SAT

Marco Kuhlmann & Guido Tack

Lecture 8

deciding **SAT**isfiability of propositional formulae

Marco Kuhlmann & Guido Tack

Lecture 8

Today

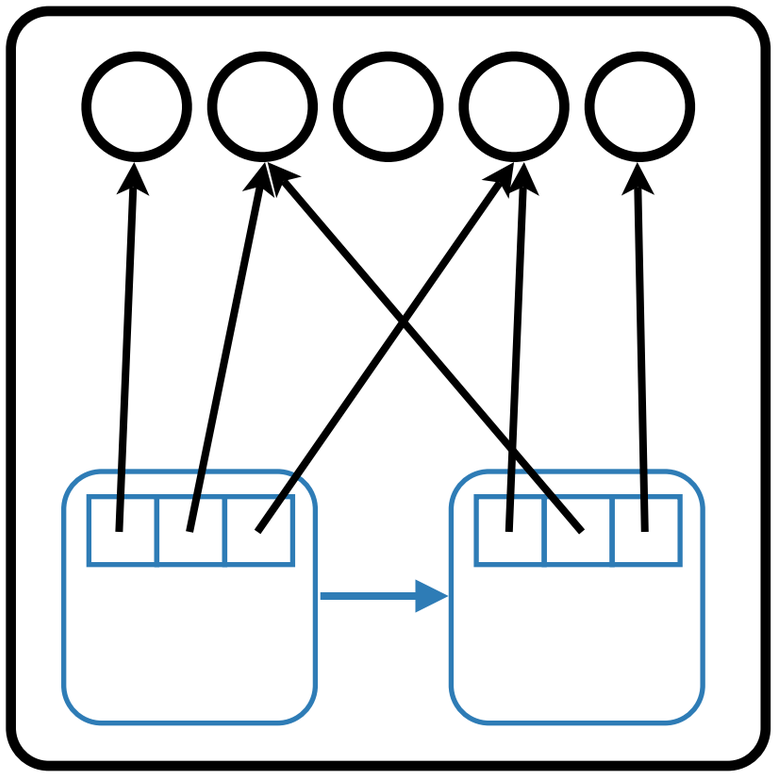
- SAT as constraint solving: the DPLL method
- Trailing
- Watched literals
- Conflict clause learning
- Special branching heuristics

Detour: copying spaces

- **so far:** search with copying/recomputation
- **today:** search with trailing
- to understand the difference:

  how are spaces copied?

Copying a space



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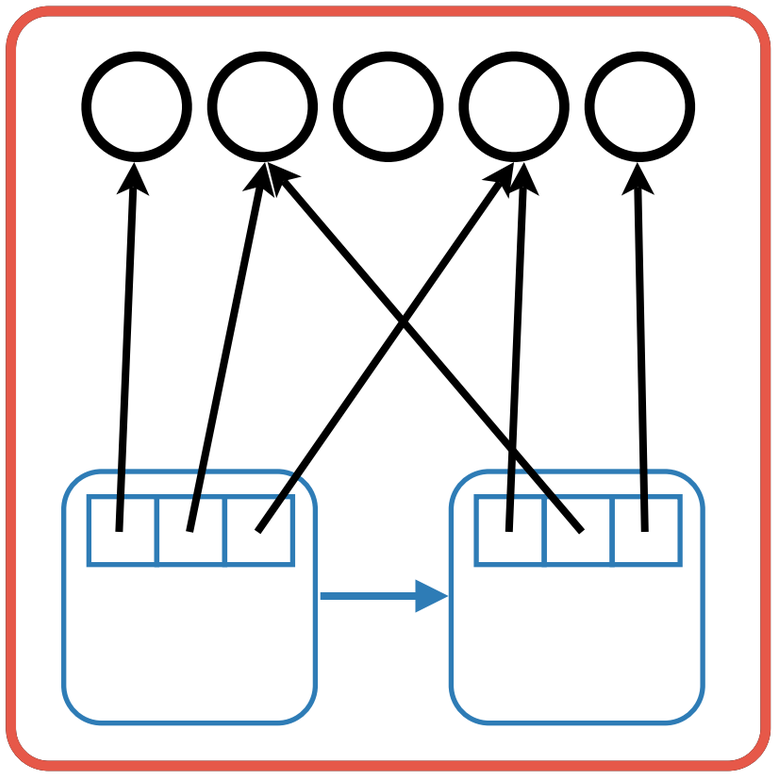

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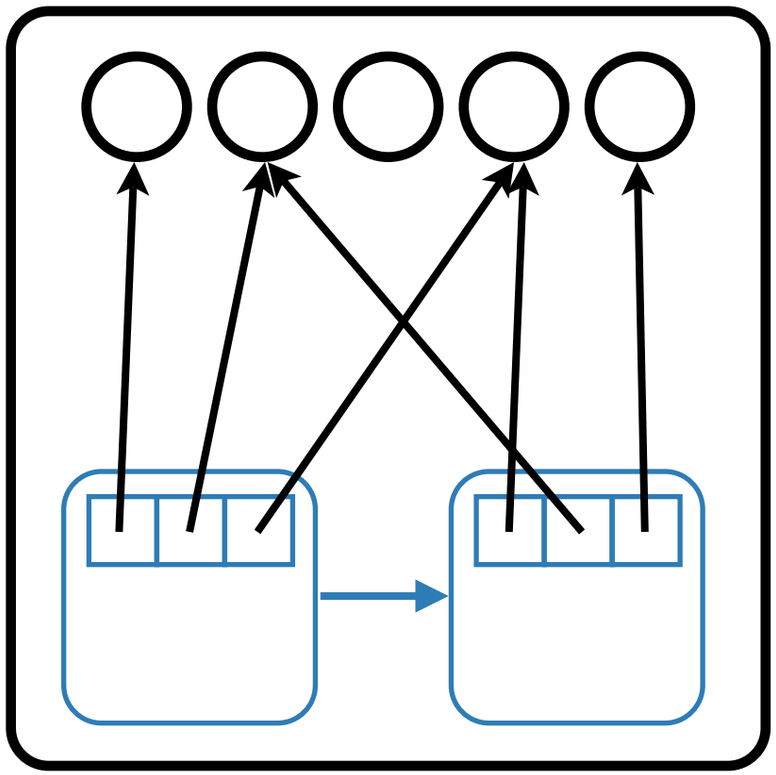

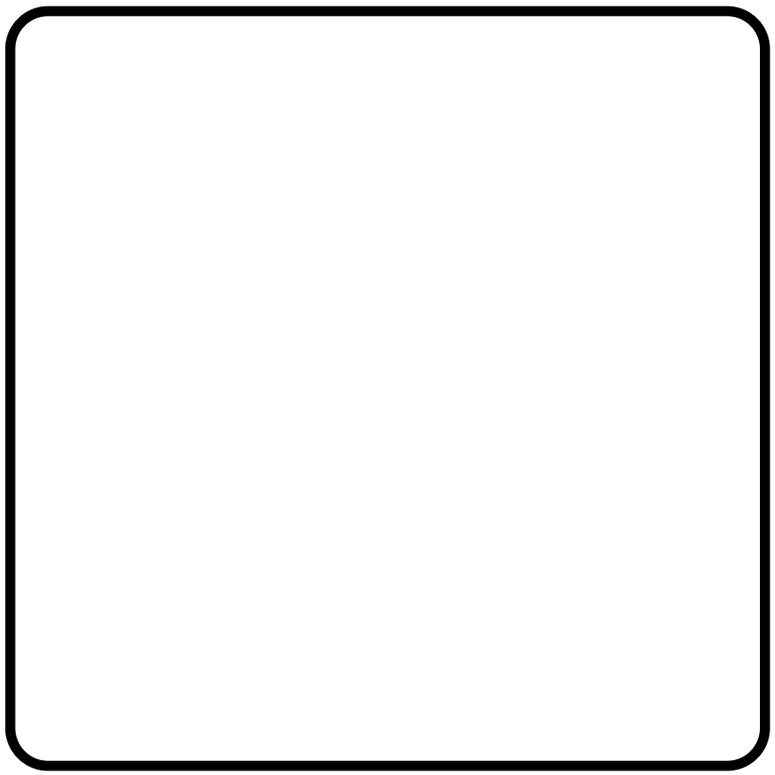

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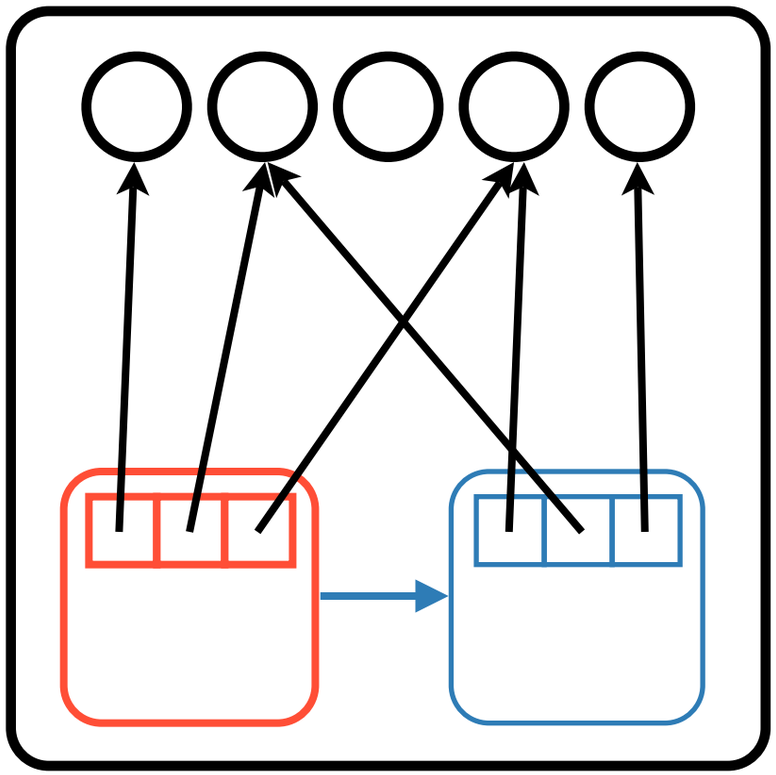

