Constraint Programming

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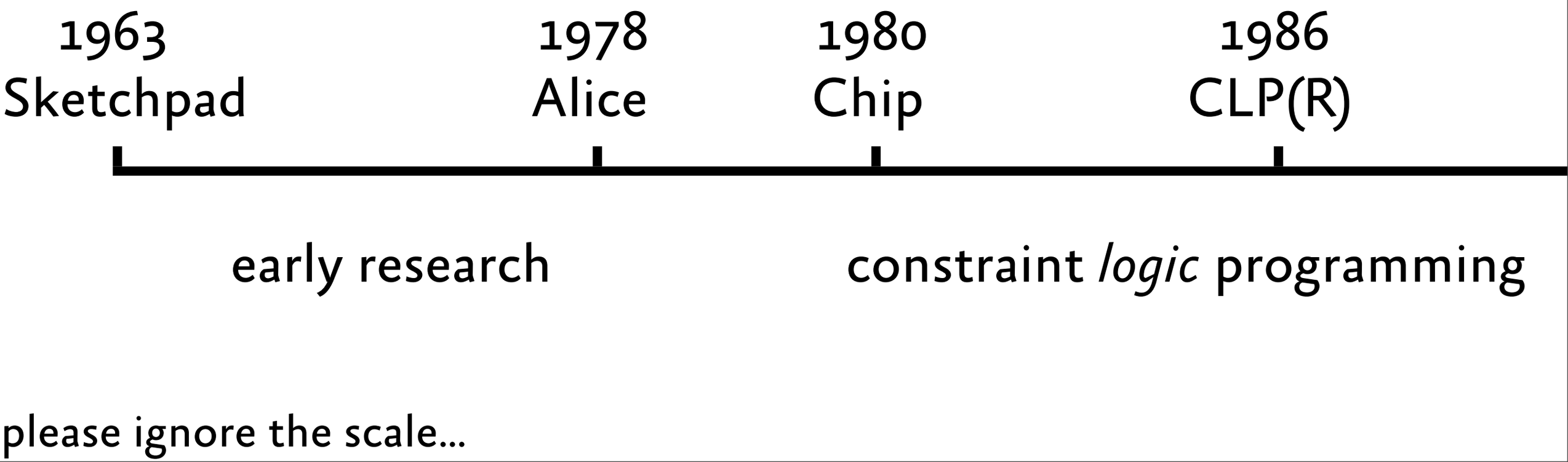
Lecture 2

Today:
History and Practice

History and Practice

- **Part I: Short historical overview**
  - where does CP come from?
- **Part II: Constraint Programming with Gecode/J**
  - give you intuition about what's under the hood
  - scratch all topics we will discuss in this course

Historical notes



please ignore the scale...

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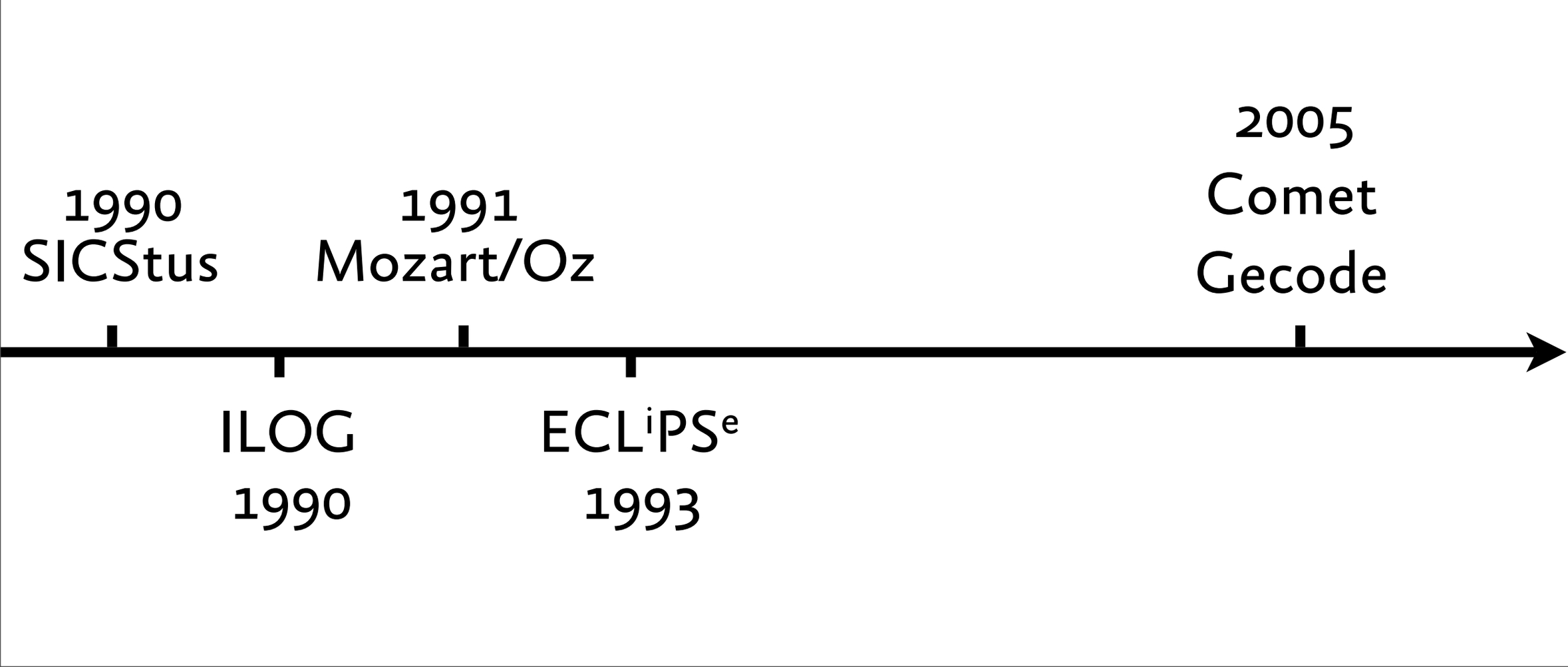

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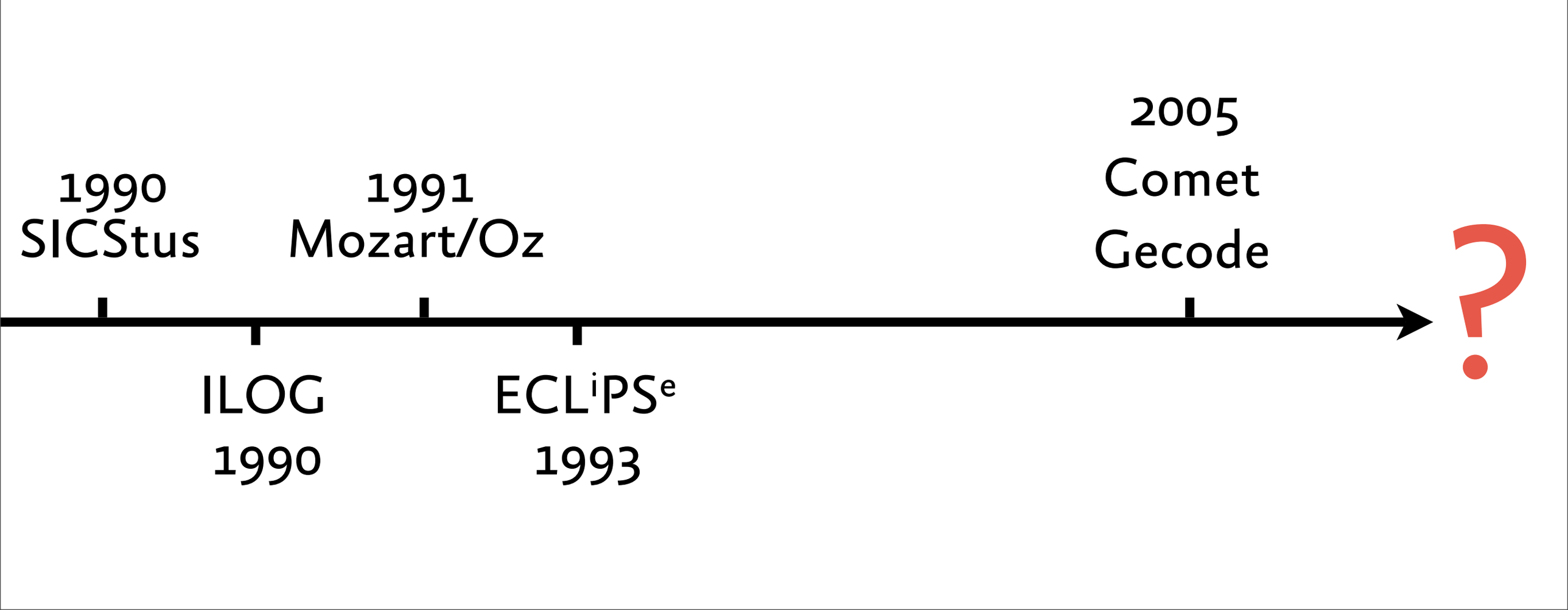

