Assignment 5
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapter 5

Exercise 5.1 Consider the proof system

$$S = \{(\{x, y\}, z) \mid x, y, z \in \mathbb{N} \wedge x \cdot y = z\}.$$

a) Determine $S[\emptyset]$.

b) Determine $S[\{2\}]$.

c) Give a proof step $(\{x\}, y) \in S$ (one premise only).

d) Derive the proof step $(\{2, 3\}, 12)$.

e) Does $S[\{2x \mid x \in \mathbb{N}\}]$ contain an odd number?

Exercise 5.2 Derive the following rules in the basic proof system.

a)

$$\textbf{Abs} \quad \frac{A \vdash fx = s}{A \vdash f = \lambda x.s} \; x \notin \mathcal{N}A$$

b)

$$\textbf{Rep} \quad \frac{A \vdash s = t \qquad A \vdash C[\theta t]}{A \vdash C[\theta s]} \; C \text{ and } \theta \text{ admissible for } A$$

Exercise 5.3 The exercise concerns the natural deduction system ND for Predicate Logic.

a) Make sure you understand the rules.

b) Derive $\vdash (\forall x.fx) \to fx$

c) Derive $\vdash fx \to \exists x.fx$

d) Derive $\exists x.\forall y.fxy \vdash \forall y.\exists x.fxy$

Exercise 5.4 Let $f : BBB$, $x : B$, and $y : B$ be names. Find an interpretation that shows that

$$(\forall y.\exists x.fxy) \to \exists x.\forall y.fxy$$

is not valid.

Exercise 5.5 Find an A and an interpretation that shows that

$$\frac{A \vdash fx}{A \vdash \forall x.fx}$$

is not sound for $\vDash$. You can choose the type of f.