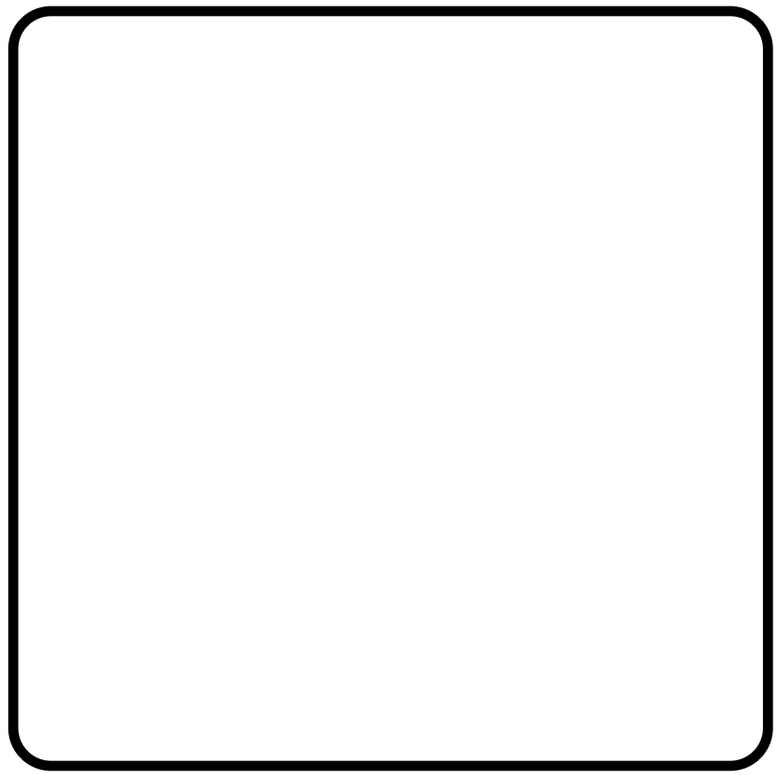

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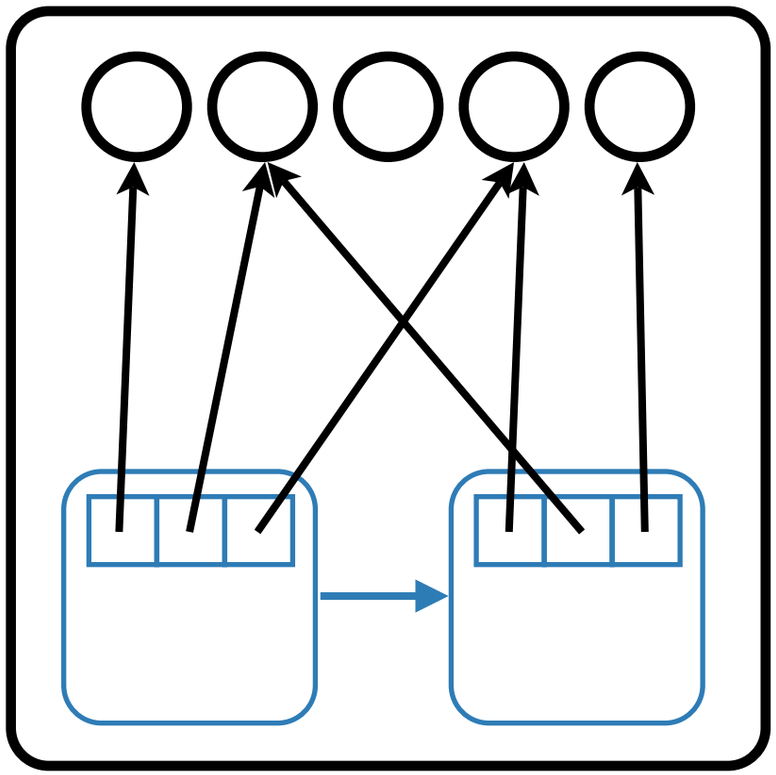

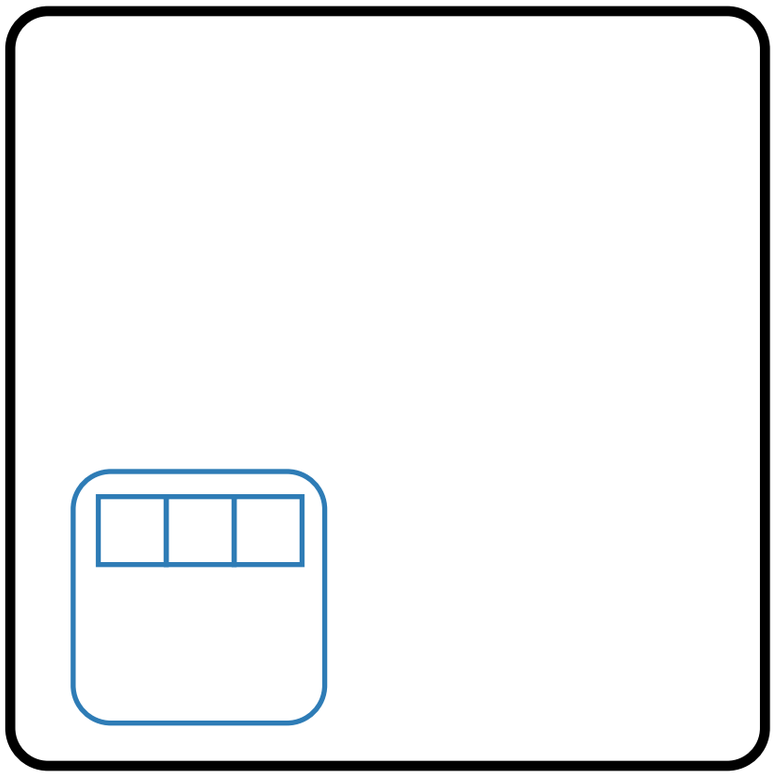

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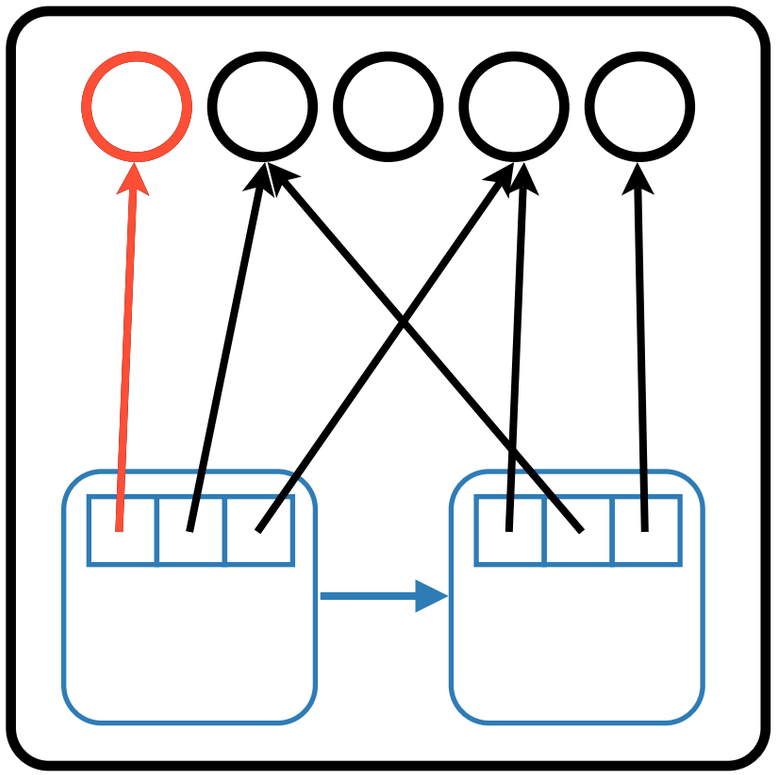

