Assignment 11
Introduction to Computational Logic, SS 2008

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Read in the lecture notes: Chapter 9

Exercise 11.1 Translate the following modal formulas into first-order formulas:

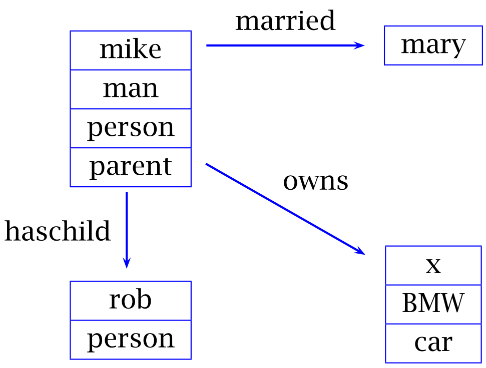
a) $(p \dot\vee q)x$

b) $\forall(p \dot\to q)$

c) $(\Box r(p \dot\to \Diamond r p))x$

d) $(\Box r \Diamond r \dot\neg p)x$

Exercise 11.2 List all the primitive formulas given by this transition graph:



Exercise 11.3 Does every Kripke set contain an individual variable?

Exercise 11.4 For each of the following formulas find a Kripke set satisfying it.

a) $(\Box r(p \dot\vee q) \dot\to \Box r p \dot\vee \Box r q)x$

b) $\neg(\Box r(p \dot\vee q) \dot\to \Box r p \dot\vee \Box r q)x$

c) $(\Diamond r p \dot\wedge \Diamond r q \dot\to \Diamond r(p \dot\wedge q))x$

Exercise 11.5 For each of the following formulas find a Kripke set that doesn't satisfy the formula. Your sets should employ only two individual variables. First draw the sets as transition diagrams and then list them explicity.

a) $\Box r(p \dot\vee q) \;=\; \Box r p \dot\vee \Box r q$

b) $\Diamond r(p \dot\wedge q) \;=\; \Diamond r p \dot\wedge \Diamond r q$

Exercise 11.6 Which of the following formulas is modally valid? If the formula is not valid, find a Kripke set that doesn't satisfy it.

a) $\forall\,(\Diamond r(p \dot\wedge q) \dot\to \Diamond rp \dot\wedge \Diamond rq)$

b) $\forall\,(\Box r(p \dot\vee q) \dot\to \Box rp \dot\vee \Box rq)$

c) $\Box r \dot\top = \dot\top$

d) $\Diamond r \dot\bot = \dot\top$

e) $\Diamond r \dot\bot = \dot\bot$

f) $\Diamond r \dot\top = \Box rp \dot\to \Diamond rp$

Exercise 11.7 (Model Checking) There is a straightforward procedure that checks whether a finite Kripke set satisfies a modal formula. We name the procedure $\Vdash$ and write $K \Vdash s$ for its application to a Kripke set K and a modal formula s. Complete the following equations so that they fully define the procedure $\Vdash$.

$$
\begin{aligned}
K \Vdash px &= (px \in K) \\
K \Vdash \dot\bot x &= 0 \\
K \Vdash \dot\top x &= 1 \\
K \Vdash \dot\neg tx &= \neg(K \Vdash tx) \\
K \Vdash (t_1 \dot\wedge t_2)x &= (K \Vdash t_1 x \wedge K \Vdash t_1 x) \\
K \Vdash \Diamond rtx &= (\exists y \in \mathcal{N}K\colon\ rxy \in K \wedge K \Vdash ty)
\end{aligned}
$$

Exercise 11.8 For each of the following formulas s, show $\mathrm{M} \vdash s$ with a tableau proof.

a) $\Diamond r \dot\top = \Box rp \dot\to \Diamond rp$

b) $\dot\neg \Box rt = \Diamond r \dot\neg t$

c) $\forall(\Box r(p \dot\to q) \dot\to \Box rp \dot\to \Box rq)$

Note: Technically each expansion of the modal constant using its defining equation requires an application of the **Apply=** rule. For this exercise we allow you to freely expand the definitions of any of the modal constants and β-normalize in a single step. You also do not need to explicitly include the defining equations for the modal constants in the tableau.

Exercise 11.9 Give a procedure that yields for every modal term an equivalent negation-normal term. Write the procedure as a system of terminating equations.