Logic Programming

```
foo(a).
foo(b).
g(X) :- X=[Y, Z], foo(Y), foo(Z).
```

Logic Programming

```
foo(a).
foo(b).
g(X) :- X=[Y, Z], foo(Y), foo(Z).


| ?- g(X).
X = [a,a] ? ;
X = [a,b] ? ;
X = [b,a] ? ;
X = [b,b]
```

Constraint Logic Programming

```
foo(X) :- fd_domain(X, 1, 3).
g(Y,Z) :- foo(Y), foo(Z), Y #< Z,
          fd_labeling([Y,Z]).
```

Constraint Logic Programming

```
foo(X) :- fd_domain(X, 1, 3).
g(Y,Z) :- foo(Y), foo(Z), Y #< Z,
          fd_labeling([Y,Z]).
| ?- g(X,Y).
X = 1
Y = 2 ? ;
X = 1
Y = 3 ? ;
X = 2
Y = 3
```

Constraint Logic Programming

```
foo(X) :- fd_domain(X, 1, 3).
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| ?- g(X,Y).
X = 1
Y = 2 ? ;
X = 1
Y = 3 ? ;
X = 2
Y = 3
```

Model:

constraint program

=

logic program

=

logical formula

Constraint Logic Programming

```
foo(X) :- fd_domain(X, 1, 3).
g(Y,Z) :- foo(Y), foo(Z), Y #< Z,
          fd_labeling([Y,Z]).
| ?- g(X,Y).
X = 1
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Y = 3 ? ;
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Y = 3
```

Languages/Systems:

GNU Prolog, BProlog, SICStus Prolog, ECL^{i}PS^{e}

Concurrent Constraint Programming

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- Von-Neumann architecture: store *values*
  - operations: `read` and `write`

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- cc architecture: store *constraints*
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  - communication through variables

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- Languages/systems:
  - cc(FD), AKL, Mozart/Oz

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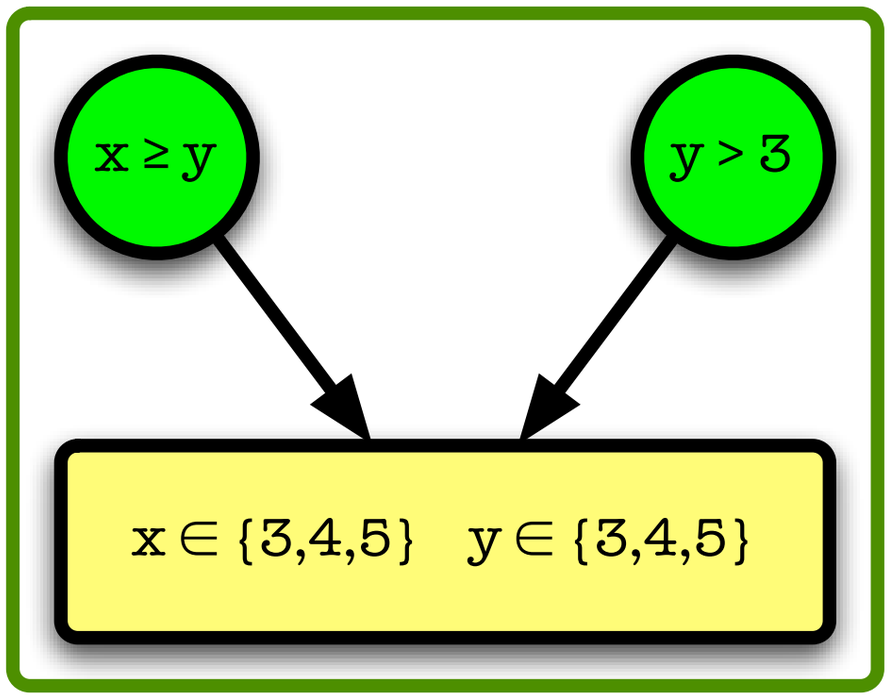
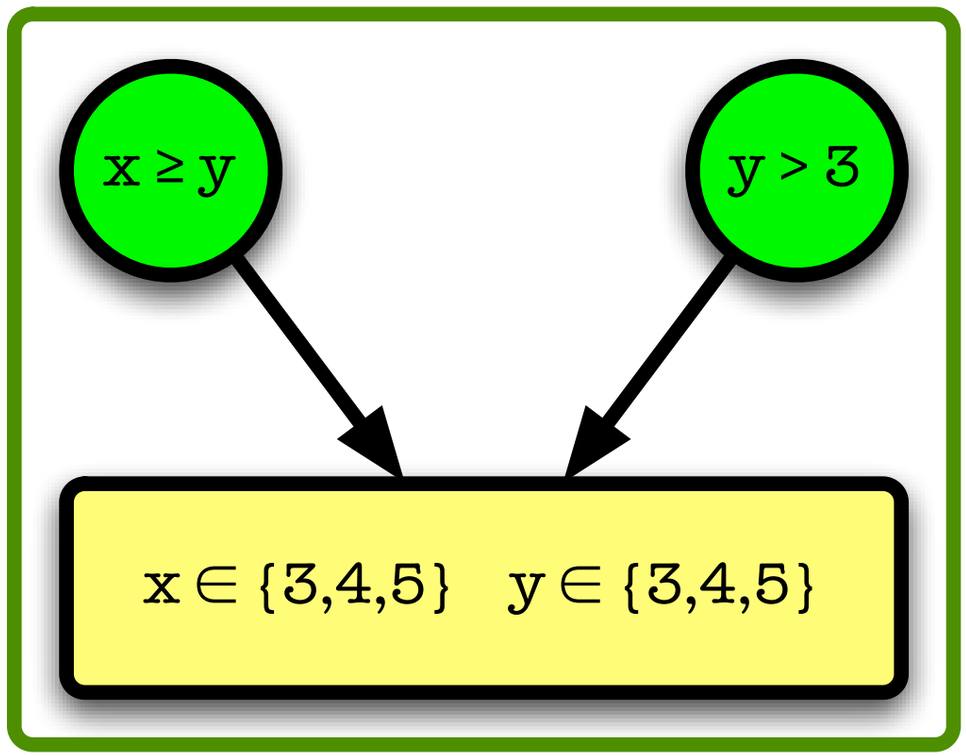