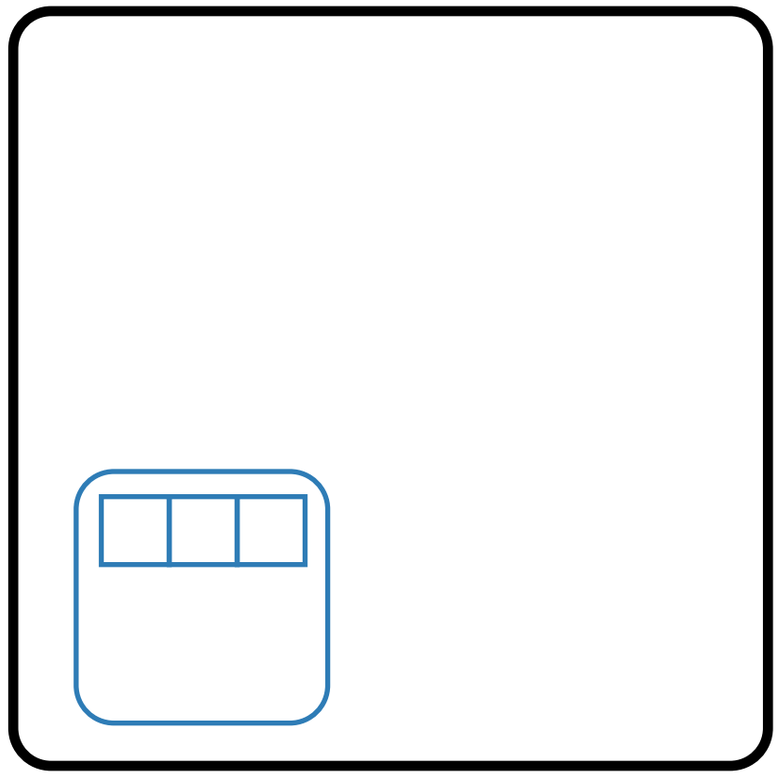

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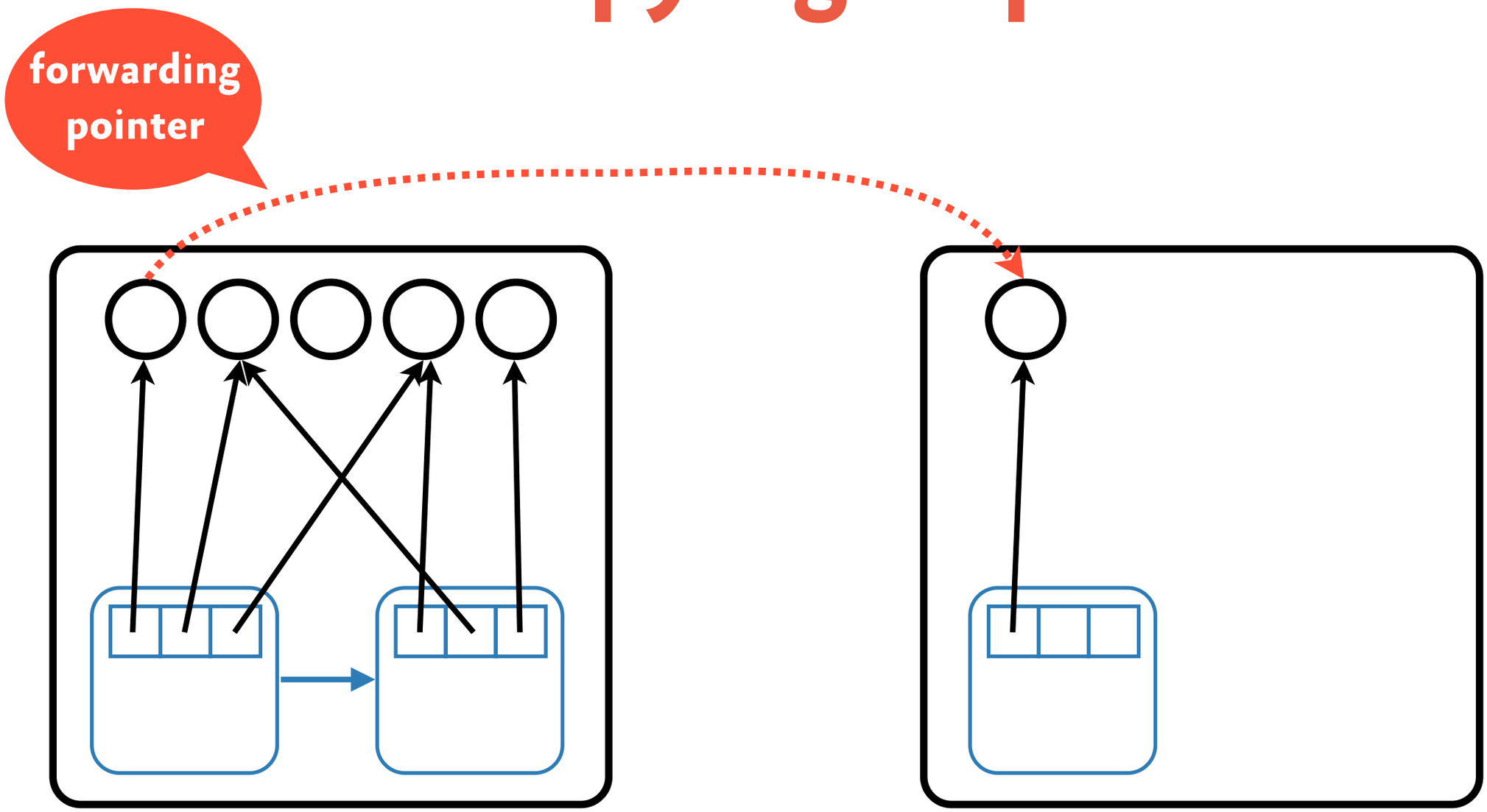

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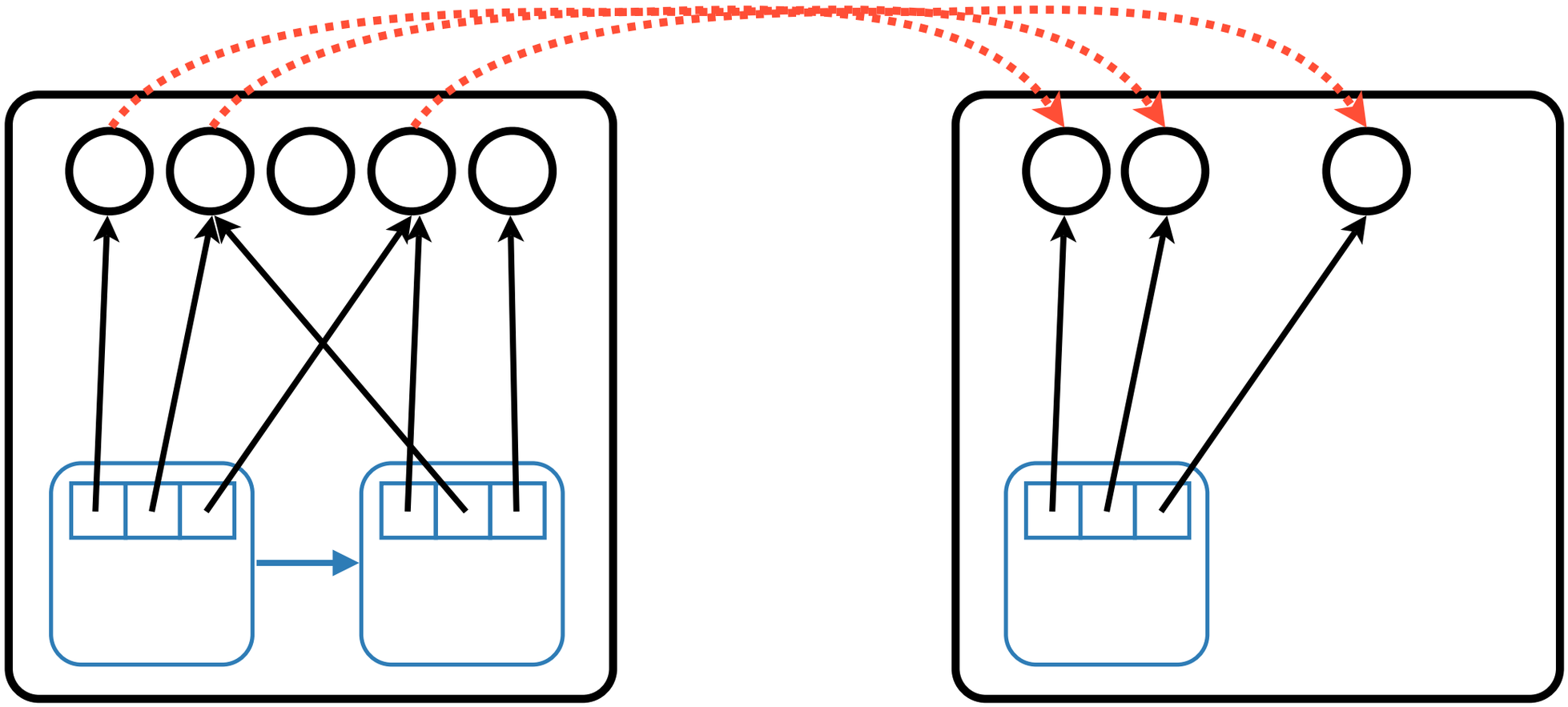

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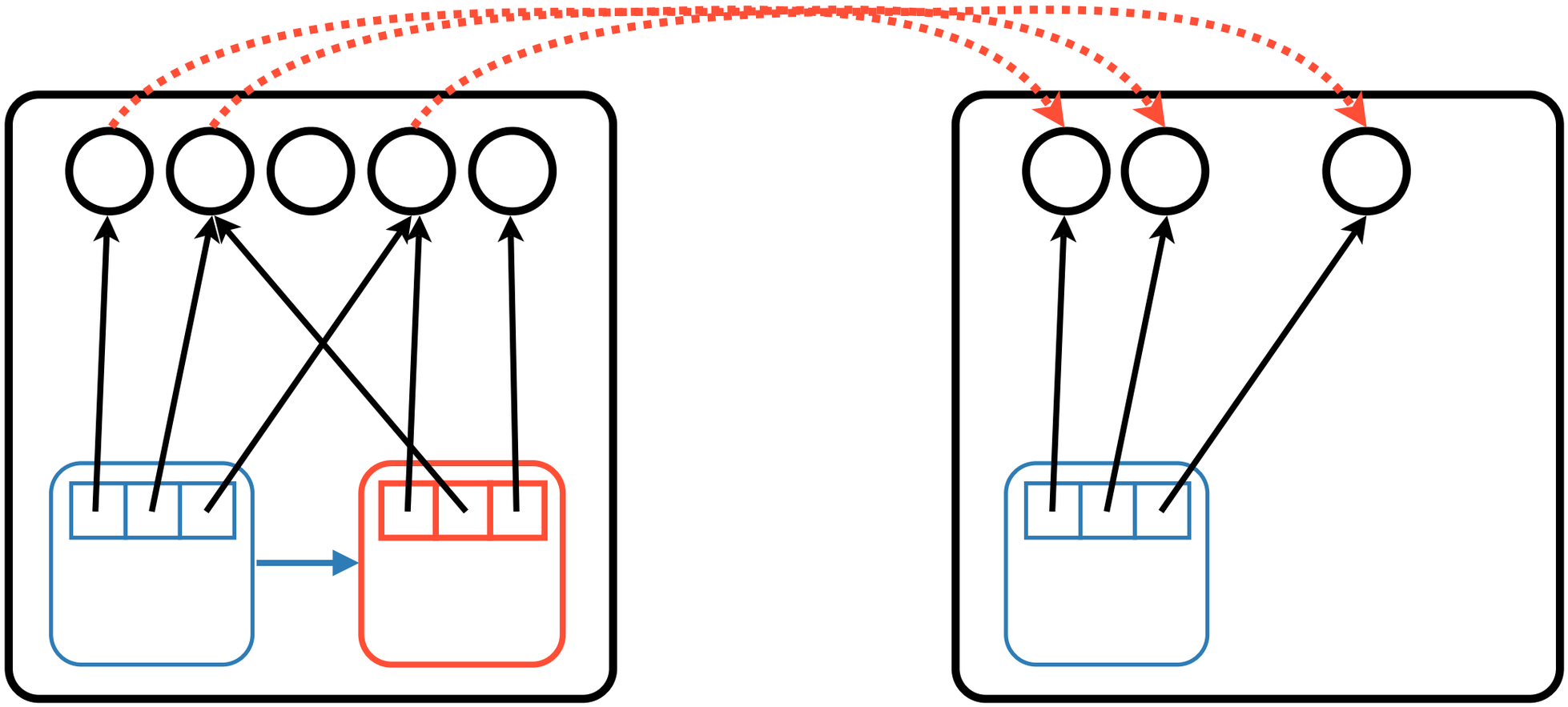

