Assignment 3
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapters 3 and 4

Exercise 3.1 Give finite relations $\to$ such that:

a) $\to$ is confluent but not terminating.
b) $\to$ is terminating but not confluent.
c) $\to$ is not confluent and not terminating.
d) $\to$ is confluent, does not terminate on x, and y is a $\to$-normal form of x.

Exercise 3.2 Consider the relation $\to := \{ (x, y) \in \mathbb{N}^2 \mid 2 \le 2y \le x \}$.

a) Is $\to$ terminating?
b) Is $\to$ confluent?
c) Give a $\to$-normal form of 7.
d) Give all $\to$-normal $n \in \mathbb{N}$.

Exercise 3.3 A relation $\to$ is *locally confluent* if for all x, y, z: $x \to y \wedge x \to z \Longrightarrow y \downarrow z$. Find a finite relation that is locally confluent but not confluent.

Exercise 3.4 Find terms s, t such that $s \to_\beta t$, s contains no η-redex, and t contains an η-redex.

Exercise 3.5 Which condition in the definition of interpretations ensures that sorts are interpreted as non-empty sets?

Exercise 3.6 (Multiplication) Extend the specification of the natural number with a formula that specifies the name $\cdot : NNN$ as multiplication.

Exercise 3.7 (Pairs) Let the names $\mathsf{pair} : \sigma\tau P$, $\mathsf{fst} : P\sigma$, and $\mathsf{snd} : P\tau$ be given. Find a formula that is satisfied by a logical interpretation $\mathcal{I}$ if and only if $\mathcal{I}P \cong \mathcal{I}\sigma \times \mathcal{I}\tau$ and pair, fst, and snd are interpreted as the pairing and projection functions.

Exercise 3.8 (Termination) Let $r : \alpha\alpha B$ be a name. Find a formula that is satisfied by a logical interpretation $\mathcal{I}$ if and only if $\mathcal{I}r$ is the functional coding of a terminating relation.

Exercise 3.9 (Finiteness) Let $f : \sigma\sigma$ be a name.

a) Find a term $\mathsf{injective} : (\sigma\sigma)B$ such that a logical interpretation satisfies the formula $\mathsf{injective}\, f$ if and only if it interprets f as an injective function.

b) Find a term $\mathsf{surjective} : (\sigma\sigma)B$ such that a logical interpretation satisfies the formula $\mathsf{surjective}\, f$ if and only if it interprets f as a surjective function.

c) Find a formula finite that is satisfied by a logical interpretation $\mathcal{I}$ if and only if $\mathcal{I}\sigma$ is a finite set.

Exercise 3.10 (Lists) Let the names $\mathsf{nil} : L$, $\mathsf{cons} : \sigma LL$, $\mathsf{hd} : L\sigma$, and $\mathsf{tl} : LL$ be given. Find a formula that is satisfied by a logical interpretation $\mathcal{I}$ if and only if L represents all lists over σ and nil, cons, hd, and tl represent the list operations. Make sure that L contains no junk elements.