Clausal Tableaux for Hybrid PDL

Mark Kaminski and Gert Smolka
Saarland University

March 16, 2010

We present the first tableau-based decision procedure for PDL with nominals. The procedure is based on a prefix-free clausal tableau system designed as a basis for gracefully degrading reasoners. The clausal system factorizes reasoning into regular, propositional, and modal reasoning. This yields a modular decision procedure and pays off in transparent correctness proofs.

1 Introduction

PDL (propositional dynamic logic) [6, 11, 8] is an expressive modal logic invented for reasoning about programs. It extends basic modal logic with expressions called programs. Programs describe relations from states to states and are used to express modalities. Programs are composed with the operators familiar from regular expressions. In addition, they may employ formulas so that conditionals and while loops can be expressed. Fischer and Ladner [6] show the decidability of PDL using a filtration argument. They also prove that the satisfiability problem for PDL is EXPTIME-hard. Pratt [15] shows that PDL satisfiability is in EXPTIME using a tableau method with an and-or graph representation. Goré and Widmann [7] address the efficient implementation of Pratt-style decision procedures.

We consider PDL extended with nominals [12, 13], a logic we call hybrid PDL or HPDL. Nominals are atomic formulas that hold exactly for one state. Nominals equip PDL with equality and are the characteristic feature of hybrid logic [2]. The satisfiability problem of HPDL is in EXPTIME [13, 16].

We are interested in a tableau system for HPDL that can serve as a basis for gracefully degrading decision procedures. We found it impossible to extend one of the existing tableau methods for PDL [15, 5, 1, 7] to nominals. The difficulties are in the correctness proofs. For Pratt-like methods [15, 7], the problem stems

from the fact that the global and-or graph representation is not compatible with nominal propagation (see Remark 5.6 in [9] for a discussion and an example).

The difficulties led us to the development of a new tableau method for modal logic. The new method is based on a prefix-free clausal form. In a previous paper [9] we used the method to give a tableau-based decision procedure for the sublogic of HPDL that restricts programs to the forms a and a^* where a is a primitive action. In the present paper we extend the clausal method to full HPDL and obtain the first tableau-based decision procedure for HPDL.

Our method factorizes reasoning into regular reasoning, propositional reasoning and modal reasoning. At each level we realize reasoning with tableau methods. Nominals are handled at the modal level. Given our approach, the integration of nominals is straightforward. The modular structure of our decision procedure pays off in transparent correctness proofs. Each level invites optimizations. The regular level, in particular, asks for further investigation. It may profit from efficient methods for translating regular expressions into deterministic automata.

In contrast to previous approaches, we do not rely on the Fischer-Ladner closure. Instead, we use the notion of a finitary regular DNF that can be obtained at the regular level.

Following Baader [3] and De Giacomo and Massacci [5], we disallow bad loops and thus avoid the a posteriori eventuality checking of Pratt's method [15].

The paper is organized as follows. First we define HPDL and outline the clausal tableau method with examples. Then we address, one after the other, regular, propositional, and modal reasoning. Finally, we prove the correctness of the decision procedure.

2 Hybrid PDL

We define the syntax and semantics of HPDL. We assume that three kinds of **names** are given:

- **nominals** (x, y, z) (denote states)
- **predicates** (p, q, r) (denote sets of states)
- **actions** (a, b, c) (denote relations from states to states).

The interpretations of HPDL are the usual transition systems where states are labelled with predicates and edges are labelled with actions. Formally, an **interpretation** $\mathcal{I}$ is a tuple consisting of the following components:

- A nonempty set $|\mathcal{I}|$ of **states**.
- A state $\mathcal{I}x \in |\mathcal{I}|$ for every nominal x.
- A set $\mathcal{I}p \subseteq |\mathcal{I}|$ for every predicate p.

- A relation $\xrightarrow{a}_{\mathcal{I}} \subseteq |\mathcal{I}| \times |\mathcal{I}|$ for every action a.

Formulas (s, t, u) and **programs** (α, β, γ) are defined as follows:

$$\begin{aligned} s &::= x \mid p \mid \neg s \mid s \wedge s \mid \langle\alpha\rangle s \\ \alpha &::= a \mid s \mid 1 \mid \alpha + \alpha \mid \alpha\alpha \mid \alpha^* \end{aligned}$$

The grammar is to be read inclusive, that is, every nominal and every predicate is a formula, and every action and every formula is a program. We write programs of the form $\alpha(\beta\gamma)$ without parentheses as $\alpha\beta\gamma$. Given an interpretation, formulas denote sets of states and programs denote relations from states to states. We use the letters X, Y, Z to denote states. The semantic relations $\mathcal{I}, X \models s$ and $X \xrightarrow{\alpha}_{\mathcal{I}} Y$ are defined by mutual induction on the structure of formulas and programs:

$$\begin{aligned} \mathcal{I}, X \models x &\iff X = \mathcal{I}x \\ \mathcal{I}, X \models p &\iff X \in \mathcal{I}p \\ \mathcal{I}, X \models \neg s &\iff \text{not } \mathcal{I}, X \models s \\ \mathcal{I}, X \models s \wedge t &\iff \mathcal{I}, X \models s \text{ and } \mathcal{I}, X \models t \\ \mathcal{I}, X \models \langle\alpha\rangle s &\iff \exists Y\colon\ X \xrightarrow{\alpha}_{\mathcal{I}} Y \text{ and } \mathcal{I}, Y \models s \end{aligned}$$

$$\begin{aligned} X \xrightarrow{a}_{\mathcal{I}} Y &\iff X \xrightarrow{a}_{\mathcal{I}} Y \\ X \xrightarrow{s}_{\mathcal{I}} Y &\iff X = Y \text{ and } \mathcal{I}, X \models s \\ X \xrightarrow{1}_{\mathcal{I}} Y &\iff X = Y \\ X \xrightarrow{\alpha+\beta}_{\mathcal{I}} Y &\iff X \xrightarrow{\alpha}_{\mathcal{I}} Y \text{ or } X \xrightarrow{\beta}_{\mathcal{I}} Y \\ X \xrightarrow{\alpha\beta}_{\mathcal{I}} Y &\iff \exists Z\colon\ X \xrightarrow{\alpha}_{\mathcal{I}} Z \text{ and } Z \xrightarrow{\beta}_{\mathcal{I}} Y \\ X \xrightarrow{\alpha^*}_{\mathcal{I}} Y &\iff X \xrightarrow{\alpha}^*_{\mathcal{I}} Y \end{aligned}$$

$\xrightarrow{\alpha}^*_{\mathcal{I}}$ denotes the reflexive transitive closure of $\xrightarrow{\alpha}_{\mathcal{I}}$

Given a set A of formulas, we write $\mathcal{I}, X \models A$ if $\mathcal{I}, X \models s$ for all formulas $s \in A$. An interpretation $\mathcal{I}$ **satisfies** (or is a **model** of) a formula s or a set A of formulas if there is a state $X \in |\mathcal{I}|$ such that $\mathcal{I}, X \models s$ or, respectively, $\mathcal{I}, X \models A$. A formula s (a set A) is **satisfiable** if s (A) has a model.