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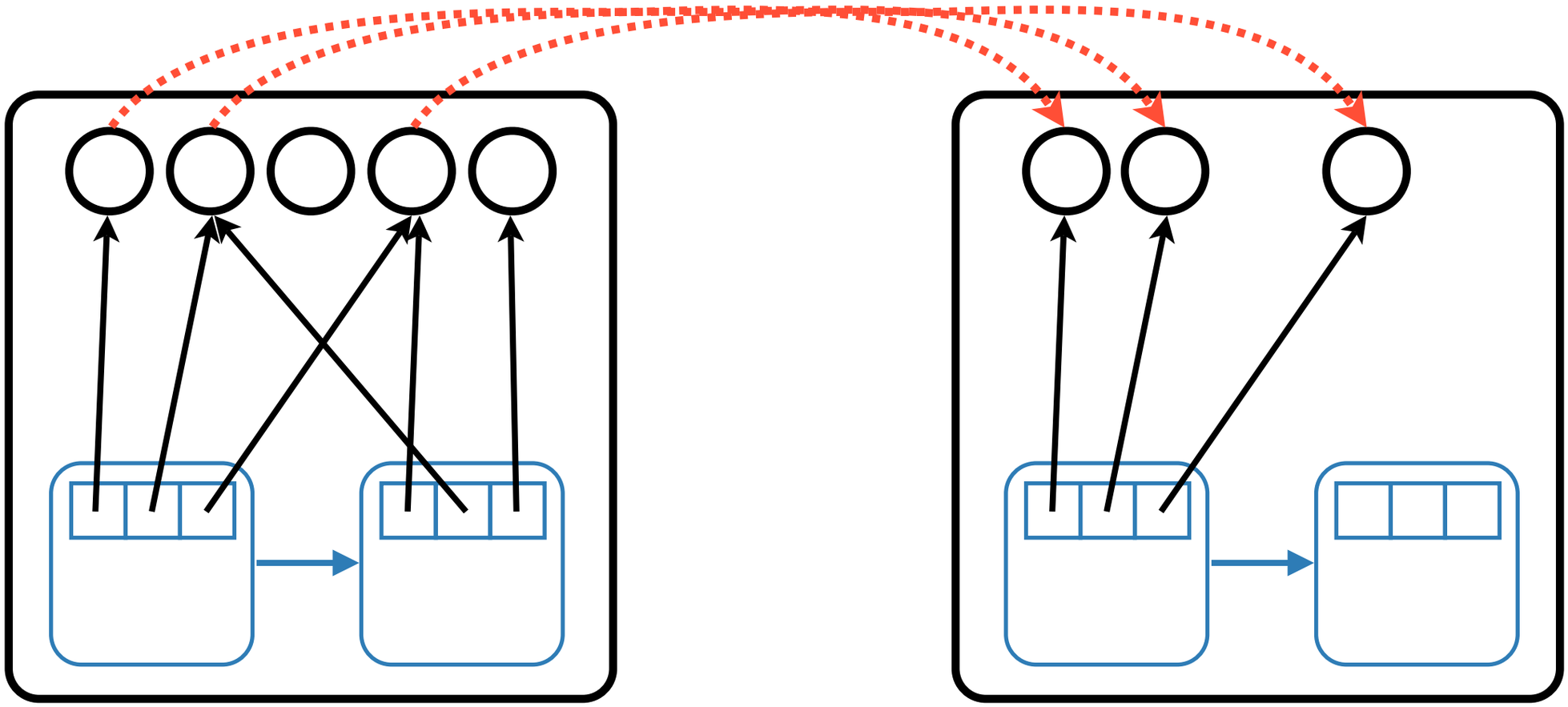

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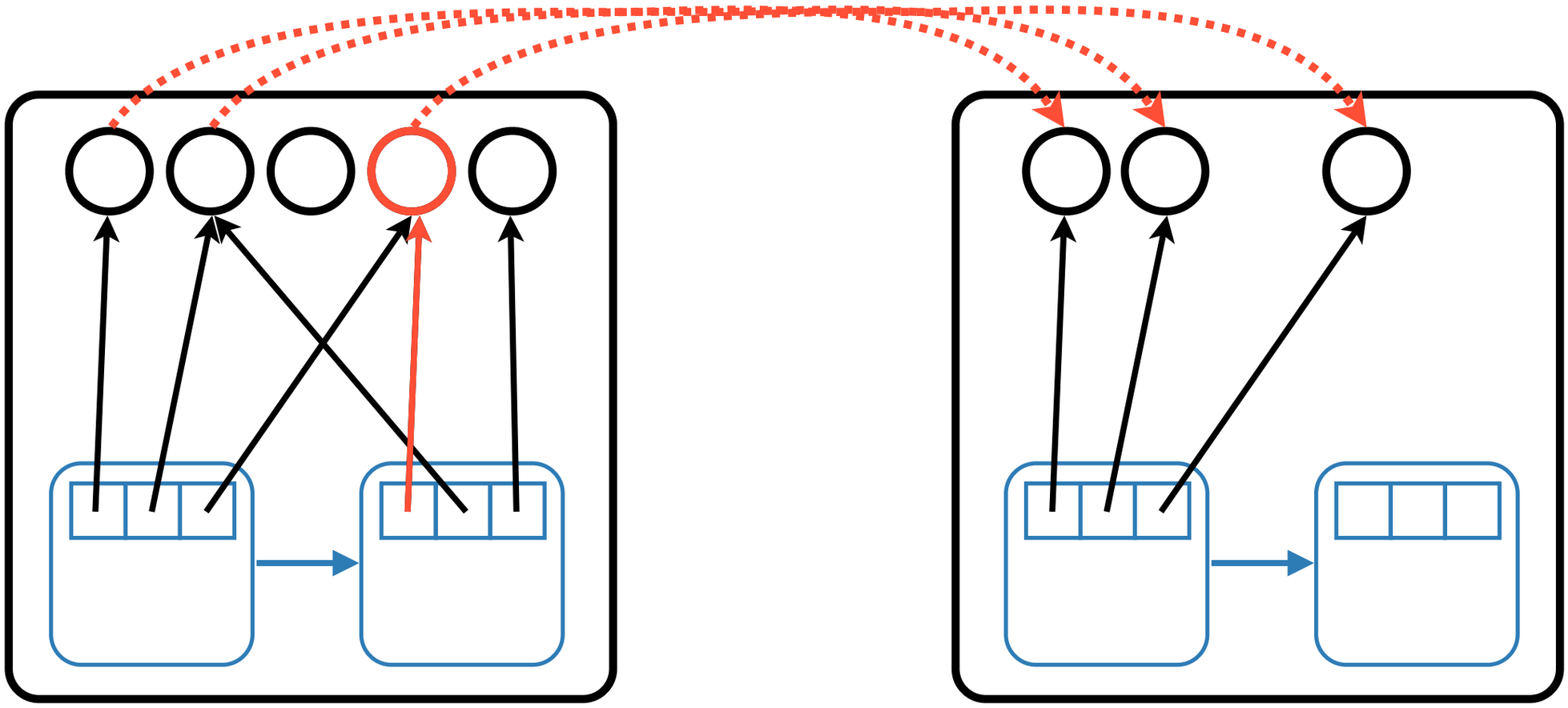

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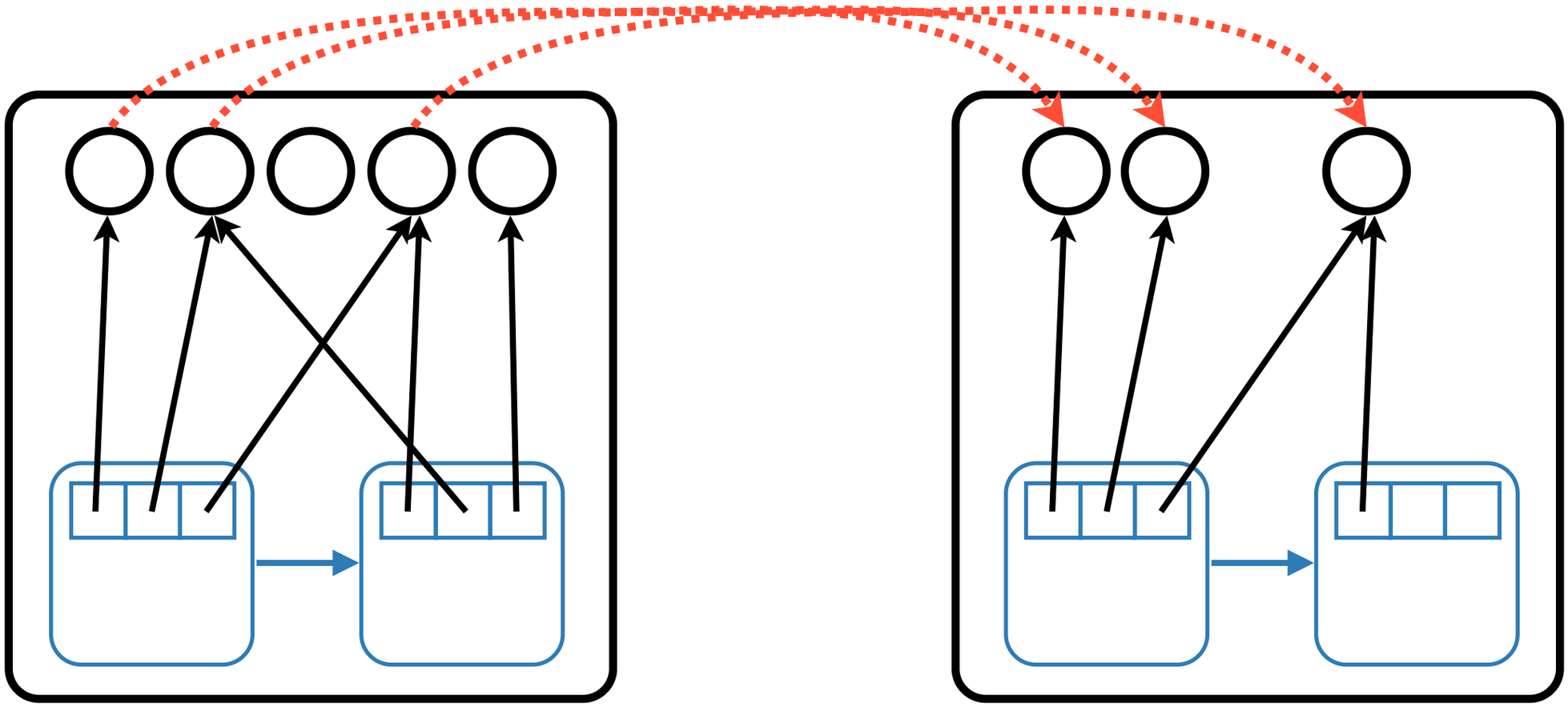