Constraint Programming with Gecode/J

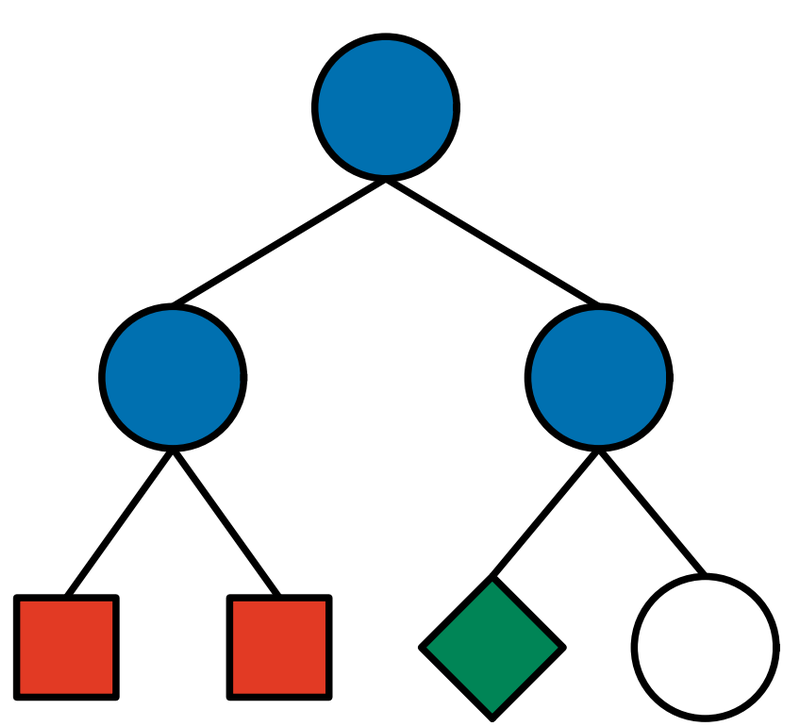
- **Quick reminder of last lecture**
- **Walk-through for *Send More Money***
- **Some modeling techniques**
- **Presentation of first graded lab**

Computation Space



constraint store with connected propagators

Search



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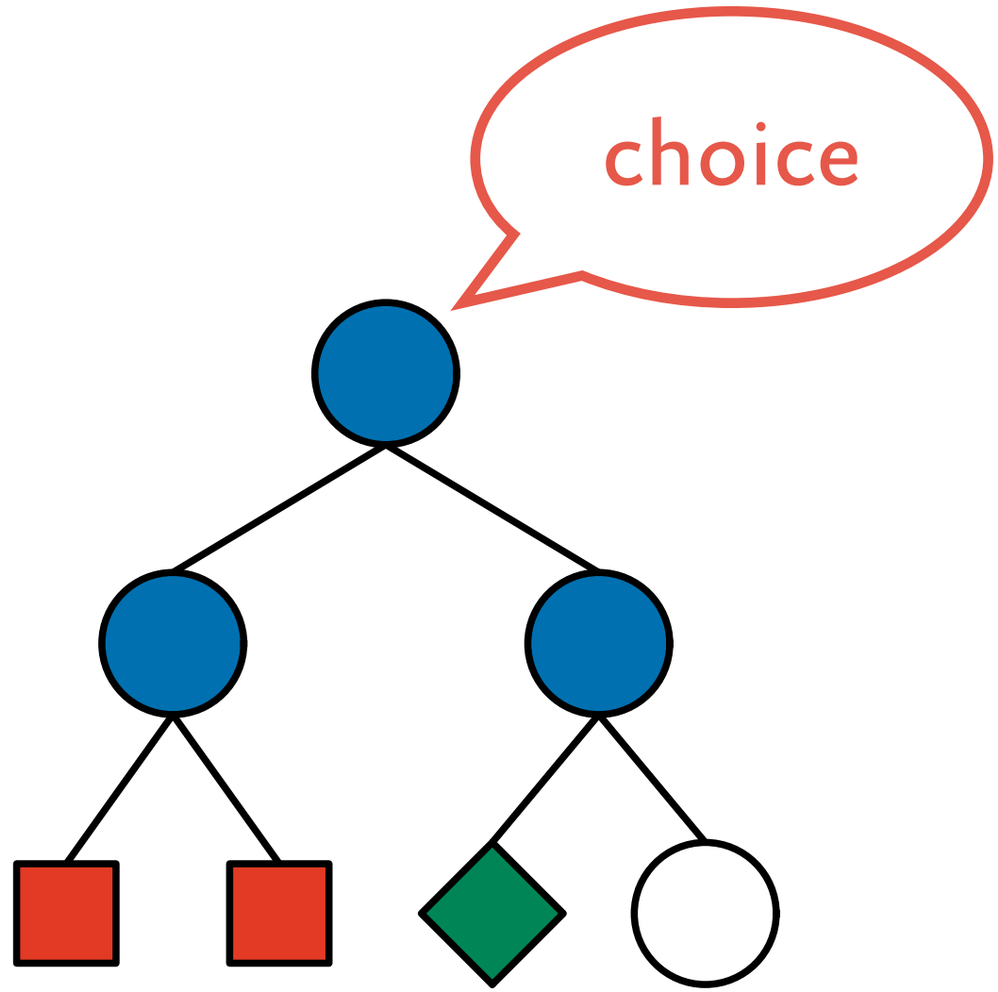

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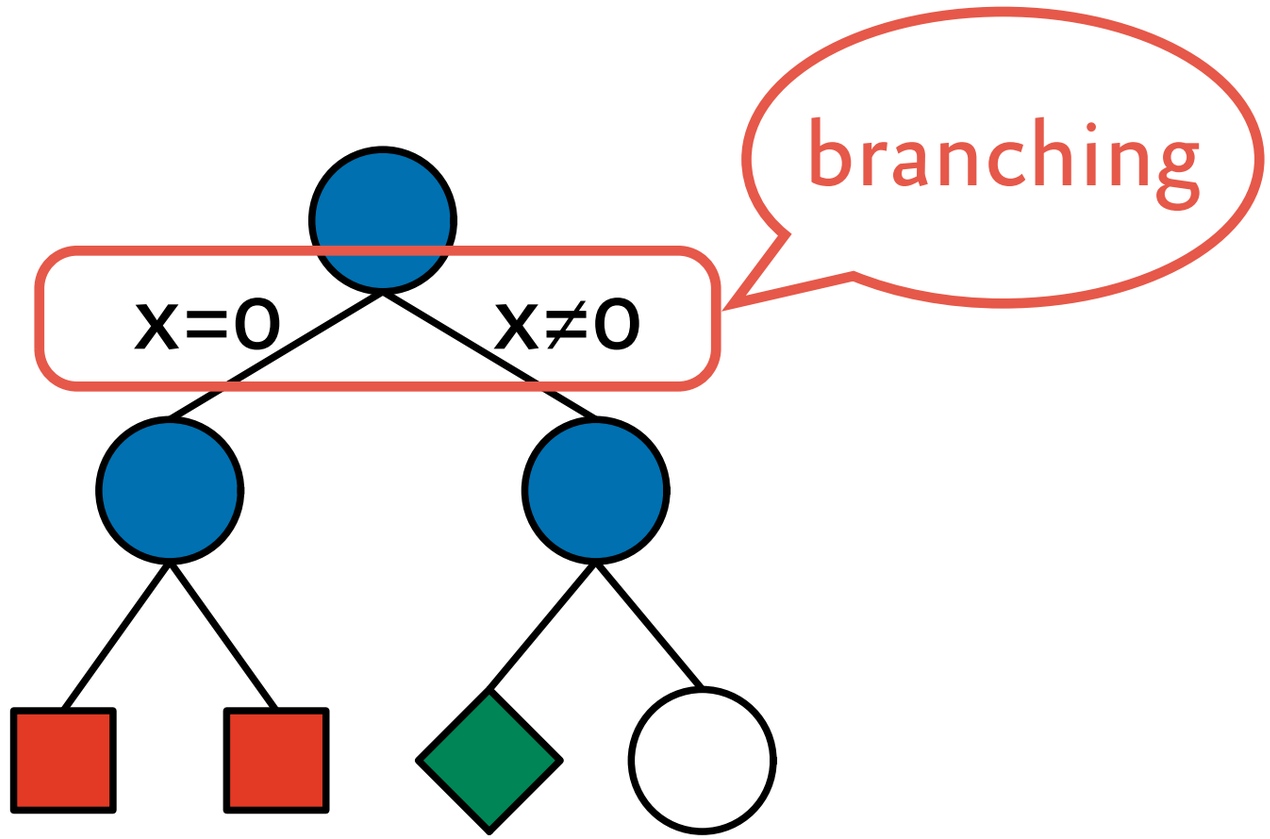