The **complement** $\sim s$ of a formula s is t if $s = \neg t$ and $\neg s$ otherwise. Note that $\sim\sim s = s$ if s is not a double negation. We use the notations $s \vee t := \neg(\sim s \wedge \sim t)$ and $[\alpha]s := \neg\langle\alpha\rangle\sim s$. Note that $\sim\langle\alpha\rangle p = [\alpha]\neg p$ and $\sim\langle\alpha\rangle\neg p = [\alpha]p$. The **size** $|s|$ and $|\alpha|$ of formulas and programs is defined as the size of the abstract syntax tree. For instance, $|ap| = |\langle p\rangle q| = 3$. Note that $|s \vee t| > |s|, |t|$ and $|[\alpha]s| \geq |\langle\alpha\rangle s| > |\alpha|, |s|$.

We use $\mathcal{F}\alpha$ to denote the set of all formulas that occur in α as subprograms. For instance, $\mathcal{F}(a\neg p + b\langle ap\rangle q) = \{\neg p,\ \langle ap\rangle q\}$. Formulas that occur as programs are called **tests**.

Proposition 2.1 If $s \in \mathcal{F}\alpha$, then $|s| < |\neg s| < |\langle\alpha\rangle t| \leq |[\alpha]t|$.

3 Outline of the Method

Our tableau method is based on a clausal form, which provides for the separation of regular, propositional, and modal reasoning. We start with a few definitions and three examples.

A **basic formula** is a formula of the form p, x, or $\langle a\alpha\rangle s$. A **literal** is a basic formula or the complement of a basic formula. A **clause** (C, D) is a finite set of literals that contains no complementary pair. A **claim** is a pair C^s consisting of a clause C and a diamond formula s. The **request** of a clause C for an action a is $\mathcal{R}_a C := \{\,[\alpha]s \mid [a\alpha]s \in C\,\}$. As an example, consider the clause $C = \{\langle ab^*\rangle p,\ \langle bb^*\rangle p,\ [a(a+b)^*]\neg p\}$. We have $\mathcal{R}_a C = \{[(a+b)^*]\neg p\}$ and $\mathcal{R}_b C = \emptyset$.

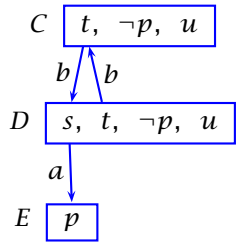
We interpret clauses conjunctively. Thus **satisfaction of clauses** (i.e., $\mathcal{I}, X \vDash C$) is a special case of satisfaction of sets of formulas (i.e., $\mathcal{I}, X \vDash A$), which was defined in §2. For instance, the clause $\{p, \neg p\}$ is unsatisfiable.

The method is implemented with three reasoners. The propositional reasoner determines for every set A of formulas a set of clauses such that $\mathcal{I}, X \vDash A$ if and only if $\mathcal{I}, X \vDash C$ for one of the clauses. Given the formula $\langle a^*\rangle p \wedge [b^*]\neg p$, for instance, the propositional reasoner determines the single clause $\{\langle aa^*\rangle p,\ \neg p,\ [bb^*]\neg p\}$. The modal reasoner constructs for every satisfiable clause a finite model whose states are clauses and where every state C satisfies the clause C. To do so, the modal reasoner starts with the initial clause and derives further clauses until every diamond literal s in every clause C is realized with a *link* $C^s D^t$ where D is one of the clauses. The modal reasoner calls the regular reasoner to determine the successor formula t and the propositional reasoner to determine the successor clause D. The tableau method terminates since the derived clauses must take their literals from a finite set that can be determined from the initial formulas.

Example 3.1 Consider the following literals and clauses:

$$\begin{aligned} s &:= \langle a(a+b)^*\rangle p & C &:= \{t, \neg p, u\} \\ t &:= \langle b(a+b)^*\rangle p & D &:= \{s, t, \neg p, u\} \\ u &:= [bb^*](\neg p \wedge t) & E &:= \{p\} \end{aligned}$$

We start the modal reasoner with the satisfiable clause C. There is one claim C^t to be realized. We need a clause that supports the formulas $\langle(a+b)^*\rangle p$ and $[b^*](\neg p \wedge t)$. The regular reasoner and the propositional reasoner determine C^t and D^s as possible successor clauses and successor literals. The modal reasoner rejects C^t since it would introduce the loop C^tC^t. The pair D^s is fine and adds the clause D and the link C^tD^s. The claim C^t is now realized. However, the new clause D has two unrealized claims D^s and D^t. To realize D^s, we need a clause that supports the formula $\langle(a+b)^*\rangle p$. The regular and the propositional reasoner yield the pairs $E^{\langle 1\rangle p}$, $\{s\}^s$, and $\{t\}^t$. We choose $E^{\langle 1\rangle p}$ and add the clause E and the link $D^tE^{\langle 1\rangle p}$. It remains to realize D^t. To do so, we need a clause that supports the formulas $\langle(a+b)^*\rangle p$ and $[b^*](\neg p \wedge t)$. As before, the regular and the propositional reasoner yield C^t and D^s. Both are fine. We choose C^t and add the link D^tC^t. This gives us a model for the initial clause C. A graphical representation of the model looks as follows:



□

Example 3.2 Consider the following literals:

$$s := \langle a(a+b)^*\rangle \neg p \qquad u := [a(b+a)^*]p$$
$$t := \langle b(a+b)^*\rangle \neg p \qquad v := [b(b+a)^*]p$$

Here is a closed tableau for the unsatisfiable clause $\{s,u\}$:

$$C_1 = \{s,u\}$$

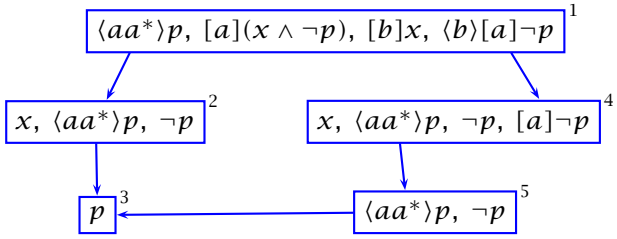
$C_2 = \{s,p,u,v\}$	$C_3 = \{t,p,u,v\}$
$C_1^sC_2^s$	$C_1^sC_3^t$
$C_4 = \{t,p,u,v\}$	$C_5 = \{s,p,u,v\}$
$C_2^sC_4^t$	$C_3^tC_5^s$

The tableau is closed since all possible links for the claims C_4^t and C_5^s introduce loops. For instance, for C_4^t the regular and the propositional reasoner yield the links $C_4^tC_2^s$ and $C_4^tC_4^t$. Note that the clause names C_i do not act as prefixes. They are only used for explanatory purposes. □

Example 3.3 Due to the clausal form, the extension of our tableau method to nominals is straightforward. When we add a new clause to a branch, we add to

the new clause all literals that occur in clauses of the branch that have a nominal in common with the new clause. This takes care of nominal propagation. Clauses and links that are already on the branch remain unchanged.

