Assignment 4
Introduction to Computational Logic, SS 2008

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www.ps.uni-sb.de/courses/cl-ss08/

Read in the lecture notes: Chapter 5

Exercise 4.1 Prove the following statements without using derived rules.

a) $x = y, y = z \vdash x = z$

b) $\vdash x = y \to y = z \to x = z$

c) $s \to t, s \vdash t$

d) $s^x_\bot, s^x_\top \vdash s$

e) $\vdash x \to y \to x$

f) $\vdash x \to (x \to y) \to y$

Exercise 4.2 Show that the following proof steps are admissible without using derived rules. (Note: Here $\vdash$ is used as the sequent constructor.)

a)

$$\text{Ded}^- \frac{A \vdash s \to t}{A, s \vdash t}$$

b)

$$\text{Lam'} \frac{A, s \vdash t}{A, s' \vdash t}\ s \sim_\lambda s'$$

c)

$$\frac{s \vdash t \qquad A \vdash s}{A \vdash t}$$

d)

$$\frac{s_1, s_2 \vdash t \qquad A \vdash s_1 \qquad A \vdash s_2}{A \vdash t}$$

e)

$$\frac{A \vdash s}{A \vdash \theta(s)}\ \theta \text{ admissible for } A$$

Exercise 4.3 Prove the following statements. Use BCAR but no other derived rules.

a) $\vdash (x \to y) \to (y \to x) \to (x \equiv y)$

b) $\vdash x \to \neg\neg x$

c) $\vdash \neg\neg x \to x$

d) $\vdash \neg\neg x \equiv x$

e) The following proof step is admissible:

$$\text{Contra}\ \frac{A, \neg s \vdash \bot}{A \vdash s}$$

f) $\bot \vdash x$

Exercise 4.4 Prove the following.

a) If $s \vdash t$, then the following proof step is admissible:

$$\frac{A \vdash s}{A \vdash t}$$

b) If $s, t \vdash u$, then the following proof step is admissible:

$$\frac{A \vdash s \qquad A \vdash t}{A \vdash u}$$