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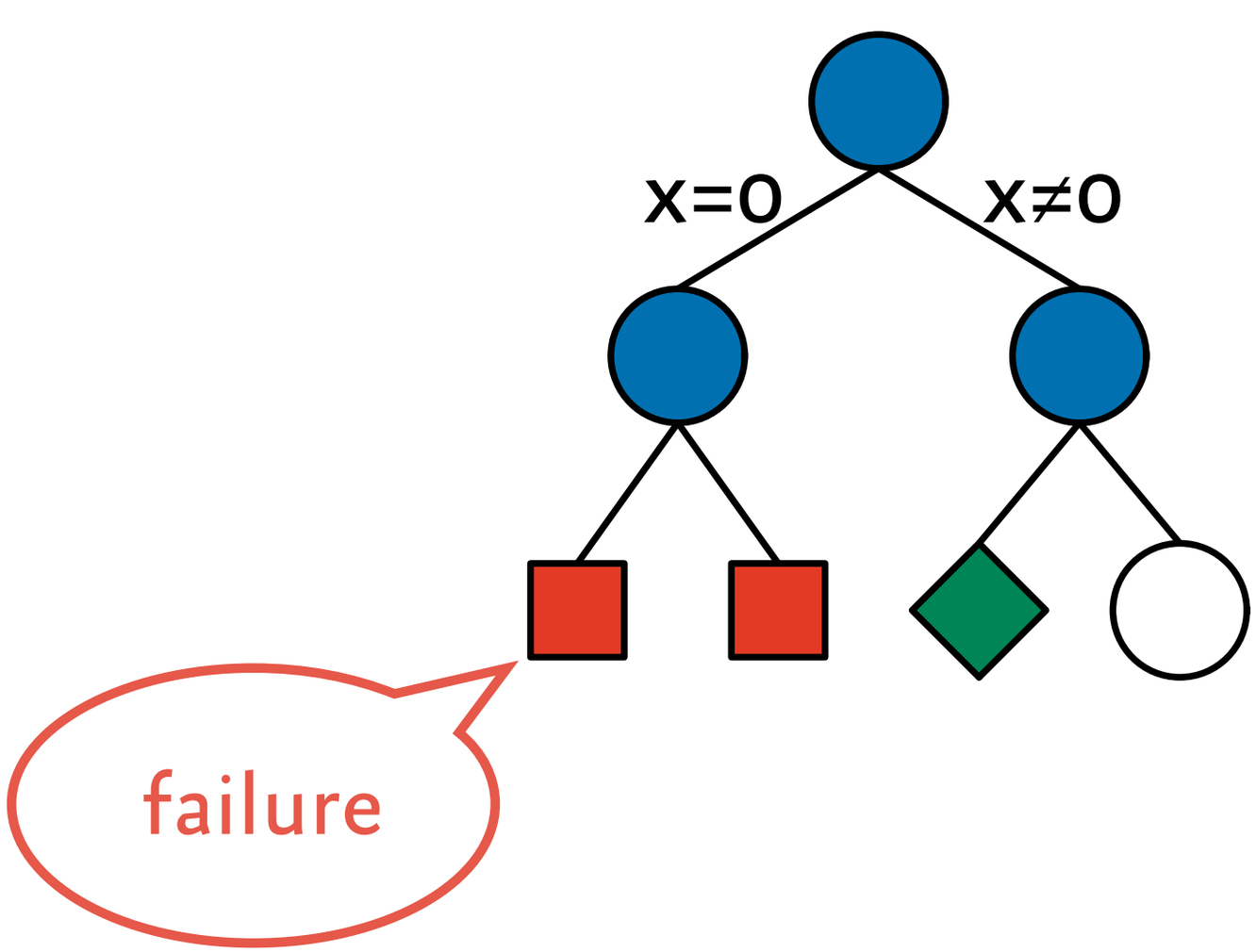

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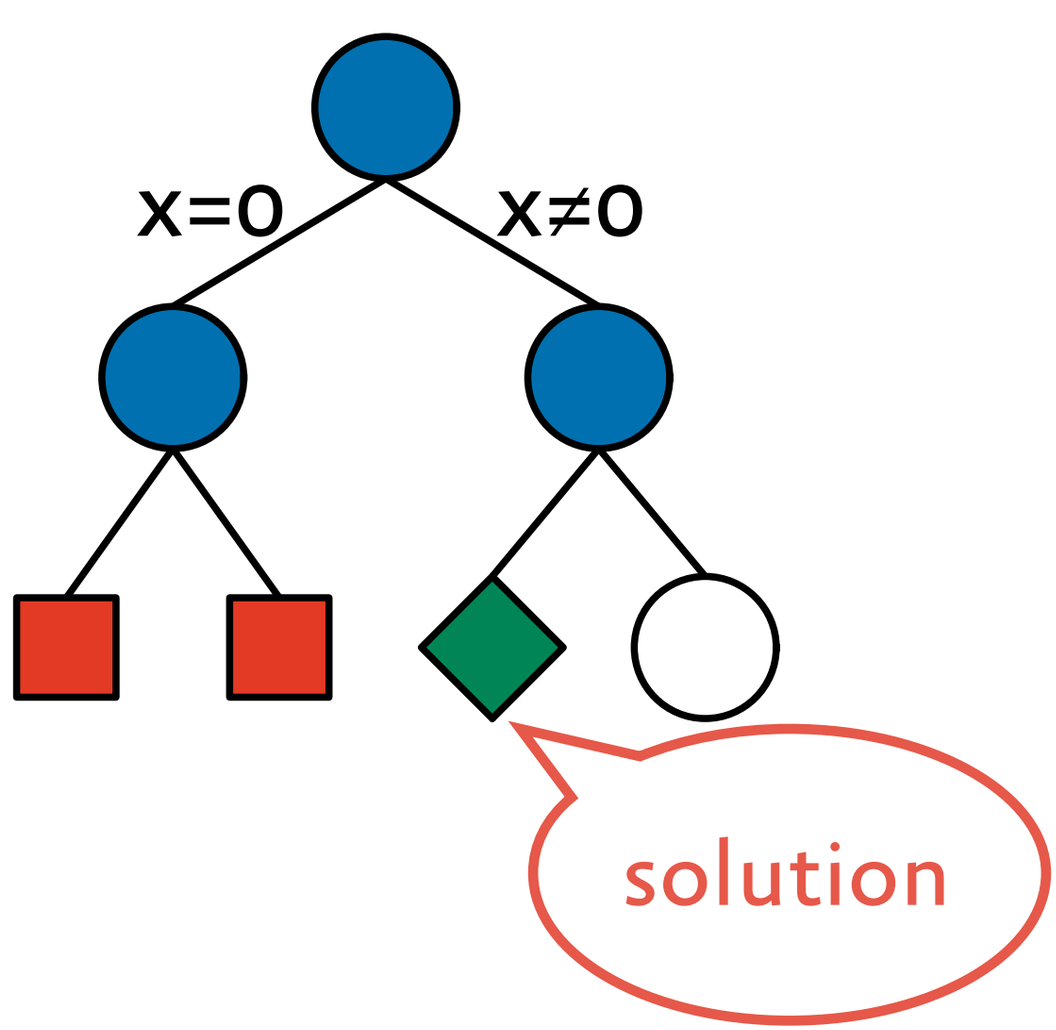