Consider the clause $C = \{\langle aa^*\rangle p,\ [a](x \wedge \neg p),\ [b]x,\ \langle b\rangle[a]\neg p\}$. The initial tableau just consisting of C can be developed into a maximal branch as follows (graphical representation):



The numbers indicate the order in which the clauses are introduced. When clause 4 is introduced, nominal propagation from clause 2 takes place. Note that we obtain a model of all clauses on the branch by taking the clauses 1, 3, 4, and 5 as states and the triples $1a4$, $1b4$, $4a5$, and $5a3$ as transitions. □

4 Language-Theoretic Semantics

We define a language-theoretic semantics for programs that treats formulas as atomic objects. This semantics is the base for the regular reasoner and decouples it from the propositional reasoner. It is also essential for the correctness proofs of the modal reasoner. The semantics is an adaption of the language-theoretic model of Kleene algebras with tests [10].

The letters A, B range over finite sets of formulas. A **guarded string** is a finite sequence $Aa_1A_1 \ldots a_nA_n$ where $n \geq 0$. The letters σ and τ range over guarded strings. The **length $|\sigma|$ of a guarded string** $\sigma = Aa_1A_1 \ldots a_nA_n$ is n. We use **For** to denote the set of all formulas. A **language** is a set of guarded strings. For languages L and L' and sets of formulas A we define the following:

$$\begin{array}{rcl@{\qquad}rcl} \mathcal{L}A & := & \{\, B \mid A \subseteq B \subseteq \mathrm{For} \,\} & L^0 & := & \mathcal{L}\emptyset \\ L \cdot L' & := & \{\, \omega A \omega' \mid \omega A \in L,\ A\omega' \in L' \,\} & L^{n+1} & := & L \cdot L^n \\ & & & L^* & := & \bigcup_{n \in \mathbb{N}} L^n \end{array}$$

Note that $L^* = \mathcal{L}\emptyset \cup (L - \mathcal{L}\emptyset) \cdot L^*$. We assign to every program α a **language $\mathcal{L}\alpha$**:

$$\begin{array}{rcl@{\qquad}rcl} \mathcal{L}a & := & \{\, AaB \mid A, B \subseteq \mathrm{For} \,\} & \mathcal{L}(\alpha + \beta) & := & \mathcal{L}\alpha \cup \mathcal{L}\beta \\ \mathcal{L}s & := & \mathcal{L}\{s\} & \mathcal{L}(\alpha\beta) & := & \mathcal{L}\alpha \cdot \mathcal{L}\beta \\ \mathcal{L}1 & := & \mathcal{L}\emptyset & \mathcal{L}\alpha^* & := & (\mathcal{L}\alpha)^* \end{array}$$

Note that $\mathcal{L}(s^*) = \mathcal{L}1 = \mathcal{L}((s+t)^*)$.

Given an interpretation $\mathcal{I}$, we define the relations $\xrightarrow{\sigma}_{\mathcal{I}} \subseteq |\mathcal{I}| \times |\mathcal{I}|$ by induction on the structure of σ:

$$X \xrightarrow{A}_{\mathcal{I}} Y \iff X = Y \text{ and } \mathcal{I}, X \vDash A$$
$$X \xrightarrow{Aa\sigma}_{\mathcal{I}} Y \iff \mathcal{I}, X \vDash A \text{ and } \exists Z\colon\ X \xrightarrow{a}_{\mathcal{I}} Z \text{ and } Z \xrightarrow{\sigma}_{\mathcal{I}} Y$$

Proposition 4.1

1. $X \xrightarrow{\alpha}_{\mathcal{I}} Y \iff \exists\, \sigma \in \mathcal{L}\alpha\colon\ X \xrightarrow{\sigma}_{\mathcal{I}} Y$
2. $\mathcal{I}, X \vDash \langle\alpha\rangle s \iff \exists\, \sigma \in \mathcal{L}\alpha\ \exists\, Y\colon\ X \xrightarrow{\sigma}_{\mathcal{I}} Y$ and $\mathcal{I}, Y \vDash s$
3. $\mathcal{I}, X \vDash [\alpha]s \iff \forall\, \sigma \in \mathcal{L}\alpha\ \forall\, Y\colon\ X \xrightarrow{\sigma}_{\mathcal{I}} Y$ implies $\mathcal{I}, Y \vDash s$

5 Regular DNF

We now describe the regular reasoner. The regular reasoner relies on the language-theoretic semantics and ignores the propositional and modal aspects of the language.

A program is **basic** if it has the form $a\alpha$, and **normal** if it is 1 or basic. A **guarded program** is a pair $A\alpha$ where A is a set of formulas and α is a program. A guarded program $A\alpha$ is **normal** if α is normal. The language of a guarded program is $\mathcal{L}(A\alpha) := \mathcal{L}A \cdot \mathcal{L}\alpha$. A **regular DNF** is a function $\mathcal{D}$ that maps every program α to a finite set $\mathcal{D}\alpha$ of normal guarded programs such that:

1. $\mathcal{L}\alpha = \bigcup_{B\beta \in \mathcal{D}\alpha} \mathcal{L}(B\beta)$
2. If $B\beta \in \mathcal{D}\alpha$, then $B \cup \mathcal{F}\beta \subseteq \mathcal{F}\alpha$.

Let $\mathcal{D}$ be a regular DNF. A set P of programs is **$\mathcal{D}$-closed** if $\beta \in P$ whenever $\alpha \in P$ and $B\beta \in \mathcal{D}\alpha$. A regular DNF $\mathcal{D}$ is **finitary** if for every program α there exists a finite $\mathcal{D}$-closed set of programs that contains α.

