

To explore the remaining search space, one of the variables that have more than one possible value is selected and speculatively constrained to these values. In order to speculatively constrain a variable to a value, we impose this constraint on a copy of the current constraint store, including all the propagators. If later on, the computation fails, another value can be tried on another copy of the store.[3] We call this process *enumeration* (in the literature it is also known as *labeling*). Once a variable is speculatively constrained to an integer, some propagators typically become able to amplify the constraint store again. When the constraint store becomes stable again, the next variable is selected for enumeration, and so on. Thus, propagators allow to prune the search space, while it is being explored. This scenario makes clear why a sequential language is inappropriate for describing constraint problems. In a sequential language, the complex interaction between enumeration and propagation needs to be made explicit by the programmer, while concurrent constraint languages allow to conceptually separate propagation and enumeration. Sequential languages like ECLiPSe [ECR95] deal with that problem by introducing ad hoc concepts like freeze and demons.

The enumeration process has two degrees of freedom. Firstly, the variable to be enumerated next needs to be selected, and secondly, the order in which the remaining possible values are tried, needs to be fixed. It is an essential ingredient of Oz that both variable and value selection can be programmed in Oz achieving a high degree of flexibility. For variable selection, we apply the first-fail strategy, in which a variable with the currently smallest domain is selected.

Using the first-fail strategy with the usual value selection beginning with the smallest possible value does not lead to a solution of our problem after one day of computation on a Sun Sparc 20.[4] The so-called *first-fit* strategy, which tries to place a course in the day with the fewest already placed courses as described in [BGJ94], also does not lead to a solution in reasonable time. Instead, we

[3] Instead of copying the store, as it is done in Oz, one can also trail the previous domain of the variable and restore it on failure.

[4] The usual first-fail strategy performs poorly because this strategy tries to place courses in a compact timetable. Due to the topology of our search space, this results in a behavior we call "thrashing" when constraints are violated because the enumeration strategy is not 'clever enough' to find the responsible variables. An approach using intelligent backtracking could help here [BP84].

enumerate the courses of each year starting from a different time of the week. The domain from 1 through 180 is divided in 10 blocks representing mornings and afternoons of school days. These blocks can be individually ordered for each course (in the implementation we use the same ordering for each year). By carefully choosing these blocks, we can come to a first solution very fast.

We can make use of this additional flexibility to optimize the timetable according to the criteria in Section 2. We simply order the blocks such that the preferred times are tried earlier than others. The first solution is now more likely to be better with respect to the criteria.

7 Optimization

In the previous section we have seen how we approach the optimization criteria by choosing a suitable enumeration strategy. However, experimentation shows that enumeration alone cannot guarantee that the first solution will fulfill the criteria sufficiently well. A way to achieve optimal solutions in constraint logic programming is the use of branch-and-bound. Branch-and-bound starts out with one solution and imposes that every next solution must be better than the previous one using a suitably defined cost function.

Soft constraints, i.e., constraints which should hold, but might be dropped, if necessary, can be modeled by including their reified version in the cost function. As an example consider that `Course` should be scheduled on Monday. The cost function will use the result `B` of the reified constraint `B = CourseNew::Monday >:CourseOld::Monday`.

In timetabling, as in many constraint problems in the real world, there is rarely a unique cost-function to optimize. For example, the goals to achieve compact timetables for students and instructors may conflict with each other. Hence, we have chosen general criteria that students and instructors can agree upon, such as minimizing the number of courses being held after the lunch break.

After the first (fairly good) solution is found with customized first-fail, we use branch-and-bound search to further optimize starting from this solution. Since due to the topology of our search space, going for the globally best solution is not feasible, the user can define a limit on the number of enumeration steps leading to a resource-limited branch-and-bound search. The user can interrupt the optimization at any time and request the currently best solution. This solution can be inspected and the user can decide whether it is good enough or whether she likes to search for a better solution. This process can be continued arbitrarily often. In this sense, we can say that we implement an anytime algorithm.

8 Implementation Issues

Figure 1 shows the top-level graphical user interface to our timetabling program. By using the object-oriented features of Oz and the interface to the window programming toolkit Tcl/Tk [Meh94] it was straightforward to implement the interface. The interoperability features of Oz [Sch94] allow the integration of tools to display, type-set (using LaTeX) and print the resulting timetable.