Send More Money

- variables: $S,E,N,D,M,O,R,Y \in \{0,\ldots,9\}$
- constraints: $S \neq 0,\ M \neq 0$

  $\text{distinct}(S,E,N,D,M,O,R,Y)$

$$
\begin{array}{r}
1000\times S + 100\times E + 10\times N + D \\
+\ 1000\times M + 100\times O + 10\times R + E \\
\hline
= 10000\times M + 1000\times O + 100\times N + 10\times E + Y
\end{array}
$$

Modelling in Gecode/J

- **Implement model as a *script***
  - declare variables
  - post constraints (create propagators)
  - define branching
- **Solve script**
  - basic search strategy (DFS)
  - interactive, graphical search tool (Gist)

Script: Overview

- **Inherit from class Space**
- **Constructor**
  - initialize variables
  - post propagators
  - define branching
- **Copy constructor**
  - copy a space
- **Main function**
  - invoke search engine

Script: Structure

```
import static org.gecode.Gecode.*;
import static org.gecode.GecodeEnumConstants.*;
import org.gecode.*;

public class Money extends Space {
  public VarArray<IntVar> letters;
  public Money() {...}
  public Money(Boolean share, Money money) {...}
  public static void main(String[] args) {...}
}
```

Script: Structure

import all we need

```
import static org.gecode.Gecode.*;
import static org.gecode.GecodeEnumConstants.*;
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public class Money extends Space {
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}
```

problem variables

Script: Structure

```
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  public Money() {...}
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```

constructor

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copy constructor

Script: Structure

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public class Money extends Space {
  public VarArray<IntVar> letters;
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  public static void main(String[] args) {...}
}
```

Script: Constructor

```
public Money() {
   // Call superclass constructor
   super();
   // Initialize variables
   letters = new VarArray<IntVar>(this,
                                  8,
                                  IntVar.class,
                                  0, 9);
   ...
}
```

Script: Constructor

```
...
// Refer to the letters by name
IntVar s = letters.get(0);
IntVar e = letters.get(1);
IntVar n = letters.get(2);
IntVar d = letters.get(3);
IntVar m = letters.get(4);
IntVar o = letters.get(5);
IntVar r = letters.get(6);
IntVar y = letters.get(7);
...
```

Script: Constructor

```
...
// Initial letters non-zero
rel(this, s, IRT_NQ, 0);
rel(this, m, IRT_NQ, 0);
// IRT: Integer relation type
...
```

Posting Constraints

- **Defined in the class `org.gecode.Gecode`**
  - accessed with `import static`
- **Check the documentation**
- **All constraints take a space as first argument**
  - where is the constraint to be installed?
  - Scripts are subclasses of Space!

Linear equation constraints

- Propagator for equations of the form

$$\sum_{i=1}^{n} a_i x_i = d$$

- Specified as arrays

```
int[] a
```

```
VarArray<IntVar> x
```

- Supported relations:

```
IRT_EQ, IRT_NQ, IRT_LE, IRT_GR, IRT_LQ, IRT_GQ
```

Script: Constructor

```
...
// Post linear equation
int a[]={              1000,    100,     10,       1,
                       1000,    100,     10,       1,
           -10000,  -1000,   -100,    -10,      -1};
VarArray<IntVar> x =
    new VarArray<IntVar>(    s, e, n, d,
                             m, o, r, e,
                          m, o, n, e, y);
linear(this, a, x, IRT_EQ, 0);
...
```

Script: Constructor

```
...
// Letters take distinct values
distinct(this, letters);

// Find values using first-fail
branch(this, letters,
       BVAR_SIZE_MIN,
       BVAL_MIN);

...
```

Branching

- **Choose variable**
  - smallest domain size: `BVAR_SIZE_MIN`
  - smallest minimum: `BVAR_MIN_MIN`
  - given order: `BVAR_NONE`
- **Choose value**
  - try smallest value: `BVAL_MIN`
  - split (lower first) `BVAL_SPLIT_MIN`

Script: Copying

```
public Money(Boolean share, Money m) {
  super(share, m);
  letters = new VarArray<IntVar>(this,
                                 share, m.letters);
}
```

Script: Copying

```
public Money(Boolean share, Mo
  super(share, m);
  letters = new VarArray<IntVar>(this,
                                 share, m.letters);
}
```

copy all variables you need for output!