The regular reasoner computes a finitary regular DNF. Kleene's theorem (regular expressions translate into finite automata) [14] suggests that finitary regular DNFs exist. We give a naive algorithm that computes a finitary DNF. For space reasons we omit the correctness proof. The algorithm employs the following inference rules for guarded programs.

$$\frac{Aa}{Aa1} \qquad \frac{As}{(A;s)1} \qquad \frac{As\beta}{(A;s)\beta} \qquad \frac{A1\beta}{A\beta} \qquad \frac{A(\alpha_1+\alpha_2)}{A\alpha_1\,,\ A\alpha_2} \qquad \frac{A(\alpha_1+\alpha_2)\beta}{A\alpha_1\beta\,,\ A\alpha_2\beta}$$

$$\frac{A(\alpha_1\alpha_2)\beta}{A\alpha_1\alpha_2\beta} \qquad \frac{A\alpha^*}{A1\,,\ A\alpha\alpha^*} \qquad \frac{A\alpha^*\beta}{A\beta\,,\ A\alpha\alpha^*\beta}$$

The notation $A\,;s$ stands for the set $A \cup \{s\}$. Given a set G of guarded programs, we denote the closure of G under the rules with $\mathcal{R}G$. One can show that $\mathcal{R}G$ describes the same language as G, and that $\mathcal{R}G$ is finite if G is finite. If G is a set of guarded programs, we call a guarded program $A\alpha \in G$ **minimal in** G if $A \subseteq B$ whenever $B\alpha \in G$. We obtain a finitary regular DNF $\mathcal{D}$ by taking for $\mathcal{D}\alpha$ all normal guarded programs in $\mathcal{R}\{\emptyset\alpha\}$ that are minimal in $\mathcal{R}\{\emptyset\alpha\}$.

Example 5.1 Consider the program $(a+b)^*$. We have:

$$\mathcal{R}\{\emptyset(a+b)^*\} = \{\emptyset(a+b)^*,\ \emptyset 1,\ \emptyset(a+b)(a+b)^*,\ \emptyset a(a+b)^*,\ \emptyset b(a+b)^*\}$$
$$\mathcal{D}\{(a+b)^*\} = \{\emptyset 1,\ \emptyset a(a+b)^*,\ \emptyset b(a+b)^*\}$$ □

Example 5.2 Consider the program $(p+q)^*$ where p, q are predicates. We have $\mathcal{D}\{(p+q)^*\} = \{\emptyset 1\}$. We profit from the optimization that only the minimal guarded programs are taken for the DNF. Otherwise $\mathcal{D}\{(p+q)^*\}$ would contain three further elements. □

There are efficient algorithms that translate regular expressions into deterministic finite automata [4]. For programs without tests this gives us efficient regular DNFs. We expect that efficient regular DNFs also exist for programs with tests.

We fix some computable and finitary regular DNF $\mathcal{D}$ for the rest of the paper.

Proposition 5.3

1. $\mathcal{I}, X \vDash \langle\alpha\rangle s \iff \exists\, B\beta \in \mathcal{D}\alpha\colon\ \mathcal{I}, X \vDash B\,;\langle\beta\rangle s$
2. $\mathcal{I}, X \vDash [\alpha]s \iff \forall\, B\beta \in \mathcal{D}\alpha\colon\ (\exists\, t \in B\colon\ \mathcal{I}, X \vDash \neg t)$ or $\mathcal{I}, X \vDash [\beta]s$

Proof Follows with Proposition 4.1. ∎

6 Propositional DNF

The propositional reasoner relies on a support relation from clauses to formulas that abstracts from most modal aspects of the language. We define the **support**

relation $C \rhd s$ by recursion on s.

$$
\begin{aligned}
C \rhd s &\iff s \in C \quad \text{if } s \text{ is a literal} \\
C \rhd \neg\neg s &\iff C \rhd s \\
C \rhd s \wedge t &\iff C \rhd s \text{ and } C \rhd t \\
C \rhd s \vee t &\iff C \rhd s \text{ or } C \rhd t \\
C \rhd \langle 1 \rangle s &\iff C \rhd s \\
C \rhd [1] s &\iff C \rhd s \\
C \rhd \langle \alpha \rangle s &\iff \exists B\beta \in \mathcal{D}\alpha\colon\ (\forall t \in B\colon\ C \rhd t) \text{ and } C \rhd \langle \beta \rangle s \quad \text{if } \alpha \text{ not normal} \\
C \rhd [\alpha] s &\iff \forall B\beta \in \mathcal{D}\alpha\colon\ (\exists t \in B\colon\ C \rhd \neg t) \text{ or } C \rhd [\beta] s \quad \text{if } \alpha \text{ not normal}
\end{aligned}
$$

The last two equivalences of the definition employ the finitary regular DNF $\mathcal{D}$ fixed above. The recursion terminates since either the size of the formula is reduced (verify with Proposition 2.1) or the recursion is on a formula $\langle \beta \rangle s$ or $[\beta] s$ where β is normal and s is unchanged. We say C **supports** s if $C \rhd s$. We write $C \rhd A$ and say C **supports** A if $C \rhd s$ for every $s \in A$. Note that $C \rhd D \iff D \subseteq C$ if C and D are clauses.

Proposition 6.1 If $C \rhd A$ and $C \subseteq D$ and $B \subseteq A$, then $D \rhd B$.

Proposition 6.2 If $\mathcal{I}, X \vDash C$ and $C \rhd A$, then $\mathcal{I}, X \vDash A$.

Proof Follows with Proposition 5.3. ∎

A **propositional DNF** is a function $\mathcal{D}$ that maps every finite set A of formulas to a finite set of clauses such that:

1. $\mathcal{I}, X \vDash A \iff \exists D \in \mathcal{D}A\colon\ \mathcal{I}, X \vDash D$.
2. $C \rhd A \iff \exists D \in \mathcal{D}A\colon\ D \subseteq C$.

For the termination of the modal reasoner the propositional DNF must have some additional finiteness property. We need a few preparatory definitions. The **variants** of a program α are the basic programs β such that $B\beta \in \mathcal{D}\alpha$ for some B. A **base** is a set U of basic formulas such that $\langle \beta \rangle s \in U$ whenever $\langle a\alpha \rangle s \in U$ and β is a variant of α. A base U **supports a formula** s if the following conditions are satisfied:

1. U contains every basic formula that occurs in s.
2. If $\langle \alpha \rangle t$ occurs in s, α is not basic, and β is a variant of α, then $\langle \beta \rangle t \in U$.

A base **supports a set of formulas** A if it supports every formula $s \in A$.

Proposition 6.3 Every set of formulas is supported by a finite base.

Proof Follows from the fact that the underlying regular DNF is finitary. ∎

A propositional DNF $\mathcal{D}$ is **finitary** if for every finite set of formulas A and every base U supporting A and every clause $C \in \mathcal{D}A$ it holds that U supports C.