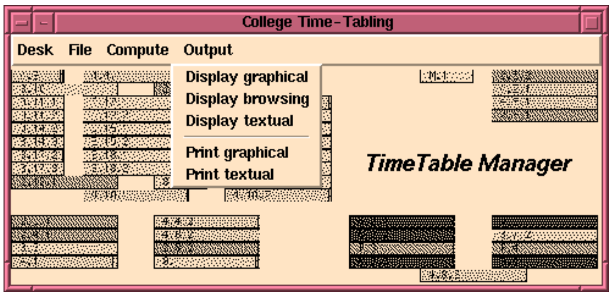


Fig. 1. Top-level graphical user interface

The visualization of the resulting timetable turned out to be extremely useful for the human user to judge the quality of the solutions during optimization. Figure 2 shows a program generated timetable visualization of a solution.

Other features of the language Oz, such as statically scoped higher-order programming and concurrent object-oriented programming, which are not available in other constraint logic languages, vastly facilitate coding and maintenance of programs.

The program deals with more than 25000 propagators. The first solution is found in less than a minute on a Sun Sparc20 with 60 MHz. A considerable optimization of 5 lectures less at afternoons is obtained after further 10 minutes.

9 Comparison to Constraint Languages

In this section we briefly compare Oz with existing constraint systems for solving combinatorial problems.

`CHIP` [DVS+88] is the forerunner for most commercial constraint systems. It lacks flexibility for search strategies and constructive disjunction. Only a small set of predefined reified constraints are supported by an if-then-else construct. Nevertheless, it is a successful commercial tool because several OR techniques

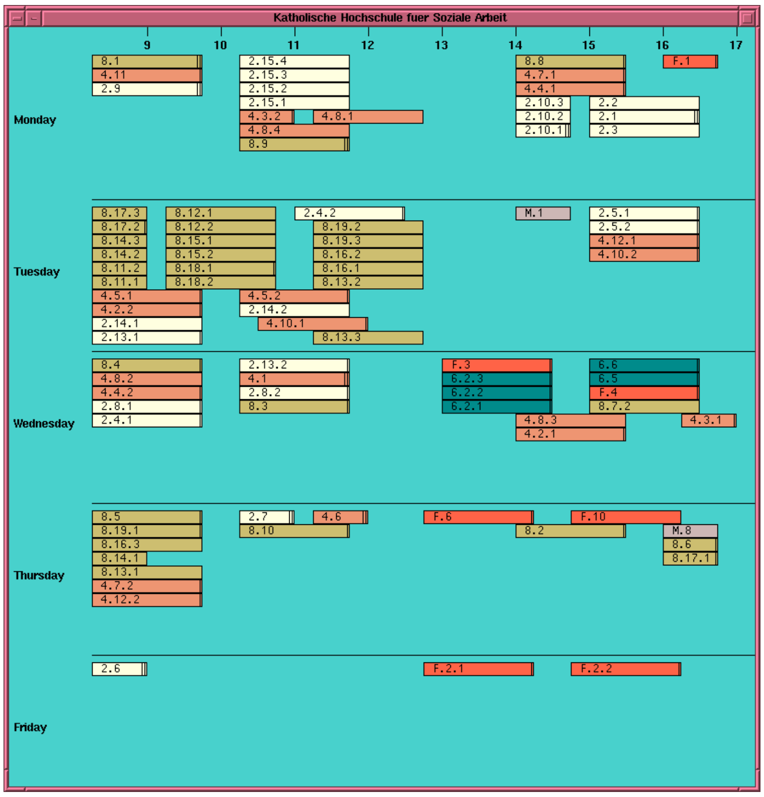


Fig. 2. Program generated timetable visualization

have been incorporated in operators dealing with disjunctive information. Disjunctive constraints not appropriate for these operators must be modeled by choice-points.

ECL^{i}PS^e[ECR95] is the research successor of `CHIP` enriched with features like attributed variables. By using attributed variables it is possible to program user-defined constraints on a rather low level.

`clp(FD)` [DC93] is a constraint language compiling to the C language. It is based on indexicals [VSD91] and is the fastest finite domain system freely available. Nevertheless it supports only basic ingredients for constraint programming (no reified constraints or conditionals).

The portation of the indexical approach to the concurrent constraint paradigm resulted in the system AKL(FD) [CCD94]. It includes means to express reified constraints and constructive disjunction. Unfortunately, the maximal domain is limited to 255 and the system is not publicly available. The ability to invent new search strategies is not given.

`cc(FD)` [VSD95] served as an inspiration for both AKL(FD) and Oz. `cc(FD)` integrates for the first time reified constraints, realized by the cardinality operator (which can be expressed in Oz), with constructive disjunction. `cc(FD)` does not support the ability to invent new search strategies. There is no implementation of `cc(FD)` available for further comparison.

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Remark

The DFKI Oz system and the documentation are available from the programming systems lab of DFKI through anonymous ftp from `ps-ftp.dfki.uni-sb.de` or through WWW from `http://ps-www.dfki.uni-sb.de/oz/`.

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