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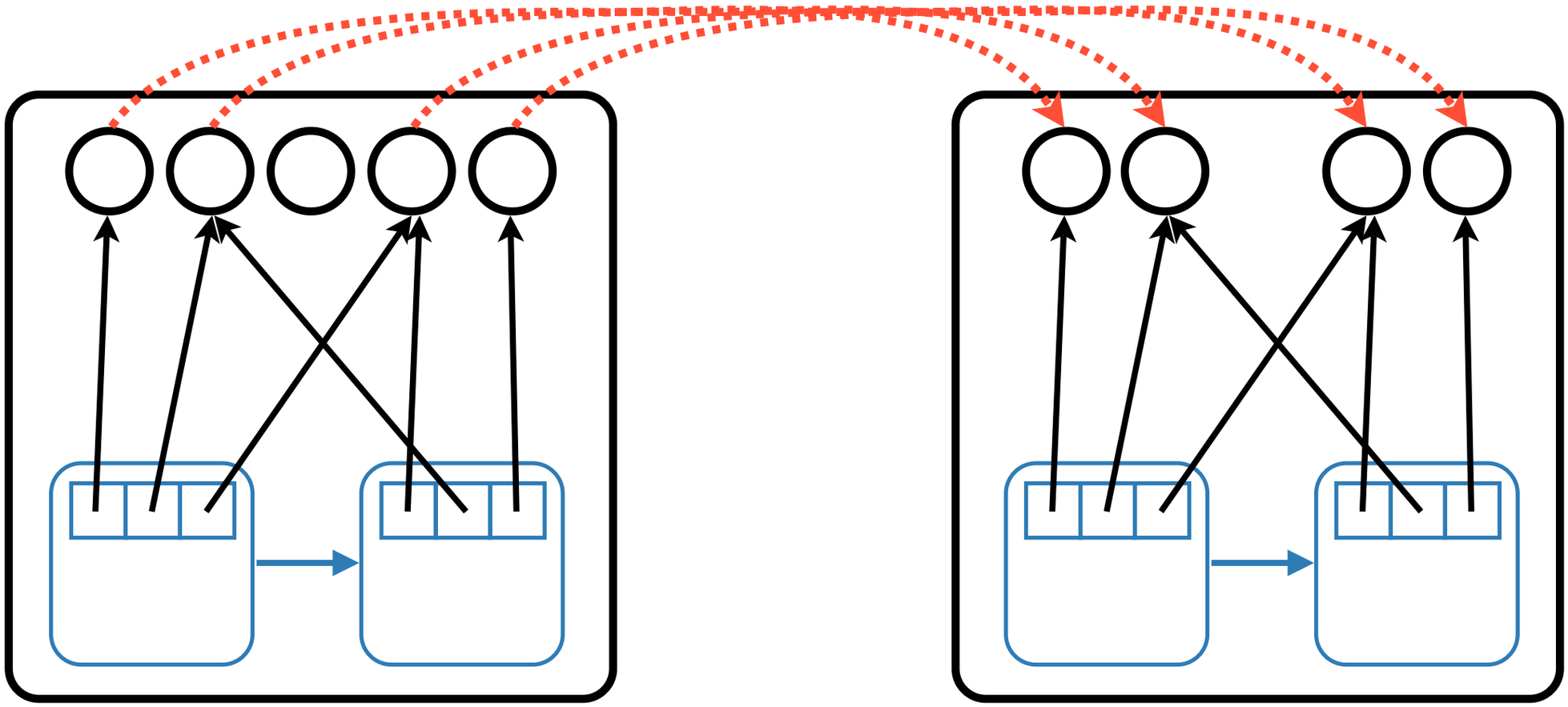

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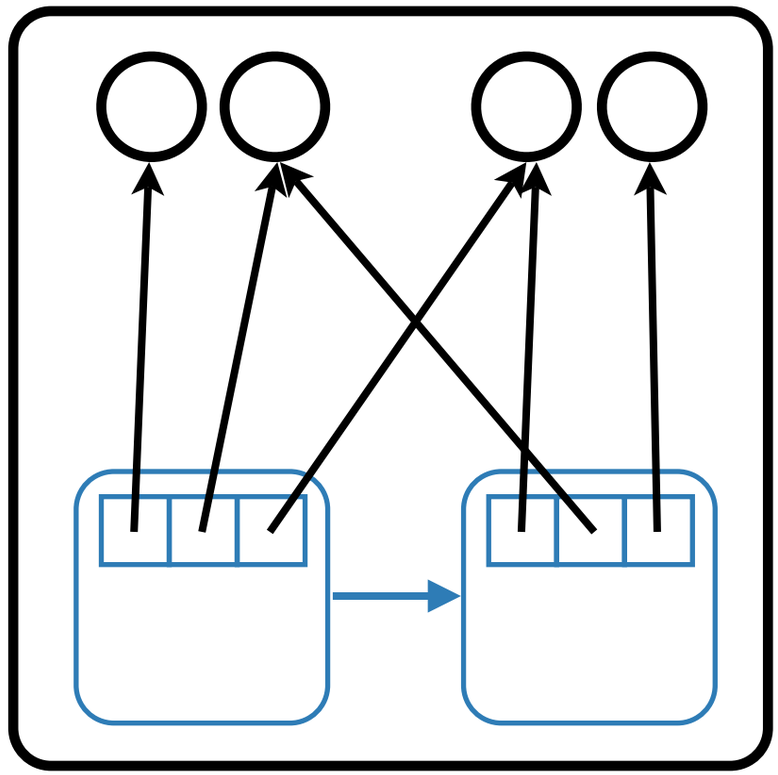