Proposition 6.4 There is a computable finitary propositional DNF.

Proof The definition of the support relation can be seen as a tableau-style decomposition procedure for formulas. The clauses of a propositional DNF can be obtained with the literals the decomposition produces. The direction "⇐" of property (1) for propositional DNFs follows with Proposition 6.2. That the DNF is finitary follows from the fact that the decomposition does not introduce new formulas except for diamond formulas obtained with the finitary regular DNF. ∎

Example 6.5 Take the regular DNF given in §5 and the propositional DNF given in the proof of Proposition 6.4. We have:

$$\begin{aligned}
\mathcal{D}\{\langle b^*\rangle p\} &= \{\{p\},\ \{\langle bb^*\rangle p\}\} \\
\mathcal{D}\{\langle b^*\rangle p,\ [b^*](q \wedge \neg p)\} &= \{\{\langle bb^*\rangle p,\ q,\ \neg p,\ [bb^*](q \wedge \neg p)\}\} \\
\mathcal{D}\{\langle a^*\rangle p,\ [a^*]\neg p\} &= \emptyset \\
\mathcal{D}\{\langle (a+b)^*\rangle p\} &= \{\{p\},\ \{\langle a(a+b)^*\rangle p\},\ \{\langle b(a+b)^*\rangle p\}\} \\
\mathcal{D}\{\langle (a+b)^*\rangle p,\ [b^*]\neg p\} &= \{\ \{\langle a(a+b)^*\rangle p,\ \neg p,\ [bb^*]\neg p\}, \\
&\qquad \{\langle b(a+b)^*\rangle p,\ \neg p,\ [bb^*]\neg p\}\ \}
\end{aligned}$$

For the third example note that $[a^*]\neg p$ is the complement of $\langle a^*\rangle p$. □

We fix some computable and finitary propositional DNF $\mathcal{D}$ for the rest of the paper.

7 Diamond Expansion and Nominal Propagation

We now return to the modal reasoner, which was first explained in §3. The modal reasoner builds a tableau where each branch contains clauses and links. The goal consists in constructing a branch where every claim is realized with a link and some further conditions are satisfied. We first make precise how the modal reasoner derives new clauses.

An **expansion** of a claim $C^{\langle a\alpha\rangle s}$ is a claim $D^{\langle\beta\rangle s}$ such that $B\beta \in \mathcal{D}\alpha$ and $D \in \mathcal{D}(B;\langle\beta\rangle s \cup \mathcal{R}_a C)$ for some B.

Proposition 7.1 Let C^s be a claim such that $s \in C$ and let $\mathcal{I}$ satisfy C. Then there exists an expansion D^t of C^s such that $\mathcal{I}$ satisfies D.

Proof We have $s = \langle a\alpha\rangle t$ and $\mathcal{I}, X \vDash \{\langle\alpha\rangle t\} \cup \mathcal{R}_a C$ since $s \in C$ and $\mathcal{I}$ satisfies C. By Proposition 5.3 we have $\mathcal{I}, X \vDash B;\langle\beta\rangle s \cup \mathcal{R}_a C$ for some $B\beta \in \mathcal{D}\alpha$. The claim follows by (1) in the definition of propositional DNFs. ∎

A **link** is a pair $C^s D^t$ of two claims such that $s \in C$ and there is an expansion E^t of C^s such that $E \subseteq D$. A **quasi-branch** is a finite set Γ of clauses and links such that $\{C, D\} \subseteq \Gamma$ whenever $C^s D^t \in \Gamma$. A quasi-branch Γ **realizes a claim** C^s if Γ contains some link $C^s D^t$. A base **supports a quasi-branch** Γ if it supports every clause of Γ. A **model of a quasi-branch** Γ is an interpretation that satisfies every clause in Γ.

We call a clause **nominal** if it contains a nominal. Let Γ be a quasi-branch and A be a set of formulas. We realize **nominal propagation** with the notation

$$A^\Gamma \;:=\; A \cup \{\, s \mid \exists x \in A\ \exists C \in \Gamma\colon\ x \in C \wedge s \in C \,\}$$

Note that A^Γ is the least set of formulas that contains A and all clauses $C \in \Gamma$ that have a nominal in common with A. Thus $(A^\Gamma)^\Gamma = A^\Gamma$. Moreover, $A^\Gamma = A$ if A contains no nominal.

Proposition 7.2 If an interpretation satisfies Γ and C, it satisfies C^Γ.

Proposition 7.3 Let U be a base that supports a quasi-branch Γ, C^s be a claim such that $s \in C \in \Gamma$, and D^t be an expansion of C^s. Then U supports D^Γ.

8 Branches and Expansion Rule

A quasi-branch that realizes all its claims does not necessarily have a model. To guarantee the existence of a model, we impose certain conditions on quasi-branches that act as invariants of the modal reasoner. One of the conditions is loop freeness.

Example 8.1 Consider the clause $C = \{\langle aa^*\rangle\neg p,\ p, q,\ [aa^*](p \wedge q)\}$. Note that C is unsatisfiable, and that $\{C,\ C^{\langle aa^*\rangle\neg p} C^{\langle aa^*\rangle\neg p}\}$ is a quasi-branch that realizes every claim. The link of this quasi-branch describes a loop. □

A **path in a quasi-branch** Γ is a sequence ${C_1}^{s_1} \ldots {C_n}^{s_n}$ of claims such that:

1. $\forall i \in [1, n]\colon\ C_i^\Gamma = C_i$.
2. $\forall i \in [1, n-1]\ \exists D\colon\ {C_i}^{s_i} D^{s_{i+1}} \in \Gamma$ and $D^\Gamma = C_{i+1}$.

A **loop in a quasi-branch** Γ is a path ${C_1}^{s_1} \ldots {C_n}^{s_n}$ in Γ such that $n \geq 2$ and ${C_n}^{s_n} = {C_1}^{s_1}$. A **branch** is a quasi-branch Γ that satisfies the following conditions:

- **Functionality:** If $C^s D^t \in \Gamma$ and $C^s E^u \in \Gamma$, then $D^t = E^u$.

- **Loop-freeness:** There is no loop in Γ.
- **Nominal coherence:** If $C \in \Gamma$, then $C^\Gamma \in \Gamma$.

The **core of a branch** Γ is $C\Gamma := \{C \in \Gamma \mid C^\Gamma = C\}$. A branch Γ is **evident** if Γ realizes $C^{\langle\alpha\rangle s}$ for all $\langle\alpha\rangle s \in C \in C\Gamma$. We will show that every evident branch has a model. The modal reasoner works on branches and applies the following expansion rule:

Expansion Rule
If $\langle\alpha\rangle s \in C \in C\Gamma$ and Γ does not realize $C^{\langle\alpha\rangle s}$,
then expand Γ to all branches $\Gamma; D^\Gamma; C^{\langle\alpha\rangle s}(D^\Gamma)^t$
such that D^t is an expansion of $C^{\langle\alpha\rangle s}$ and D^Γ is a clause.

