Assignment 2
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapter 2

Exercise 2.1 Give all subterms of the term $\lambda x.fxx$. For each subterm give a corresponding context. Is there a subterm with more than one corresponding context?

Exercise 2.2 Determine all pairs C, s such that $C[s] = xxxx$ and s is a application.

Exercise 2.3 We say that a name x occurs **bound** in a term s if s has a subterm $\lambda x.t$ such that x is free in t. Give a term s such that x is free in s and also occurs bound in s.

Exercise 2.4 Apply the following substitutions.

a) $((\lambda x.y)y)^y_x$

b) $(\lambda x.y)^y_{fxy}$

c) $(\lambda x.y)^x_{fxy}$

Exercise 2.5 Which of the following terms are α-equivalent?

$$\lambda xyz.xyz, \quad \lambda yxz.yxz, \quad \lambda zyx.zyx, \quad \lambda xyz.zyx, \quad \lambda yxz.zxy$$

Exercise 2.6 Determine $S_0\epsilon t$ for the following terms t. Assume $x \cong 0$, $y \cong 1$, and $z \cong 2$.

a) $\lambda z.z$

b) $\lambda yx.yx$

c) $\lambda xy.yx$

d) $\lambda xy.y$

e) $\lambda zxy.xyz$

f) $\lambda z.x$

Exercise 2.7 Find counterexamples that falsify the following statements.

a) $\lambda x.s \sim_\alpha \lambda y.t \iff \exists z : s^x_z \sim_\alpha t^y_z$

b) $\lambda x.s \sim_\alpha \lambda y.t \iff s^x_y \sim_\alpha t$

Exercise 2.8 Give the β-normal forms of the following terms.

a) $(\lambda xy.fyx)ab$
b) $(\lambda fxy.fyx)(\lambda xy.yx)ab$
c) $(\lambda x.xx)((\lambda xy.y)((\lambda xy.x)ab))$
d) $(\lambda xy.y)((\lambda x.xx)(\lambda x.xx))a$
e) $(\lambda xx.x)yz$

Exercise 2.9 Give the $\beta\eta$-normal forms of the following terms.

a) $\lambda xy.fx$
b) $\lambda xy.fy$
c) $\lambda xy.fxy$

Exercise 2.10 Determine all pairs C, s such that $C[s] = \lambda xyz.(\lambda x.x)yzx$ and s is a β- or η-redex.

Exercise 2.11 Find terms as follows.

a) A term that has no β-normal form.
b) A term that has a β-normal form but is not terminating.

Exercise 2.12

a) Find a term that has more than one β-normal form.
b) Find a term s such that there infinitely many terms t such that $s \to_\beta^* t$.

Exercise 2.13 For each of the following terms finds types for the names occurring in the term such that the term becomes well-typed.

a) $\lambda xy.x$
b) $\lambda f.fyx$
c) $\lambda fgx.fx(gx)$

Exercise 2.14 Find closed terms that have the following types.

a) $\alpha\alpha$
b) $\alpha\beta\alpha$
c) $(\alpha\beta)(\beta\gamma)\alpha\gamma$
d) $\alpha(\alpha\beta)\beta$

Exercise 2.15 Find terms s, t such that $s \to_\beta t$, s is ill-typed, and t is well-typed.