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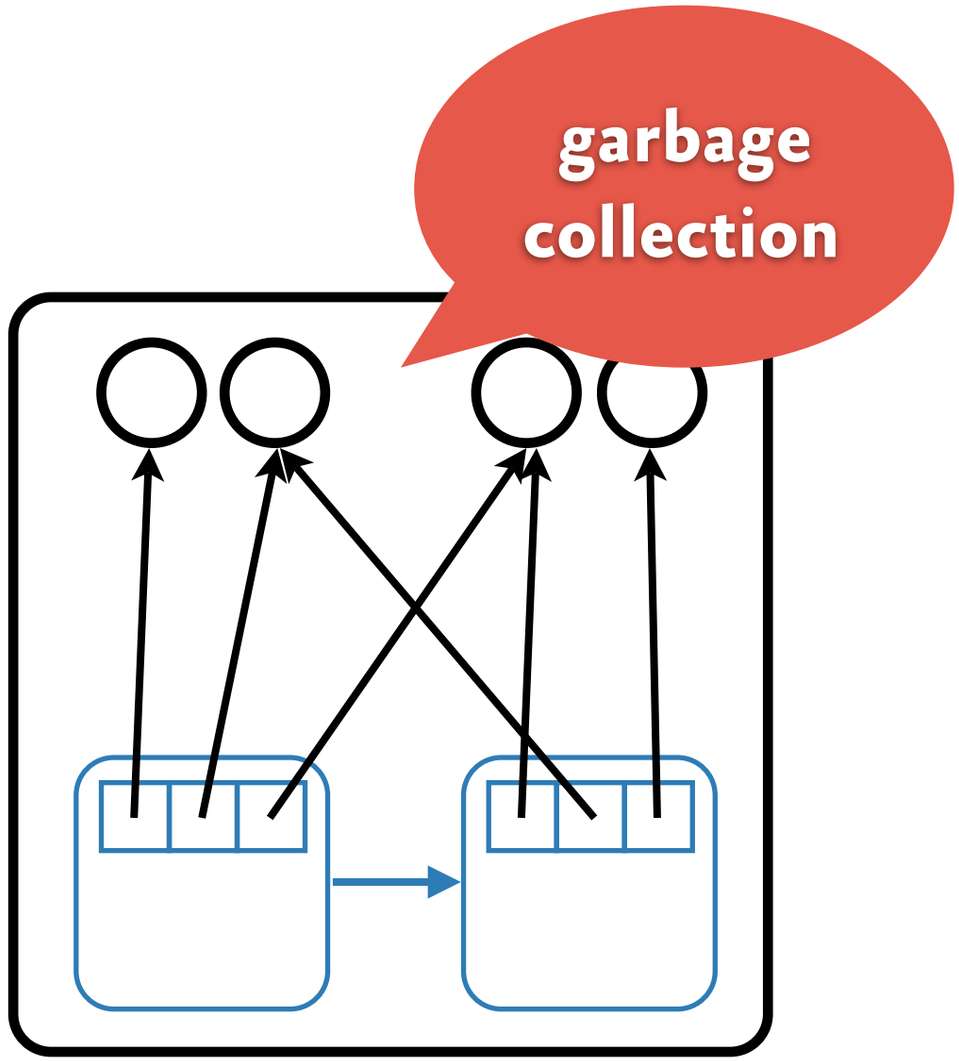
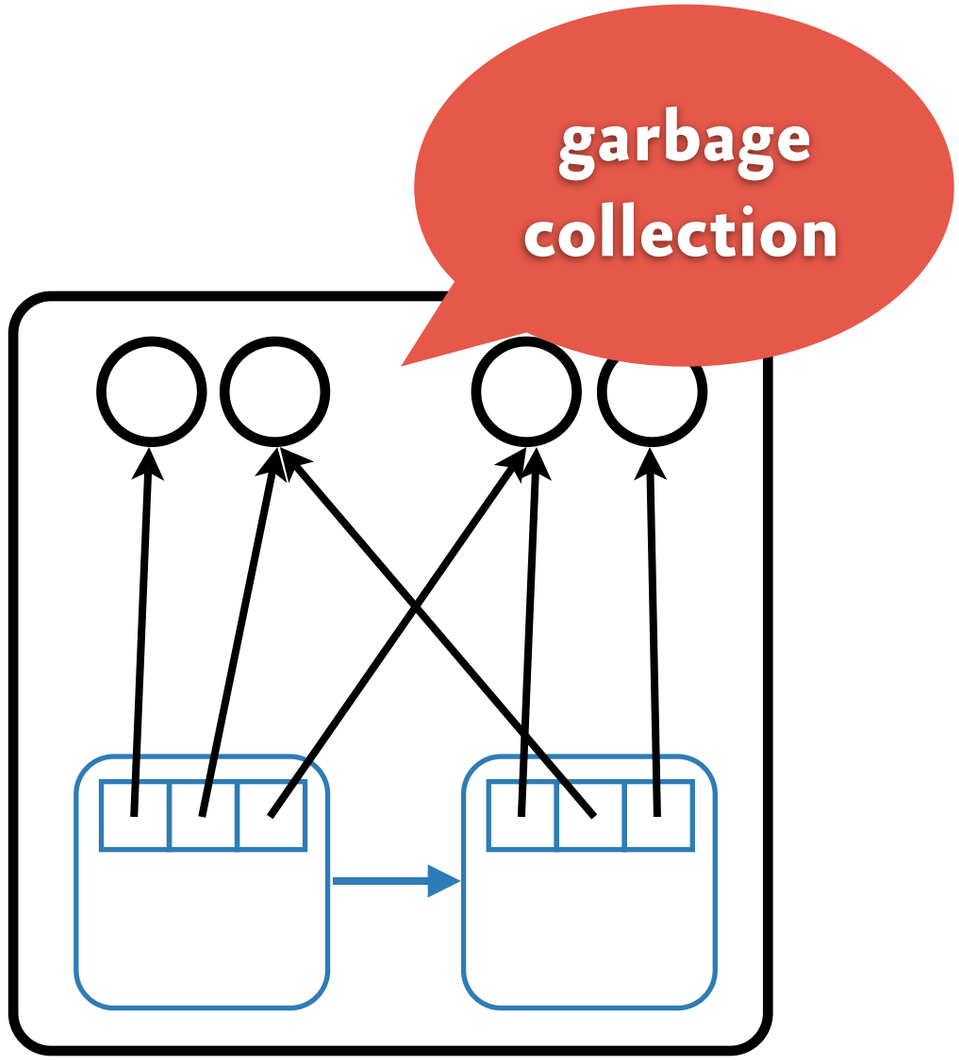


Copying a space

- copy only **live variables**
- copy only **live propagators** (not entailed)
- user can **keep variables live** by copying them explicitly
- **sharing** is preserved (essential!)
- inspired by **copying garbage collection**



Why copy propagators?

- **internal state**

  (e.g. variable-value graph)

- **self-contained spaces**

  (important for multi-threaded applications)

- **enable propagator rewriting**

  (different set of propagators in different spaces)

SAT -
a constraint satisfaction problem

SAT

- D = {false,true}
- Constraints: $\bigvee_i l_i$ where $l_i = x_i$ or $l_i = \neg x_i$

  (clauses) (literals)

- Model:

  conjunctive normal form, e.g.

  $$(\neg X \vee Y \vee Z) \wedge Y \wedge (X \vee \neg Y \vee \neg Z) \wedge (X \vee \neg Z)$$

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  positive literal

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  unit clause

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positive literal

unit clause

negative literal

SAT - interesting?

- **simple language**
- **NP complete**
- **applications:**
  - model checking
  - verification
  - test generation
  - planning

Algorithms for SAT

History: SAT in the Sixties

- **Martin Davis, Hilary Putnam** (1960!)

  iteratively add **resolvents**

  refute formula by finding the empty clause

- **Martin Davis, George Logemann, Donald W. Loveland** (1962)

  very similar to DP, but using **splitting rule**

DPLL in 1962

- **rule 1:**

  remove unit clauses (and detect inconsistency)

- **rule 2:**

  remove clauses with literals that occur only
  either positively *or* negatively (*pure* literals)

- **rule 3:**

  pick a variable *x*, construct F'=F[*x*:=true] in memory, **put** F[x:=false] **on tape**

  if F' not satisfiable, get the next formula from the tape

DPLL in 1962

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cafeterial stack-of-plates scheme!

DPLL in 1962

- **transform your formula into CNF**

  (pencil and paper!)

- **create punchcards,** one for each clause

- **feed punchcards** into an IBM 704

- **run the algorithm**

DPLL in 1962

- **transform your formula into CNF**
  (pencil and paper!)
- **create punchcards,** one for each clause
- **feed punchcards** into an IBM 704
- **run the algorithm**

DPLL: Example

$$
\begin{aligned}
\omega_1 &= x_1 \vee x_2 \\
\omega_2 &= \neg x_1 \vee \neg x_2 \\
\omega_3 &= x_1 \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
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no unit clause, no pure literal

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DPLL: Example

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\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_3 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_3 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_3 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_3 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_3 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

no unit clause, no pure literal

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
\qquad
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_4 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_4 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_5 \\
\omega_5 &= \bot \vee \neg x_5 \\
\omega_6 &= \top \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_4 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_5 \\
\omega_5 &= \bot \vee \neg x_5 \\
\omega_6 &= \top \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$x_4 = \top$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_4 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_5 \\
\omega_5 &= \bot \vee \neg x_5 \\
\omega_6 &= \top \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$x_4 = \top$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$x_4 = \bot$$

$$\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \bot \vee x_5 \\
\omega_5 &= \bot \vee \neg x_5 \\
\omega_6 &= \top \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}$$

tape

$$\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}$$

$$\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_5 \\
\omega_5 &= \neg x_5 \\
\omega_6 &= \top \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_5 \\
\omega_5 &= \neg x_5 \\
\omega_6 &= \top \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_4 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_5 \\
\omega_5 &= \neg x_5 \\
\omega_6 &= \top \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_4 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_5 = \top$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= x_5 \\
\omega_5 &= \neg x_5 \\
\omega_6 &= \top \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_4 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$x_5 = \top$$

$$\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \\
\omega_5 &= \bot \\
\omega_6 &= \top \\
\omega_7 &= \top \vee x_6 \\
\omega_8 &= \bot \vee \neg x_6
\end{aligned}$$

ω_4 **is unit**

tape

$$\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}$$

$$\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \\
\omega_5 &= \bot \\
\omega_6 &= \top \\
\omega_7 &= \top \vee x_6 \\
\omega_8 &= \bot \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

tape

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

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\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \vee x_5 \\
\omega_5 &= \top \vee \neg x_5 \\
\omega_6 &= \bot \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

tape

$$
\begin{aligned}
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\end{aligned}
$$

DPLL: Example

$x_6 = \bot$

$$
\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \\
\omega_5 &= \top \\
\omega_6 &= \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

ω_6 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
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\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

DPLL: Example

$x_6 = \bot$

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\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \\
\omega_5 &= \top \\
\omega_6 &= \top \\
\omega_7 &= x_5 \vee \bot \\
\omega_8 &= \neg x_5 \vee \top
\end{aligned}
$$

ω_6 is unit

tape

$$
\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
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DPLL: Example

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\end{aligned}
$$

DPLL: Example

$$x_5 = \top$$

$$\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \\
\omega_5 &= \top \\
\omega_6 &= \top \\
\omega_7 &= x_5 \vee \bot \\
\omega_8 &= \neg x_5 \vee \top
\end{aligned}$$

tape

$$\begin{aligned}
\omega_1 &= \top \vee x_2 \\
\omega_2 &= \bot \vee \neg x_2 \\
\omega_3 &= \top \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}$$

DPLL: Example

$$x_5 = \top$$

$$\begin{aligned}
\omega_1 &= \top \\
\omega_2 &= \top \\
\omega_3 &= \top \\
\omega_4 &= \top \\
\omega_5 &= \top \\
\omega_6 &= \top \\
\omega_7 &= \top \\
\omega_8 &= \top
\end{aligned}$$

tape

$$\begin{aligned}
\omega_1 &= \top \vee x_2 \\
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DPLL: variable selection, 1962

- **observation:**

  efficiency depends on variable chosen for splitting

- **strategy:**

  if no solution within time bounds,

DPLL: variable selection, 1962

- **observation:**

  efficiency depends on variable chosen for splitting

- **strategy:**

  if no solution within time bounds,
  shuffle the punchcards!

DPLL today

- **basically unchanged!**
- **but:**
  - store variable assignments, not formulae
  - use trailing, not copying
  - singleton removal: unit propagation
  - slightly advanced splitting heuristic

DPLL today: unit propagation

- **idea:**

  if all but one literal are false, the remaining literal must be true

- **simple propagator:**
  - failure if no literal is true
  - propagate if all but one literal are false
  - (entailed if one literal is true)

DPLL today: trailing

- **basic idea:**
  - remember all modifications
  - undo them on backtracking
- **what to trail for SAT:**
  - assigning a variable to true or false
  - each variable is modified exactly once on one path!