Note that a single clause always yields a branch. So the modal reasoner can start with any clause.

Proposition 8.2 The modal reasoner terminates on every branch.

Proof Since branches are finite by definition, we know by Proposition 6.3 that the initial branch is supported by a finite base. By Proposition 7.3 we know that the expansion rule only adds clauses that are supported by the initial base. The claim follows since a finite base can only support finitely many clauses. ∎

Given termination, the correctness of the modal reasoner can be established by showing two properties:

1. *Model Existence:* Every evident branch has a model.
2. *Soundness:* Every satisfiable clause can be developed into an evident branch.

9 Model Existence

Proposition 9.1 Let Γ be an evident branch and $\langle\alpha\rangle s \in C \in C\Gamma$. Then there exists a unique path $C^{\langle\alpha\rangle s} \dots D^{\langle 1\rangle s}$ in Γ.

Proof The path exists since Γ is loop-free and realizes every claim with a clause in $C\Gamma$. The path is unique since Γ is functional. ∎

Lemma 9.2 If $X \xrightarrow{a}_{\mathcal{I}} Y$ and $\mathcal{I}, Y \vDash B; \langle\beta\rangle s$ and $B\beta \in \mathcal{D}\alpha$, then $\mathcal{I}, X \vDash \langle a\alpha\rangle s$.

Proof Follows with Propositions 4.1 and 5.3. ∎

The model existence proof requires a somewhat involved induction, which we realize with the following lemma.

Lemma 9.3 Let Γ be an evident branch and $\mathcal{I}$ be an interpretation such that:

- $|\mathcal{I}| = C\Gamma$
- $C \xrightarrow{a}_{\mathcal{I}} D \iff \exists \alpha, s, t, E\colon\ C^{\langle a\alpha\rangle s}E^{t} \in \Gamma$ and $D = E^{\Gamma}$ for all actions a
- $C \in \mathcal{I}p \iff p \in C$ for all predicates p
- $\mathcal{I}x = C \iff x \in C$ for all nominals x that occur in Γ

Let $|\mathcal{F}\alpha| := \max\{\, |s| \mid s \in \mathcal{F}\alpha \,\}$. Then for all $n \in \mathbb{N}$:

1. For every path $C^{\langle\alpha\rangle s} \dots D^{\langle 1\rangle s}$ in Γ such that $|\mathcal{F}\alpha|, |s| < n$:
   If $C \rhd \langle\alpha\rangle s$, then $\mathcal{I}, C \vDash \langle\alpha\rangle s$.
2. For all C, D, σ, α, s such that $|\mathcal{F}\alpha|, |s| < n - 1$:
   If $C \rhd [\alpha]s$, $\sigma \in \mathcal{L}\alpha$, and $C \xrightarrow{\sigma}_{\mathcal{I}} D$, then $D \rhd s$.
3. For all C, s such that $C \in C\Gamma$ and $|s| = n$:
   If $C \rhd s$, then $\mathcal{I}, C \vDash s$.

Proof By induction on n. Let $n \in \mathbb{N}$.

1. Let $\pi = C^{\langle\alpha\rangle s} \dots D^{\langle 1\rangle s}$ be a path in Γ such that $|\mathcal{F}\alpha|, |s| < n$ and $C \rhd \langle\alpha\rangle s$. We show $\mathcal{I}, C \vDash \langle\alpha\rangle s$ by induction on the length of π. Case analysis.

- $\alpha = 1$. Then $C \rhd s$ and thus $\mathcal{I}, C \vDash s$ by (3) of the outer inductive hypothesis. The claim follows.
- $\alpha = a\beta$. Then $\pi = C^{\langle a\beta\rangle s}E^{\langle\gamma\rangle s} \dots D^{\langle 1\rangle s}$ for some E and γ. It follows that $E \rhd \langle\gamma\rangle s$, $C \xrightarrow{a}_{\mathcal{I}} E$, $B\gamma \in \mathcal{D}\beta$, and $E \rhd B$ for some B. We have $\mathcal{I}, E \vDash \langle\gamma\rangle s$ by the inner inductive hypothesis and $\mathcal{I}, E \vDash B$ by (3) of the outer inductive hypothesis. The claim follows by Lemma 9.2.

2. Let $C \rhd [\alpha]s$, $\sigma \in \mathcal{L}\alpha$, $C \xrightarrow{\sigma}_{\mathcal{I}} D$, and $|\mathcal{F}\alpha|, |s| < n - 1$. We show $D \rhd s$ by induction on $|\sigma|$. Case analysis.

- $\alpha = 1$. Then $C \rhd s$ and $C = D$. The claim follows.
- $\alpha = a\beta$. Then $\sigma = Aa\tau$, $\tau \in \mathcal{L}\beta$, and $[a\beta]s \in C$ for some A and τ. Moreover, $\mathcal{I}, C \vDash A$ and $C \xrightarrow{a}_{\mathcal{I}} E \xrightarrow{\tau}_{\mathcal{I}} D$ for some E. Thus $E \rhd \mathcal{R}_a C$. Hence $E \rhd [\beta]s$ since $[a\beta]s \in C$. The claim follows by the inner inductive hypothesis.
- α not normal. Then $B\beta \in \mathcal{D}\alpha$ and $\sigma \in \mathcal{L}(B\beta)$ for some B and β. Thus $\sigma \in \mathcal{L}\beta$ and $\mathcal{I}, C \vDash B$. Hence we know by (3) of the outer induction hypothesis that $C \rhd \neg u$ for no $u \in B$. Thus $C \rhd [\beta]s$ since $C \rhd [\alpha]s$. Since β is normal, we now obtain the claim by arguing as in the first two cases.

3. Let $C \in C\Gamma$ such that $C \rhd s$ and $|s| = n$. We show $\mathcal{I}, C \vDash s$. Case analysis:

- $s = p$. Then $p \in C$ and hence $C \in \mathcal{I}p$. The claim follows.
- $s = \neg p$. Then $\neg p \in C$. Hence $p \notin C$ and so $C \notin \mathcal{I}p$. The claim follows.
- $s = x$ and $s = \neg x$. Analogously to the above two cases.
- $s = \neg\neg t$. Then $C \rhd t$. The claim follows by (3) of the inductive hypothesis.