Script: Copying

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Script: Copying

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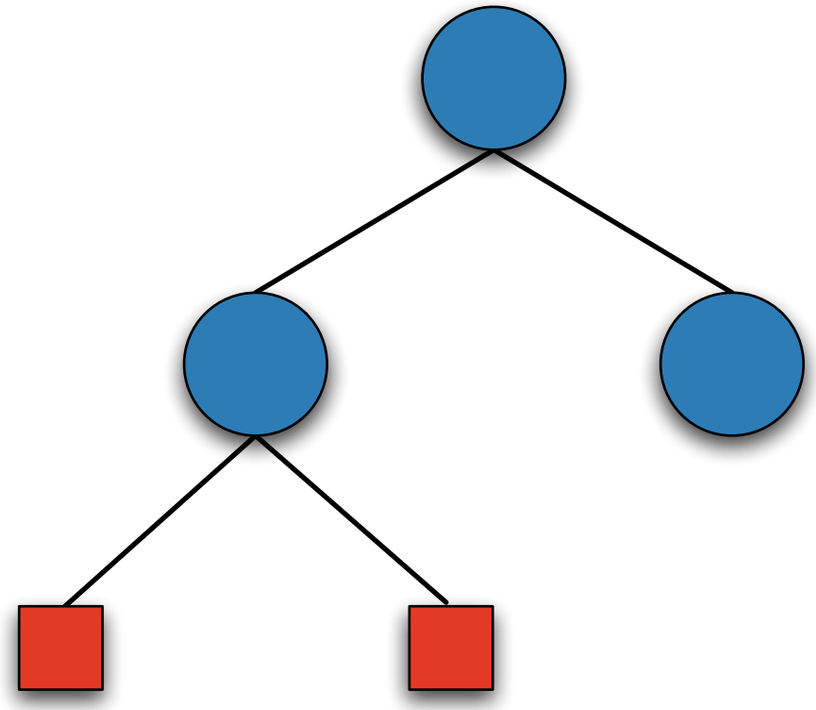
}
```

copying of single variables also possible!

Why copy? Search!



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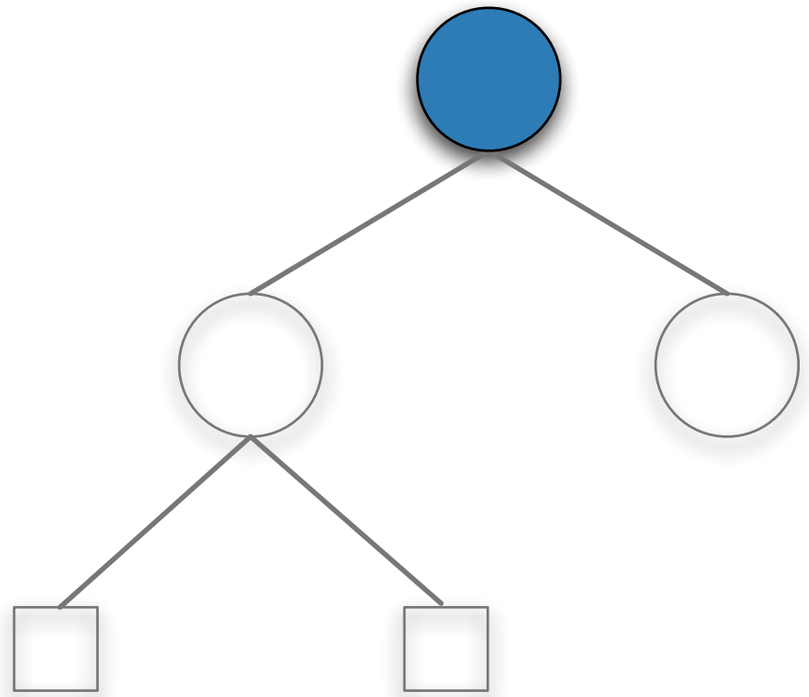

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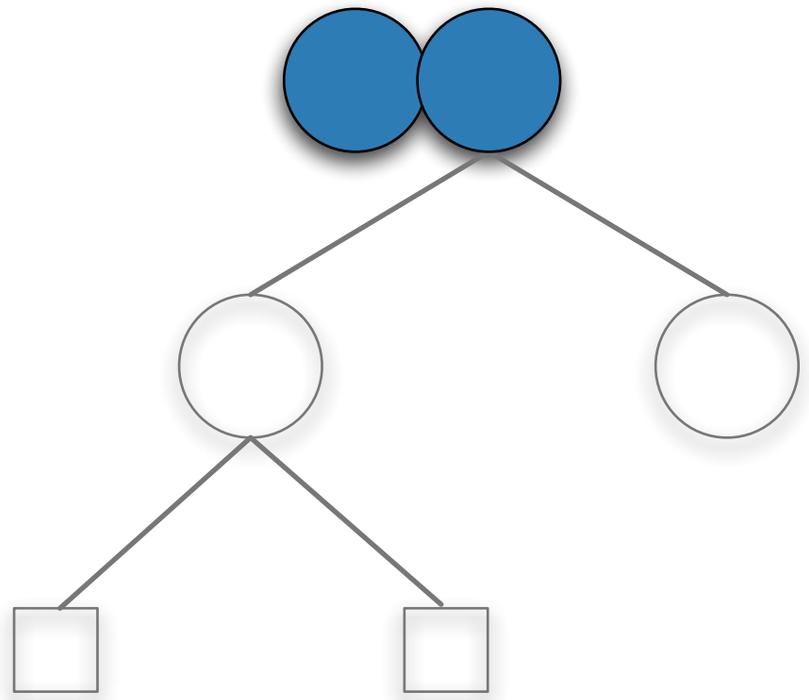

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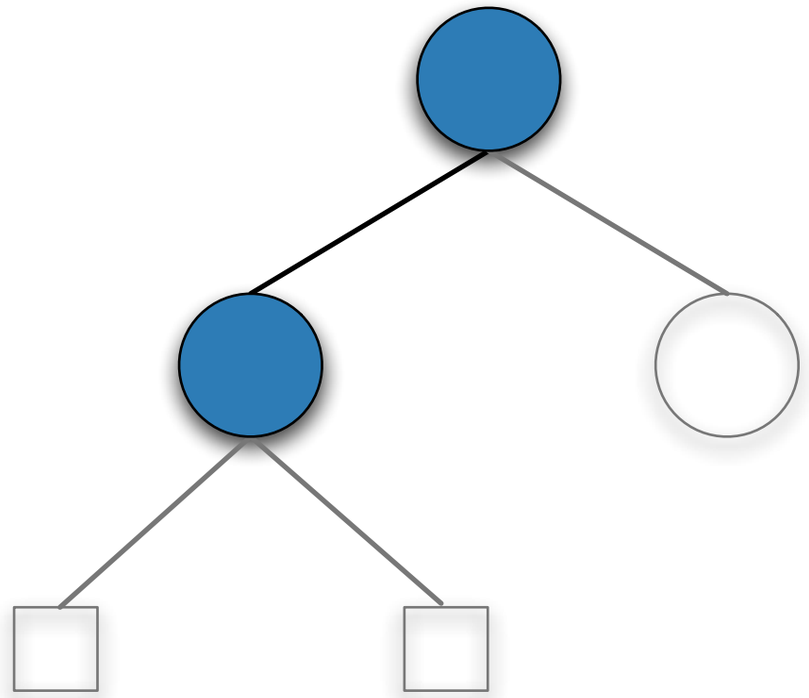

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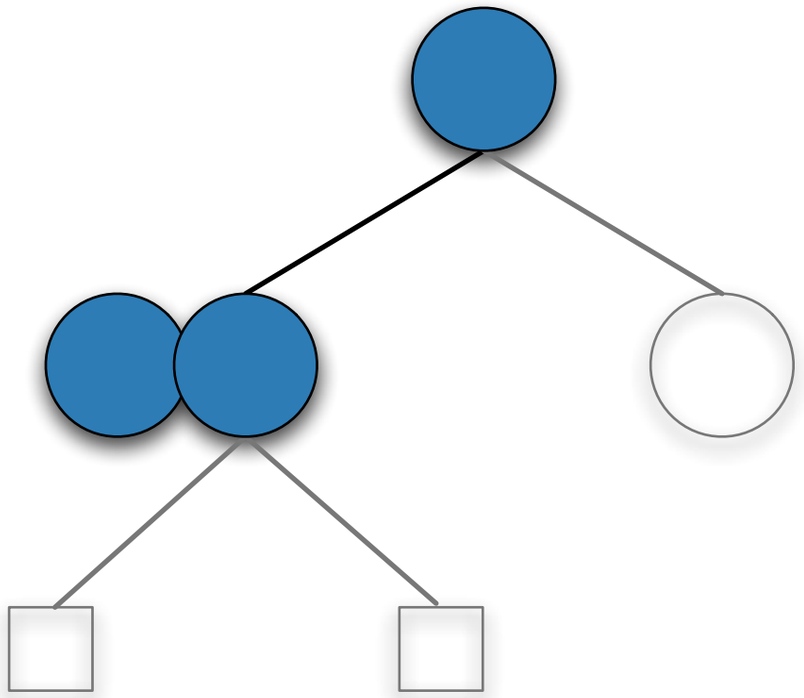