Example: DPLL with trailing

$$
\begin{aligned}
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\omega_3 &= x_1 \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
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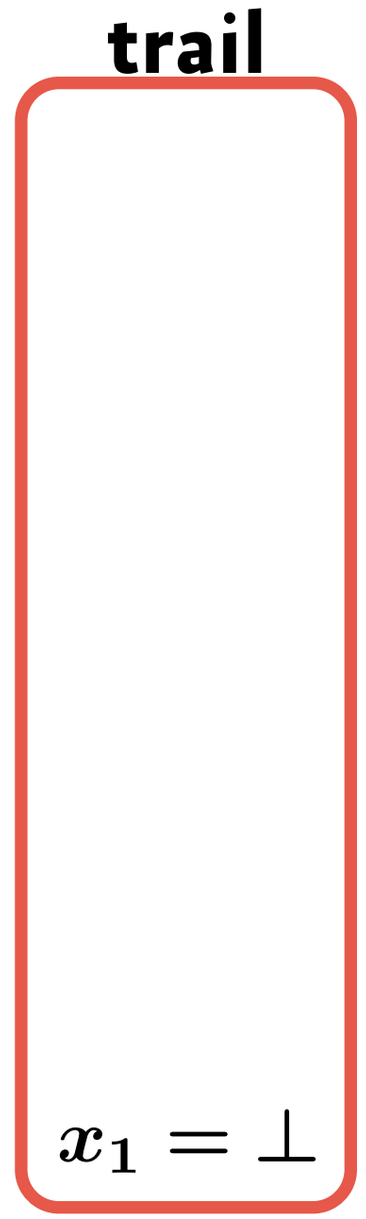
$$



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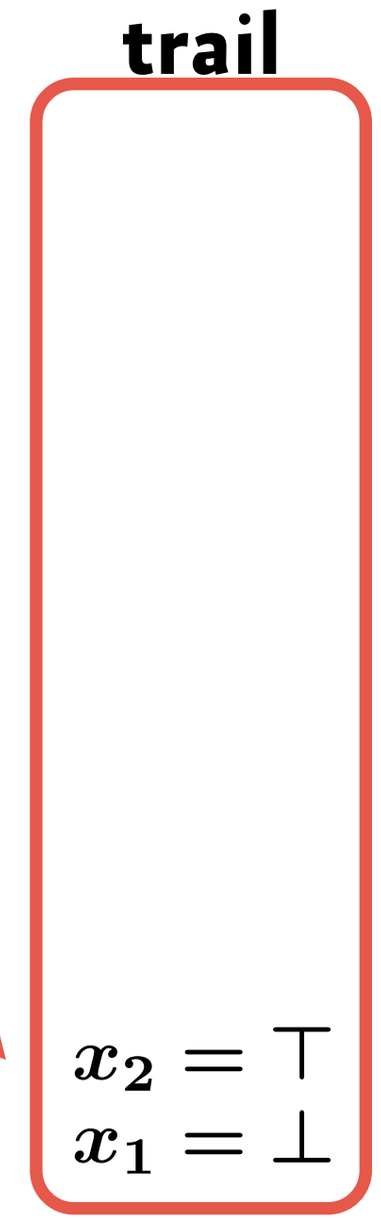

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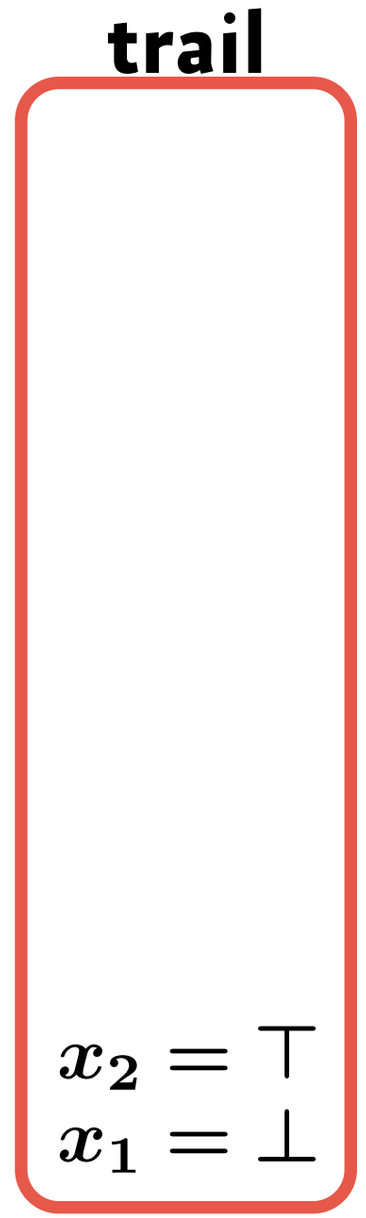

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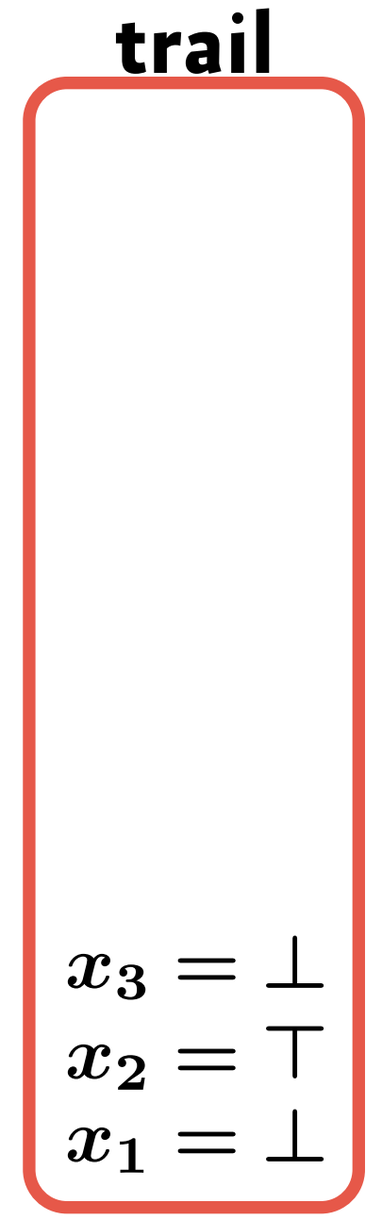

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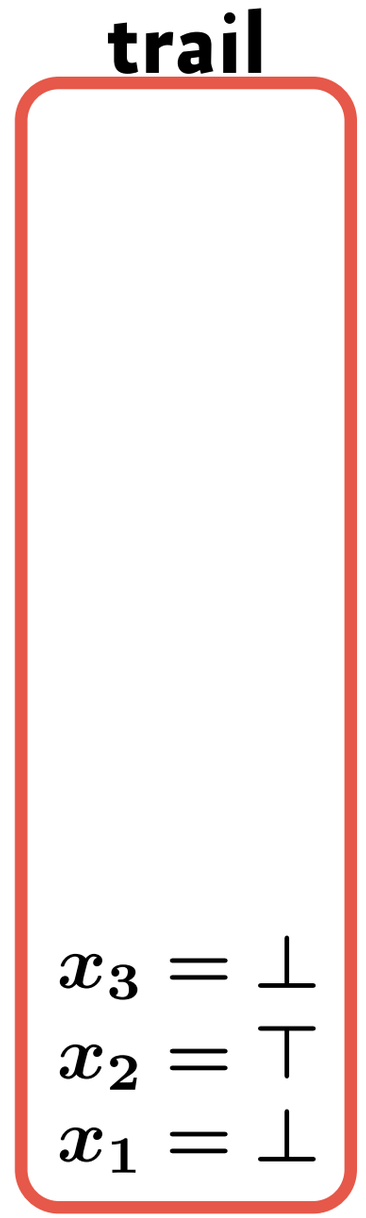

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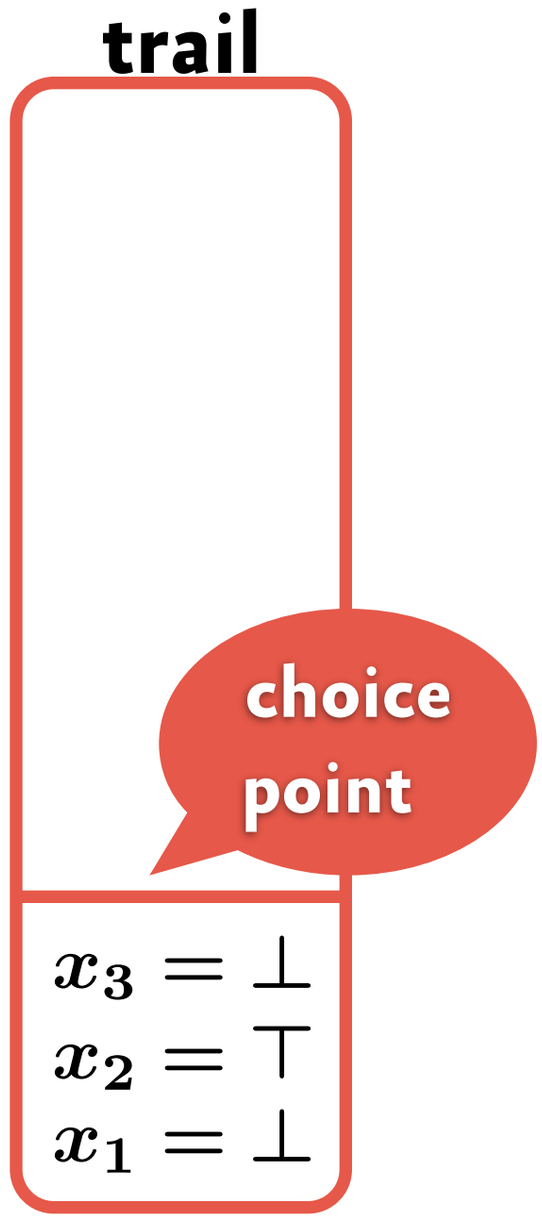

