Constraint Programming: Assignment no. 4

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In this week's assignment, the exercises will help you build intuition for propagators, fixpoints, and the main propagation loop as introduced in the lecture.

Exercise 4.1 (Propagator rewriting)

In some situations, a propagator p can be replaced by a set Q of simpler propagators.

a) Change the propagation algorithm from the lecture to support rewriting of propagators. The basic idea is to not only return a status message and the resulting store, but also the set Q.
b) What property is Q required to have with respect to p?

Exercise 4.2 (Propagator for maximum)

In the lecture, you have seen propagators for inequality ($p_{\leq}$), equality ($p_{=}$) and disequality ($p_{\neq}$). Along the same lines, define a propagator for $\max(x, y) = z$.

a) Define the propagator, taking subsumption into account.
b) Is your propagator idempotent?
c) In which situations can the propagator be rewritten to a simpler propagator?

Exercise 4.3 (Propagators for addition)

We want to define a set of propagators that implements the constraint $x + y = z$. For each variable, we will define one propagator that changes that variable based on the domains of the two others. For example, a propagator $p_{+,x}$ would use the domains of y and z as input and narrow the domain of x.

a) Define propagators $p_{+,x}, p_{+,y}, p_{+,z}$ that only propagate if the input variables are assigned.
b) Define propagators $p_{+,x}, p_{+,y}, p_{+,z}$ that only use the bounds of the input variables, i.e. their maximum and minimum.
c) Define propagators $p_{+,x}, p_{+,y}, p_{+,z}$ that use the full domains of the input variables.

Exercise 4.4 (Changing order of propagation)

The main reason why we require propagators to be monotonic is that this guarantees us *unique fixpoints*. That means, the fixpoint does not depend on the order in which propagators are executed.

One might thus come to the conclusion that for all constraint stores s and two propagators p_1 and p_2,

$$p_1(p_2(s)) = p_2(p_1(s))$$

Provide a proof or a counterexample for this statement!

Exercise 4.5 (Idempotent propagators)

a) Prove that for two propagators p_1 and p_2, $p_1 \circ p_2$ (i.e. the functional composition) is again a propagator.
b) Is the following true? For any given propagator p, there exists $n \in \mathbb{N}$ such that p^n is idempotent. Give a proof or a counterexample.

Exercise 4.6 (Propagating on paper)

Consider an initial store $s = \{x \mapsto \{0, 1, 2, 3\}, y \mapsto \{0, 1, 2, 3\}\}$ and two propagators for $x < y$ and $y < x$. Exercise the propagation algorithm from the lecture using pen and paper.

Exercise 4.7 (Generalized linear equations)

This exercise is difficult. Don't become desperate, only solve it if you feel like a little challenge!

Define a propagator for an arbitrary linear equation $\sum_{i=1}^{n} a_i x_i = c$, where the a_i and c are constant integers and the x_i are integer variables. Hints:

a) Generalize the $x + y = z$ propagator that uses only bounds information to an arbitrary number of variables, but without coefficients.
b) Generalize the $x + y = z$ propagator that uses only bounds information to $ax + by - cz = 0$. Pay particular attention to rounding and sign!
c) Now you should know how to combine the ideas you used for the previous two propagators into a general propagator for linear equations.