Assignment 8
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapter 6

Exercise 8.1 Recall that we defined $V_0 = \emptyset$, $V_{n+1} = \wp(V_n)$ and $V_\omega = \bigcup_{n\in\omega} V_n$. Find a set $X \in V_\omega$ such that $X \notin \wp(X)$.

Exercise 8.2 Make sure you can prove the following version of Cantor's theorem in the tableau system:

$$\neg\exists g.\forall f.\exists x.gx = f$$

where $g : IIB$ is a name, $f : IB$ is a name, and $x : I$ is a name. (See Jitpro Exercise 4.2, but BE CAREFUL: Jitpro does some β-reductions implicitly that you must do explicitly!)

Exercise 8.3 Prove the following three statements are equivalent:

(1) X is transitive.

(2) For every $A \in X$ we know $A \subseteq X$.

(3) $X \subseteq \wp(X)$.

You can prove these are equivalent by proving (1) implies (2), proving (2) implies (3), and proving (3) implies (1). After proving these three facts, formally prove them using Jitpro (see Jitpro Exercise 4.3).

Exercise 8.4 If X is transitive, then $\wp(X)$ is transitive. After proving this on paper, formally prove it using Jitpro (see Jitpro Exercise 4.4).

Exercise 8.5 Suppose I is a sort with $I \neq B$ and $\in: IIB$ is a name. As in the lecture notes, we write $\in$ in infix notation. Let p be a name of type IB and let x and y be names of type I. Give a tableau refutation of

$$A = \{\forall p.\exists y.\forall x.x \in y \equiv px\}$$

(See Jitpro Exercise 4.5.)

Exercise 8.6 Let $p : BB$ be a name and $a, b : B$ be names. Give a tableau proof of $p(a \wedge b) \to p(b \wedge a)$. (See Jitpro Exercise 4.7.)