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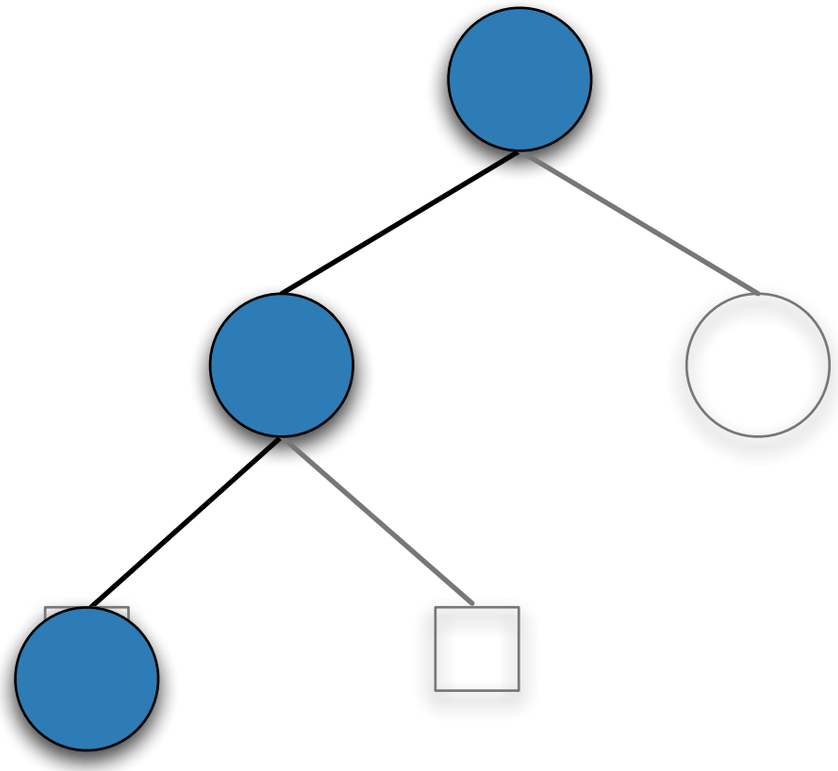

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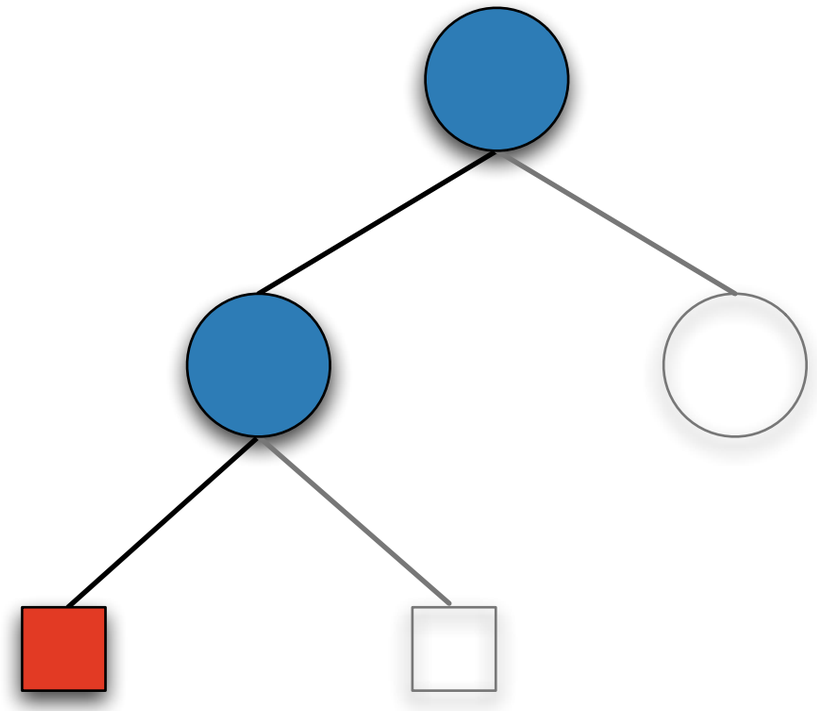

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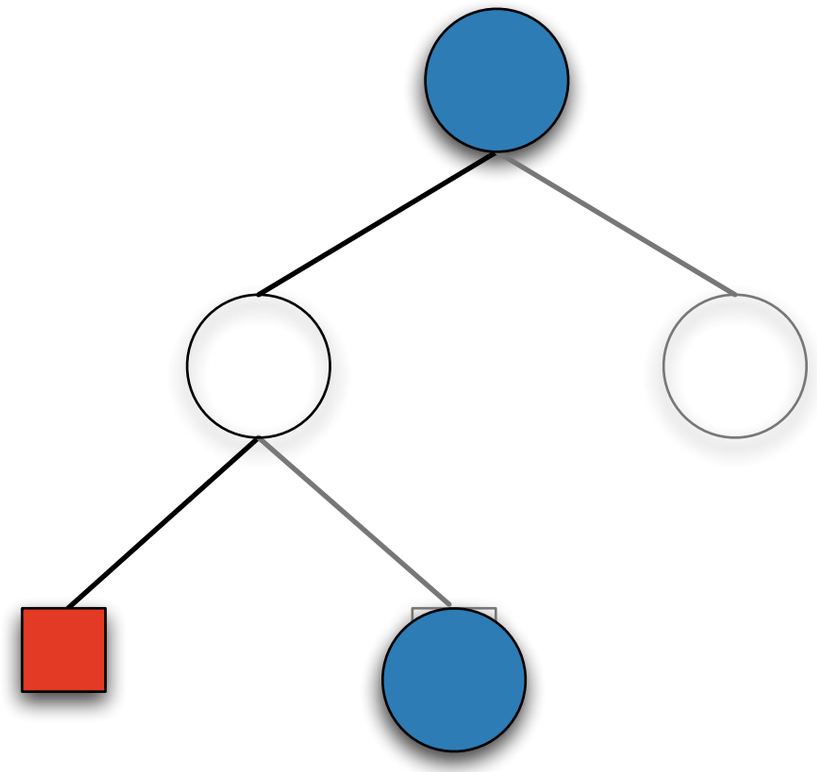

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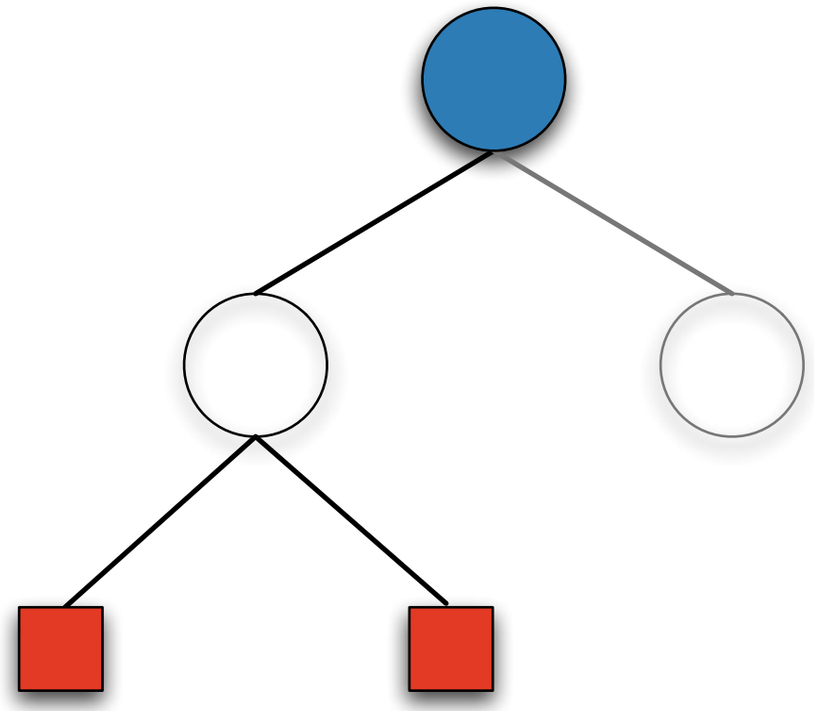

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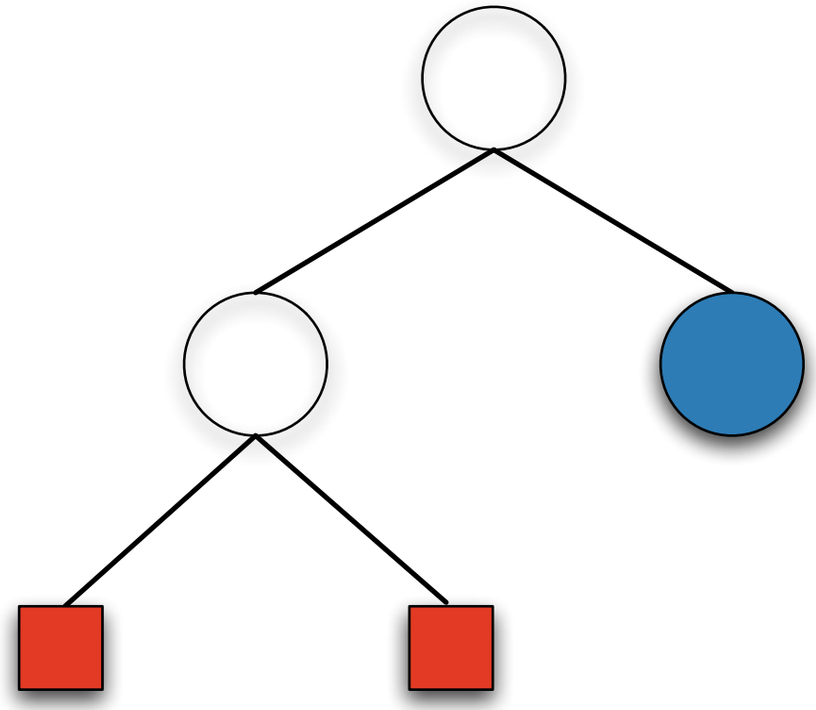

Script

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  public Money() {...}
  public Money(Boolean share, Money money) {...}
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}
```

