Assignment 9
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapter 6

Exercise 9.1 (Ping Pong) Let I be a sort, $f, g : II$ be variables, and $x, y : I$ be variables. Give a tableau proof of

$$(\exists f.\forall y.f(gy) = y) \to \exists f.\forall y.\exists x.fx = y$$

(See Jitpro Exercise 5.1)

Exercise 9.2 (Double Instantiation) Let I be a sort, $f : IIB$ be a variable, $h : (IB)B$ be a variable, $g : IB$ be a variable, and $x : I$ be a variable. Give a tableau proof of

$$\exists g.\forall x.\exists h.h(fx) \land \neg hg$$

Hint: You will need to instantiate h twice. (See Jitpro Exercise 5.2)

Exercise 9.3 (Kaminski Equation (Special Case)) Let $f : BB$ be a variable. Give a tableau proof of

$$f(f(f\bot)) = f\bot$$

assuming the lemma

$$\forall p.p = \top \lor p = \bot.$$

In other words, complete the following tableau:

$$f(f(f\bot)) \neq f\bot$$
$$\forall p.p = \top \lor p = \bot$$

(See Jitpro Exercise 5.3)

Exercise 9.4 (Boolean Connectives) In this exercise you will use tableau to formally prove two of the solutions to Exercise 1.2 of Assignment Sheet 1 are correct. (See Jitpro Exercise 5.4) Let $x, y : B$ be variables.

a) Give a tableau proof of

$$\forall xy.\mathsf{and}xy = (x \land y)$$

where and is notation for

$$\lambda xy.\mathsf{neg}(x \to \mathsf{neg}y)$$

and neg is notation for

$$\lambda x.x \to \bot.$$

b) Give a tableau proof of

$$\forall xy.\mathsf{imp}xy = (x \to y)$$

where imp is notation for

$$\lambda xy.\neg x \lor y.$$

Exercise 9.5 (Sets As Functions) In this exercise you will formally solve two parts of Exercise 1.3 of Assignment Sheet 1. (See Jitpro Exercise 5.5) Let X be a sort, $f, g : XB$ be variables, $x : X$ be a variable, union : $(XB)(XB)XB$ be a variable, and subseteq : $(XB)(XB)B$ be a variable.

a) Give a tableau proof of

$$\exists \mathsf{union}.\forall fgx.\mathsf{union}fgx = (fx \lor gx)$$

b) Give a tableau proof of

$$\exists \mathsf{subseteq}.\forall fg.\mathsf{subseteq}fg = \forall x.fx \to gx$$

Exercise 9.6 (Identities and Quantifiers) In this exercise you will use tableau to formally prove four of the solutions to Exercise 1.4 of Assignment Sheet 1 are correct. (See Jitpro Exercise 5.6.) Let X and Y be sorts, $f : XB$ be a variable, $x, y : X$ be variables, and $g, h : XY$ be variables.

a) Give a tableau proof of

$$\forall f.\mathsf{allX}f = \forall x.fx$$

where allX is notation for $\lambda f.f = \lambda x.\top$.

b) Give a tableau proof of

$$\forall f.\mathsf{exX}f = \exists x.fx$$

where exX is notation for $\lambda f.\neg\forall x.\neg fx$.

c) Give a tableau proof of

$$\forall gh.\mathsf{eqXY}gh = (g = h)$$

where exXY is notation for $\lambda gh.\forall x.gx = hx$.

d) Give a tableau proof of

$$\forall xy.\mathsf{eqX}xy = (x = y)$$

where exX is notation for $\lambda xy.\forall f.fx \to fy$.

Exercise 9.7 (Henkin's Reduction) In this exercise you will use tableau to formally prove two of the solutions to Exercise 1.5 of Assignment Sheet 1 are correct. (See Jitpro Exercise 5.7)

a) Give a tableau proof of

$$\neg\mathsf{False}$$

where False is notation for $(\lambda x.x) = \lambda x.\top$

b) Give a tableau proof of

$$\forall xy.\mathsf{and}xy = (x \wedge y)$$

where and is notation for $(\lambda g.gxy) = \lambda g.g\top\top$.