Assignment 6
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapter 6

Exercise 6.1 Prove the following rules are derivable. You may use any of the basic or derived rules from Chapter 5.

a) Prove **Contra⁻** is derivable. (Hint: It is enough to use **Triv**, **Weak**, **Sub**, **D¬**, **Rew** and **MP**.)

$$\textbf{Contra}^{-}\ \frac{A \vdash s}{A, \neg s \vdash \bot}$$

b) Prove the **Imp** rule is derivable. (Hint: Use **Contra**, **MP**, **Cut** and **Triv**.)

$$\textbf{Imp}\ \frac{A, t \vdash \bot \qquad A, \neg s \vdash \bot}{A \vdash \bot}\ s \to t \in A,\ t \notin A,\ \neg s \notin A$$

c) Prove the **NegImp** rule is derivable. (Hint: Use **Contra**, **Contra⁻** and **Ded**.)

$$\textbf{NegImp}\ \frac{A, s, \neg t \vdash \bot}{A \vdash \bot}\ \neg(s \to t) \in A,\ \{s, \neg t\} \notin A$$

Exercise 6.2 Which of the following are instances of the refutation rule **NegImp**.

a)

$$\frac{\neg p, p, \neg q \vdash \bot}{\neg p, \neg(p \to q) \vdash \bot}$$

b)

$$\frac{\neg p, \neg(p \to q), p, \neg q \vdash \bot}{\neg p, \neg(p \to q) \vdash \bot}$$

c)

$$\frac{\neg(p \to q), p, \neg q \vdash \bot}{\neg(p \to q), p \vdash \bot}$$

d)

$$\frac{\neg(p \to q), p, \neg q \vdash \bot}{\neg(p \to q), p, \neg q \vdash \bot}$$

Exercise 6.3 Let p, q, r be names of type B. For each of the following propositional sequents, use tableaux to determine if the sequent is valid or satisfiable. If the sequent is valid, give a tableau for the sequent where every branch is closed. If the sequent is satisfiable, give a satisfying assignment. **You should use Jitpro to solve the problem, but make sure you can also solve the problem by hand. You will need to solve such problems by hand on the exam!**

a) $\vdash \neg(p \to p) \to q \to p$

b) $\vdash \neg(q \to p) \to q \to p$

c) $\vdash \neg(p \to q) \to q \to p$

d) $\vdash (p \to q) \vee (q \to p)$

e) $\vdash (p \to r) \to (p \wedge r \to q) \to (r \to q)$

f) $\vdash (p \to q) \to (p \wedge r \to q) \to (r \to q)$

g) $\vdash (r \to p) \to (p \wedge r \to q) \to (r \to q)$

h) $\vdash p \to q \equiv \neg p \vee q$

i) $\vdash p \to q \equiv \neg q \to \neg p$