Solving

- **Hidden in `Options` class, but...**
- **Search engines:**
  - `DFSSearch` (depth first search)
  - `BABSearch` (branch-and-bound search)
- **Interactive search tool**
  - Gist

First Solution Search

```
public static void main(String[] args)
{
  Money m = new Money();
  DFSSearch s = new DFSSearch(m);
  Money sol = (Money) s.next();
  if (sol != null) {
    System.out.println(sol.toString());
  }
}
```

First Solution Search

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```

create root space

First Solution Search

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  if (sol != null) {
    System.out.println(sol.toString());
  }
}
```

create
search engine

First Solution Search

```
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  if (sol != null) {
    System.out.println(sol.toString());
  }
}
```

search

All Solution Search

```
public static void main(String[] args)
{
  Money m = new Money();
  DFSSearch s = new DFSSearch(m);
  Money sol = (Money) s.next();
  while (sol != null) {
    System.out.println(sol.toString());
    sol = (Money) s.next();
  }
}
```

Gecode/J Gist

- **Graphical search tree explorer**
  - explore step by step
  - search next solution, search all solutions
  - "inspect" nodes (double-click)
  - hide failed subtrees
  - ...
- **Try it out!**

Invoking Gist

```
import org.gecode.gist.*;

public static void main(String[] args)
{
  Money m = new Money();
  Gist g = new Gist(m);
  g.exploreOne();
  // Also possible: g.exploreAll()
}
```

Best Solution Search

- **Reminder: SMM+**

  ```
    SEND
  +  MOST
  =  MONEY
  ```

- **Constraints:**

  Find distinct numbers such that the equation holds and MONEY is maximal

SMM+: Order

- **Branch-and-bound principle:**

  after each solution, constrain remaining search to find only better solutions

- **Script provides method:**

```
public void constrain(Space s) { ... }
```

- **Invoked on Space object to be constrained**
- **`Space s:` so far best solution**
  - Only use *values* from `s`! Never mix variables!

SMM+: constrain method

```
IntVar money;
public SMM() {
   ...
   int [] c = {10000,1000,100,10,1};
   VarArray<IntVar> l = {m,o,n,e,y}; // pseudocode!
   linear(this, c, l, IRT_EQ, money);
   ...
}

public void constrain(Space bestSol) {
   SMM smm = (SMM) bestSol;
   rel(this, money, IRT_GR, smm.money.val());
}
```

SMM+: main method

```
public static void main(String[] args) {
   SMM m = new SMM();

   BABSearch s = new BABSearch(m);
   SMM best = null;
   SMM sol = (SMM) s.next();
   while (sol != null) {
      best = sol; sol = (SMM) s.next();
   }
   if (best != null) {
      System.out.println(best.toString());
   }
}
```

SMM+: Gist

```
public static void main(String[] args)
{
  SMM m = new SMM();

  Gist g = new Gist(m, true);
  g.exploreAll();
}
```

SMM+: Gist

```
public static void main(String[] args)
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  g.exploreAll();
}
```

switch on b&b

Solving: Summary

- **Non-interactive search:**

  `DFSSearch`, `BABSearch`

- **Interactive search:**

  Gist

- **Best solution search:** provide constrain method

- **Search engines independent of script**

Some modeling techniques

- **Global constraints**
- **Symmetry breaking**
- **Reification**

Global constraints

- **Classic example:**

  $x,y,z \in \{1,2\}, \quad x \neq y, x \neq z, y \neq z$

- **No solution!**
- **But: each individual constraint still satisfiable!**

  ⇒ no propagation possible!

- **Solution:** look at several constraints at once

  distinct(x,y,z)

Distinct

- **Remember last exercise for Sudoku?**

  "Experiment with different … propagator strengths."

- `ICL_VAL`: same as x ≠ y, x ≠ z, y ≠ z
- `ICL_BND`: stronger
- `ICL_DOM`: strongest possible propagation

Grocery Puzzle

- A kid buys four items in a grocery store.
- Cashier: "That's €7,11.

  Hold on! I multiplied instead of adding.

  Wow, the sum is also €7,11!"
- What are the individual prices of the four items?

Grocery Puzzle

- **Variables:**

  A,B,C,D ∈ {1, …, 711} (price in cents)

- **Constraints:**
  - A+B+C+D = 711
  - A*B*C*D = 711 * 100 * 100 * 100

Grocery Puzzle

- **Branching:**
  - Bad idea: try values one by one
  - Better: split domains
- **Typically good strategy for problems with arithmetic constraints**
- **Still:**

Grocery Puzzle: Symmetries

- **Many solutions are equivalent!**
  - swap values of A,B,C,D
- **Let's order them:**
  - $A \leq B \leq C \leq D$
- **This is called**

Symmetry Breaking

- **So let's see...**

Grocery Puzzle: Symmetries

- **Symmetry breaking also works on symmetric failure!**
- **Example:**
  - Suppose A,B,C have been assigned 1,2,3 by search.
  - We find no value for D such that we get a solution.
  - No need to try A=2, B=1, C=3!
- **Ruled out by $A \leq B \leq C \leq D$**

Grocery Puzzle: More Symmetries

- **Observation: 711 has prime factor 79**

  711 = 79 × 9

- **So assume**

  A = 79 × X (for a "fresh" variable X)

- **Remove A ≤ B**

- **And now...**

Reified constraints

- **Constraints are in a big conjuntion**
- **How about disjunctive constraints?**

$$A+B=C \lor C=0$$

- **Solution: *reify* the constraints:**

$$(A+B=C \Leftrightarrow b0) \land$$

$$(C=0 \Leftrightarrow b1) \land$$

$$(b0 \lor b1 \Leftrightarrow true)$$

Reified constraints

```
int[] ones = [1,1,-1];
VarArray<IntVar> xs = {a,b,c};
BoolVar b0(this);
BoolVar b1(this);

linear(this, ones, xs, IRT_EQ, 0, b0);
rel(this, c, IRT_EQ, 0, b1);
bool_or(this, b0, b1, true);
```

What we've learned today

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- Use **scripts** to model constraint problems

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- **Posting a propagator** installs it inside a space

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- Use **scripts** to model constraint problems
- **Posting a propagator** installs it inside a space
- We need **copying** for backtracking during search

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- **Symmetry breaking** is essential for some problems
- **Reification** allows for arbitrary Boolean combinations of constraints

Graded lab: temporal relations

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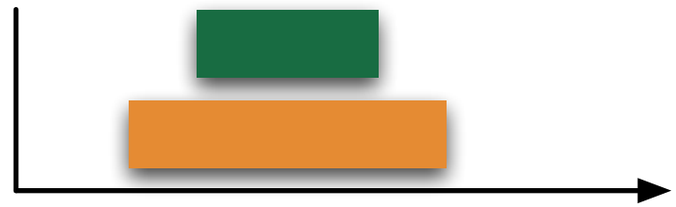

Graded lab: temporal relations

- Data structure:

```
enum Relation { PRECEDES, MEETS, OVERLAPS, DURING,
                STARTS, FINISHES, EQUALS};
class Item {
  String act1; Relation rel; String act2;
}
Vector<Vector<Item>> problem;
```

Graded lab: temporal relations

A precedes **B**

A starts **B**

A meets **B**

A finishes **B**

A overlaps **B**

A equals **B**

A during **B**

Graded lab: temporal relations

- **Your task:**
  - create script that solves temporal relation problems
- **If you have too much time:**
  - create visualization for the solutions
- **Submit**
  - by Monday, April 30, 24:00 MESZ
  - by email to tack@ps.uni-sb.de

Thanks for your attention!
See you on Thursday.