- $s = t_1 \wedge t_2$ and $s = t_1 \vee t_2$. Analogously.
- $s = \langle\alpha\rangle t$. Case analysis.
  - $\alpha = 1$. Then $C \rhd t$. Thus $\mathcal{I}, C \vDash t$ by (3) of the inductive hypothesis. The claim follows.
  - α basic. Then $\langle\alpha\rangle t \in C$. By Proposition 9.1 we know that there is a path $C^{\langle\alpha\rangle t} \dots D^{\langle 1\rangle t}$ in Γ. The claim follows by (1) of the inductive hypothesis.
  - α not normal. Then $C \rhd B;\langle\beta\rangle t$ for some $B\beta \in \mathcal{D}\alpha$. Thus $\mathcal{I}, C \vDash B$ by (3) of the inductive hypothesis. Since β is normal, we obtain $\mathcal{I}, C \vDash \langle\beta\rangle t$ by arguing as in the first two cases. The claim follows with Proposition 5.3 (1).
- $s = [\alpha]t$. Let $\sigma \in \mathcal{L}\alpha$ and $C \xrightarrow{\sigma}_{\mathcal{I}} D$. By Proposition 4.1 (3) it suffices to show that $\mathcal{I}, D \vDash t$. We have $D \rhd t$ by (2) of the inductive hypothesis. Thus $\mathcal{I}, D \vDash t$ by (3) of the inductive hypothesis. ∎

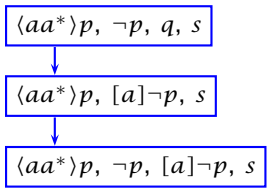
Theorem 9.4 (Model Existence) Every evident branch has a finite model.

Proof Let Γ be an evident branch. If $\Gamma = \emptyset$, the claim is trivial. Let $\Gamma \neq \emptyset$. Without loss of generality we assume that for every nominal that occurs in Γ there is a unique clause $C \in C\Gamma$ such that $x \in C$ (add clauses $\{x\}$ as necessary). Now an interpretation $\mathcal{I}$ as required by Lemma 9.3 exists. Let $C \in C\Gamma$. It suffices to show $\mathcal{I}, C \vDash C$. Let $s \in C$. Then $C \rhd s$. The claim follows with Lemma 9.3 (3). ∎

10 Soundness

We have now arrived at the crucial part of the correctness proof. Ideally, we would like to show that a satisfiable branch with an unrealized claim can always be expanded. However, this is not true.

Example 10.1 Consider the following branch where $s := [aa^*](q \vee [a]\neg p)$:



The branch is satisfiable. Still it is impossible to realize the claim for the third clause since each of the two possible expansions introduces a loop. □

Following [9], we solve the problem with the notion of a straight model. A straight model requires that all links on the branch make maximal progress

towards the fulfillment of the diamond literal they serve. Every satisfiable initial branch has a straight model, and every unrealized claim on a branch with a straight model $\mathcal{I}$ can be expanded such that $\mathcal{I}$ is a straight model of the expanded branch.

Let $\mathcal{I}$ be an interpretation and A be a set of formulas. The **depth of A and $\langle\alpha\rangle s$ in $\mathcal{I}$** is defined as

$$\delta_{\mathcal{I}} A(\langle\alpha\rangle s) := \min\{\,|\sigma| \mid \sigma \in \mathcal{L}\alpha \text{ and } \exists X, Y \in |\mathcal{I}|\colon\ \mathcal{I}, X \vDash A \text{ and } X \xrightarrow{\sigma}_{\mathcal{I}} Y \text{ and } \mathcal{I}, Y \vDash s\,\}$$

where $\min\emptyset = \infty$ and $n < \infty$ for all $n \in \mathbb{N}$. A link $C^s D^t$ is **straight** for an interpretation $\mathcal{I}$ if $\delta_{\mathcal{I}} Dt \leq \delta_{\mathcal{I}} Eu$ for every expansion E^u of C^s. A **straight model** of a quasi-branch Γ is a model of Γ such that every link $C^s D^t \in \Gamma$ is straight for $\mathcal{I}$.

Proposition 10.2 $\delta_{\mathcal{I}} As < \infty$ iff $\mathcal{I}$ satisfies $A\,;s$.

Proof Follows with Proposition 4.1. ∎

Proposition 10.3 Let $\mathcal{I}$ be a model of a quasi-branch Γ. Then $\delta_{\mathcal{I}} As = \delta_{\mathcal{I}} A^{\Gamma} s$.

Lemma 10.4 (Straightness) A quasi-branch that has a straight model contains no loops.

Proof By contradiction. Let $\mathcal{I}$ be a straight model of a quasi-branch Γ and ${C_1}^{s_1} \ldots {C_n}^{s_n}$ be a loop in Γ. Then $n \geq 2$ and ${C_1}^{s_1} = {C_n}^{s_n}$. It suffices to show that $\delta_{\mathcal{I}} C_i s_i > \delta_{\mathcal{I}} C_{i+1} s_{i+1}$ for all $i \in [1, n-1]$. Let $i \in [1, n-1]$. Then $s_i = \langle a\alpha\rangle t$, ${C_i}^{s_i} D^{s_{i+1}} \in \Gamma$, and $D^{\Gamma} = C_{i+1}$ for some a, α, t, and D. By Proposition 10.2, $\delta_{\mathcal{I}} C_i s_i < \infty$ since $\mathcal{I}$ satisfies C_i and $s_i \in C_i$. Let $\sigma \in \mathcal{L}(a\alpha)$ and $X, Y \in |\mathcal{I}|$ be such that $|\sigma| = \delta_{\mathcal{I}} C_i s_i$ and $\mathcal{I}, X \vDash C_i$ and $X \xrightarrow{\sigma}_{\mathcal{I}} Y$ and $\mathcal{I}, Y \vDash t$. Then $\sigma = Aa\tau$ for some A and some $\tau \in \mathcal{L}\alpha$. Let $Z \in |\mathcal{I}|$ be such that $X \xrightarrow{a}_{\mathcal{I}} Z$ and $Z \xrightarrow{\tau}_{\mathcal{I}} Y$. Let $B\beta \in \mathcal{D}\alpha$ such that $\tau \in \mathcal{L}(B\beta)$. Then $\mathcal{I}, Z \vDash B$ and $\mathcal{I}, Z \vDash \langle\beta\rangle t$ by Proposition 4.1 (2). Moreover, $\mathcal{I}, Z \vDash \mathcal{R}_a C_i$. Thus there is some $E \in \mathcal{D}(B\,;\langle\beta\rangle t \cup \mathcal{R}_a C_i)$ such that $\mathcal{I}, Z \vDash E$. Clearly, $E^{\langle\beta\rangle t}$ is an expansion of ${C_i}^{s_i}$ and, since $\tau \in \mathcal{L}(B\beta)$, $\delta_{\mathcal{I}} E(\langle\beta\rangle t) \leq |\tau| = |\sigma| - 1 < \delta_{\mathcal{I}} C_i s_i$. Moreover, $\delta_{\mathcal{I}} D s_{i+1} \leq \delta_{\mathcal{I}} E(\langle\beta\rangle t)$ since ${C_i}^{s_i} D^{s_{i+1}}$ is straight for $\mathcal{I}$. Hence $\delta_{\mathcal{I}} C_{i+1} s_{i+1} < \delta_{\mathcal{I}} C_i s_i$ by Proposition 10.3 since $D^{\Gamma} = C_{i+1}$. ∎

