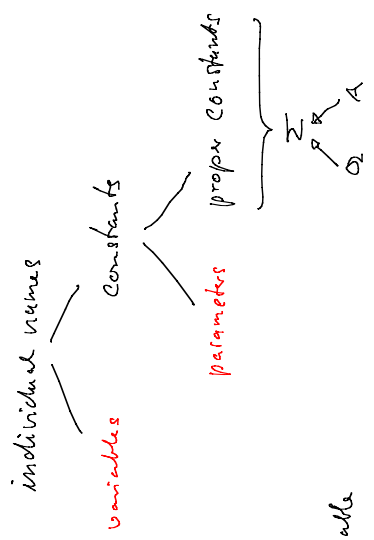


2006-6-14



variable $\hat{=}$ argument variable
 free variable $\hat{=}$ dangling argument pointer
 parameter $\hat{=}$ global variable

Examples

Specification: BA
 Constants: 0, 1, -, +, .
 Variables: x, y, z
 Parameters: a, b, c

$x + 0 = x$
 $\lambda x. x + 0 = \lambda x. x$ \leftarrow *x occurs in the description but not in the term*
 BA, $a = b \vdash a = a \cdot b \cdot b \cdot a$
 BA, $x = y \vdash 0 = 1$
 BA, $a = b \vdash 0 = 1$

Notational Conventions

$e_1, e_2 \vdash e_3 \rightsquigarrow \{e_1, e_2\} \vdash \{e_3\}$
 $\emptyset A \rightsquigarrow \emptyset A$
 $\emptyset x \rightsquigarrow \emptyset x$
 $\emptyset n \rightsquigarrow \emptyset n$
 $\eta \vDash A \rightsquigarrow \forall x \in A: \eta \vDash x$

Closed Specifications

A closed $\iff \mathcal{N}A \cap \text{Var} = \emptyset$

Extensionality $\rho = \epsilon \vdash \lambda x. \lambda = \lambda x. \epsilon$

\implies variables $\hat{=}$ dangling argument references
 \implies every specification is equivalent to a closed specification
 \implies open specifications are a notational convenience

Closed specs have nice properties

Let A be closed. Then:

- 1) $A \models e \Leftrightarrow \forall y: \neg(A \Rightarrow \neg y \models e)$
- 2) $A \models e \Rightarrow \exists A \models \emptyset \models e$
- 3) $A \models e \Rightarrow \exists A \vdash \emptyset \models e$

Semantic entailment

Stability

- \Rightarrow Generativity is a consequence of stability
- \Rightarrow Can replace vars with params and vice versa in $A \models e$ and $A \vdash e$
- \Rightarrow Variables are not essential for $A \models e$, $A \vdash e$
- \Rightarrow Generativity and stability for open specs are consequences of stability for closed specs

Example of Gödel-style proof

$\downarrow x = a \vdash \downarrow y = a$ (f, a constants, x variable)

1	$\downarrow x = a$	
2	$\lambda x. \downarrow x = \lambda x. a$	Σ 1
3	$(\lambda x. \downarrow x) y = (\lambda x. a) y$	C/R 2
4	$(\lambda x. \downarrow x) y = \downarrow y$	β
5	$(\lambda x. a) y = a$	β
6	$\downarrow y = (\lambda x. \downarrow x) y$	Sym 4
7	$\downarrow y = a$	Trans 6, 3
8	$\downarrow y = a$	Trans 7, 5

Formal proofs

- Gödel-style proofs (derivations)
 - conversion proofs
- Compile \leftarrow

Recall

$A \vdash e \Leftrightarrow \exists$ Gödel-style proof of e from A

Soundness: $A \vdash e \Rightarrow A \models e$

Def Gödel-style proof

A Gödel-style proof of e from A is a tuple (e_1, \dots, e_n) and that

1) $e_n = e$

2) $\forall i \in \{1, \dots, n\}$:

$e_i \in A$ or

exists instance (E, e_i) of a deduction rule

such that $E \subseteq \{e_1, \dots, e_{i-1}\}$

This definition will work for every set of inference rules

Example of conversion proof

$$\mathcal{B}A \vdash x = x \cdot x$$

x	$x \cdot 1$
x	$x(x + \bar{x})$
x	$x(x + x\bar{x})$
x	$x(x + 0)$
x	xx

Id

Compl

Dist

Compl

Id

$$x = x \cdot 1$$

$$x + \bar{x} = 1$$

$$x(y+z) = xy + xz$$

$$x\bar{x} = 0$$

$$x + 0 = x$$

Sym, C/R

Associativity

C/R

Associativity

Def of conversion steps

A λ -conversion step is an equation $r = t$ and that $r = t$ or $t = 0$ is a λ -reduction step

An A -conversion step is an equation $r = t$ and that $r = t$ or $t = r$ is an A -reduction step

Sym

Definition of conversion proof

A conversion proof of e from A is a tuple $(\alpha_1, \dots, \alpha_n)$ such that

1) $e = (\alpha_1, \alpha_n)$

2) $\forall i \in \{1, \dots, n-1\}$:

(α_i, α_{i+1}) is a λ -conversion step
or an A -conversion step

Trans, Def

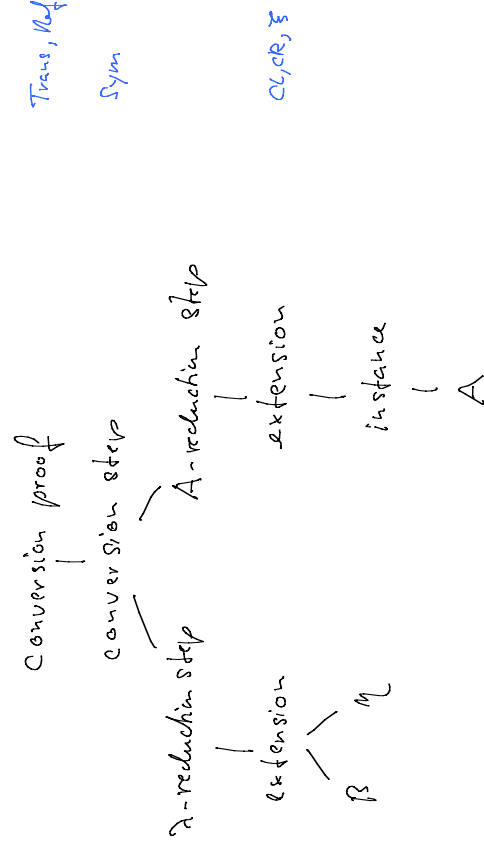
Definition of λ -reduction steps

- A β -reduction step is an extension of an equation $(\lambda x. r) t = \rho[x := t]$
- An η -reduction step is an extension of an equation $\lambda x. \rho x = \rho$ where $x \notin N\rho$
- A λ -reduction step is a β - or an η -reduction step

β, η

Definition of A-reduction steps

An A-reduction step is an extension of an instance of an equation in A



Extensions and instances

- An extension of e is an equation that can be derived from e with CL, CR, E

$$\begin{aligned} & x = a \\ & f x = f a \\ & \lambda x. f x = \lambda x. f a \\ & (\lambda x. f x) a = (\lambda x. f a) a \end{aligned}$$

- An instance of e is an equation Θe where $K \subseteq \text{Var}$

$$\begin{aligned} & f x = y \\ & f a = y \\ & (\lambda x. x) x = y \\ & (\lambda x. x) a = f y \end{aligned}$$

($\beta, E, CL, CR, \text{Sym}, \text{Trans}$) Generativity

\exists Gödel-style proof of e from A
 $\Leftrightarrow \exists$ conversion proof of e from A

Proof \Leftarrow easy
 \Rightarrow shows for every instance (E, e') of a deduction rule:
 $\forall e'' \in E: (\exists \text{ conversion proof of } e'' \text{ from } A)$
 $\Rightarrow (\exists \text{ conversion proof of } e' \text{ from } A)$