Theorem 10.5 (Soundness) Let $\mathcal{I}$ be a straight model of a branch Γ and let $\langle\alpha\rangle s \in C \in \Gamma$ such that Γ does not realize $C^{\langle\alpha\rangle s}$. Then there is an expansion D^t of $C^{\langle\alpha\rangle s}$ such that $\Gamma\,; D^{\Gamma}\,; C^{\langle\alpha\rangle s}(D^{\Gamma})^t$ is a branch and $\mathcal{I}$ is a straight model of $\Gamma\,; D^{\Gamma}\,; C^{\langle\alpha\rangle s}(D^{\Gamma})^t$.

Proof Since $\mathcal{I}$ is a model of C and $\langle\alpha\rangle s \in C$, there is an expansion D^t of $C^{\langle\alpha\rangle s}$ such that D is satisfied by $\mathcal{I}$ (by Propositions 7.1). For every such clause, $\Gamma; D^\Gamma; C^{\langle\alpha\rangle s}(D^\Gamma)^t$ is a quasi-branch that has $\mathcal{I}$ as a model (by Proposition 7.2) and satisfies the nominal coherence and functionality conditions. By Lemma 10.4, it suffices to show that we can choose D^t such that $\mathcal{I}$ is straight for $C^{\langle\alpha\rangle s}(D^\Gamma)^t$.

We choose an expansion D^t of $C^{\langle\alpha\rangle s}$ such that D is satisfied by $\mathcal{I}$ and $\delta_{\mathcal{I}} Dt$ is minimal. Let E^u be an expansion of $C^{\langle\alpha\rangle s}$. It suffices to show that $\delta_{\mathcal{I}}(D^\Gamma)t \leq \delta_{\mathcal{I}} Eu$. If $\mathcal{I}$ does not satisfy E, the claim follows by Proposition 10.2. If $\mathcal{I}$ satisfies E, we have $\delta_{\mathcal{I}} Dt \leq \delta_{\mathcal{I}} Eu$, so the claim follows by Proposition 10.3. ∎

11 Final Remarks

The main innovation of the present paper over our previous paper [9] is the notion of a finitary regular DNF. This makes it possible to cover all PDL programs and still have transparent correctness proofs.

It is straightforward to extend the clausal tableau method for HPDL to satisfaction formulas $@_x s$. To deal with such formulas, one adds an additional expansion rule at the modal level as presented in [9].

The optimizations for the modal level of clausal tableaux discussed in [9] carry over to HPDL.

Our approach yields a novel and particularly simple tableau method for hybrid logic. We are interested in extending the clausal method to difference modalities and converse modalities.

Another interesting direction for future work is the implementation of a prover based on the clausal method presented in this paper and to compare its performance to existing provers for hybrid logic and PDL.

References

[1] Pietro Abate, Rajeev Goré, and Florian Widmann. An on-the-fly tableau-based decision procedure for PDL-satisfiability. In Carlos Areces and Stéphane Demri, editors, *Proc. 5th Workshop on Methods for Modalities (M4M-5)*, volume 231 of *Electr. Notes Theor. Comput. Sci.*, pages 191–209. Elsevier, 2009.

[2] Carlos Areces and Balder ten Cate. Hybrid logics. In Patrick Blackburn, Johan van Benthem, and Frank Wolter, editors, *Handbook of Modal Logic*, volume 3 of *Studies in Logic and Practical Reasoning*, pages 821–868. Elsevier, 2007.

[3] Franz Baader. Augmenting concept languages by transitive closure of roles: An alternative to terminological cycles. Technical Report RR-90-13, DFKI, 1990.

[4] Gérard Berry and Ravi Sethi. From regular expressions to deterministic automata. *Theor. Comput. Sci.*, 48(3):117–126, 1986.

[5] Giuseppe De Giacomo and Fabio Massacci. Combining deduction and model checking into tableaux and algorithms for converse-PDL. *Inf. Comput.*, 162(1–2):117–137, 2000.

[6] Michael J. Fischer and Richard E. Ladner. Propositional dynamic logic of regular programs. *J. Comput. System Sci.*, pages 194–211, 1979.

[7] Rajeev Goré and Florian Widmann. An optimal on-the-fly tableau-based decision procedure for PDL-satisfiability. In Renate A. Schmidt, editor, *CADE 2009*, volume 5663 of *LNCS*, pages 437–452. Springer, 2009.

[8] David Harel, Dexter Kozen, and Jerzy Tiuryn. *Dynamic Logic*. The MIT Press, 2000.

[9] Mark Kaminski and Gert Smolka. Terminating tableaux for hybrid logic with eventualities. In *IJCAR 2010*. Springer, 2010. To appear.

[10] Dexter Kozen and Frederick Smith. Kleene algebra with tests: Completeness and decidability. In Dirk van Dalen and Marc Bezem, editors, *CSL'96*, volume 1258 of *LNCS*, pages 244–259. Springer, 1996.

[11] Dexter Kozen and Jerzy Tiuryn. Logics of programs. In *Handbook of Theoretical Computer Science, Volume B: Formal Models and Sematics*, pages 789–840. Elsevier, 1990.

[12] Solomon Passy and Tinko Tinchev. PDL with data constants. *Inf. Process. Lett.*, 20(1):35–41, 1985.

[13] Solomon Passy and Tinko Tinchev. An essay in combinatory dynamic logic. *Inf. Comput.*, 93(2):263–332, 1991.

[14] Dominique Perrin. Finite automata. In *Handbook of Theoretical Computer Science, Volume B: Formal Models and Sematics*, pages 1–57. Elsevier, 1990.

[15] Vaughan R. Pratt. A near-optimal method for reasoning about action. *J. Comput. System Sci.*, 20(2):231–254, 1980.

[16] Ulrike Sattler and Moshe Y. Vardi. The hybrid μ-calculus. In Rajeev Goré, Alexander Leitsch, and Tobias Nipkow, editors, *IJCAR 2001*, volume 2083 of *LNCS*, pages 76–91. Springer, 2001.