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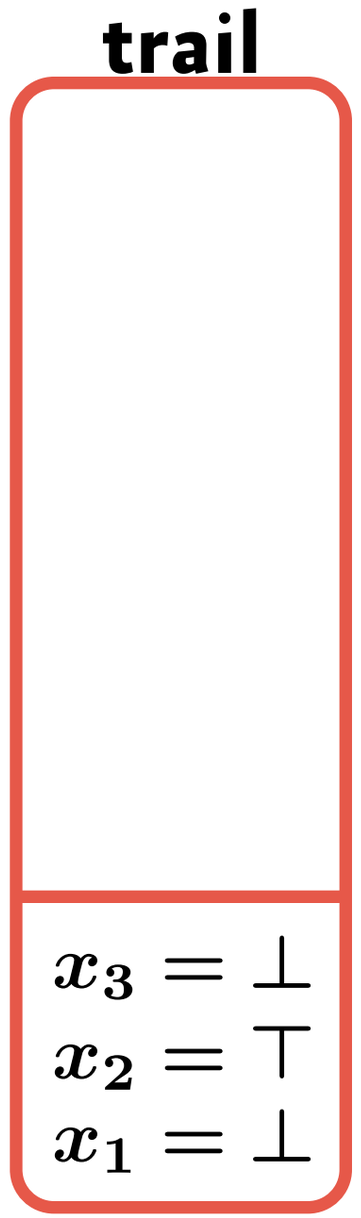

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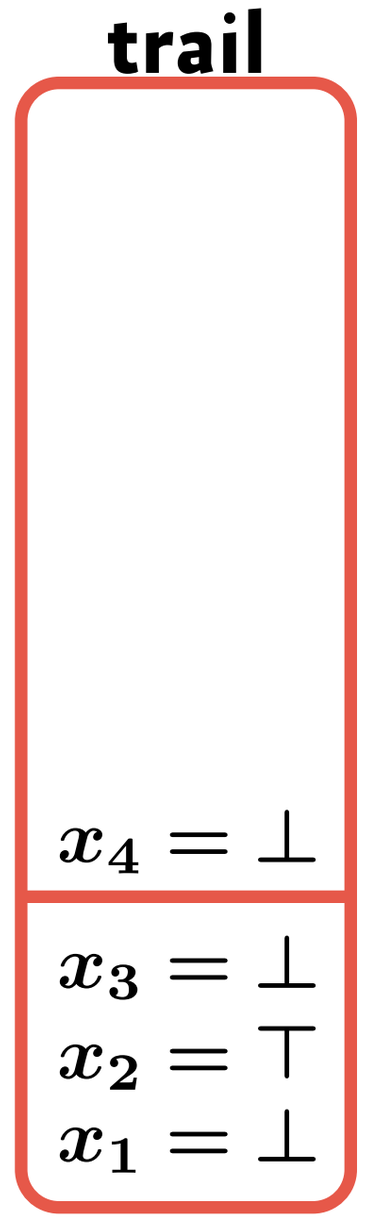

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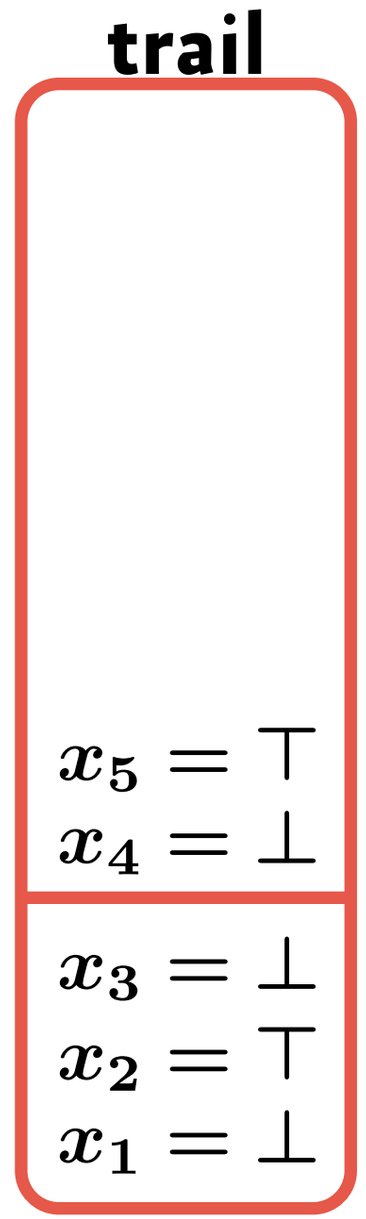

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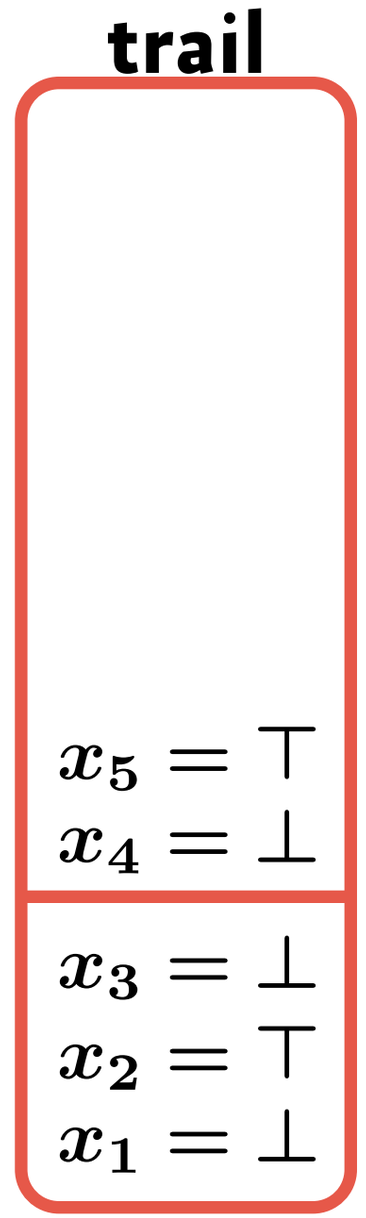

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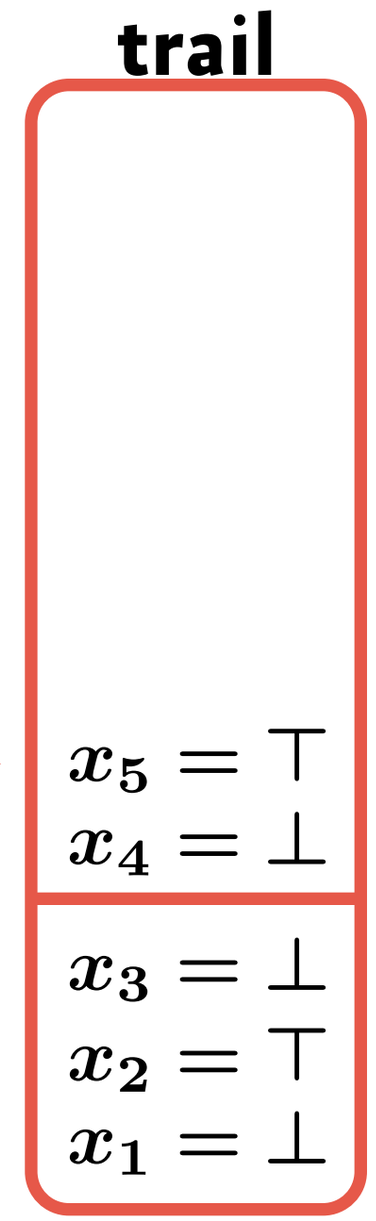

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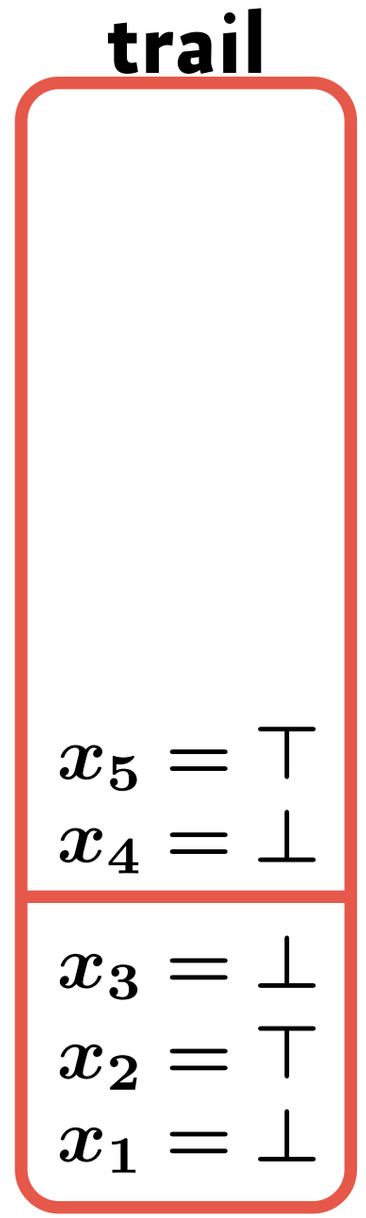

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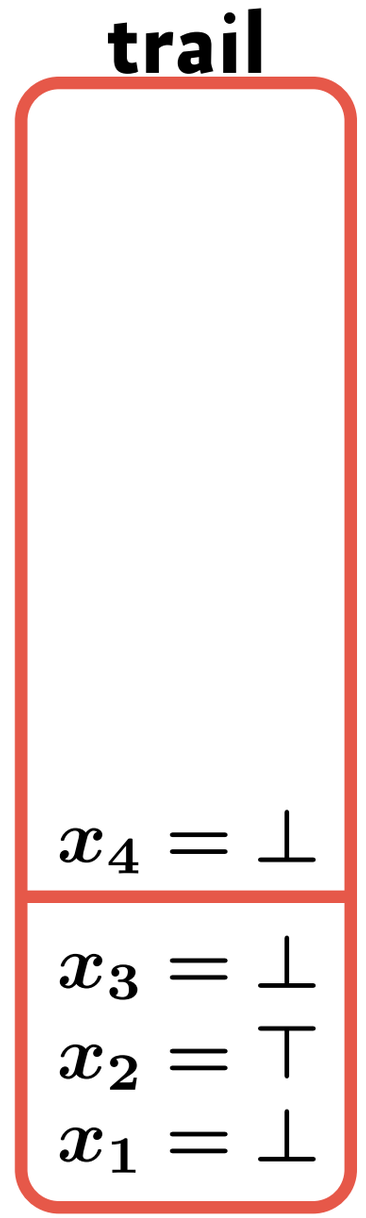

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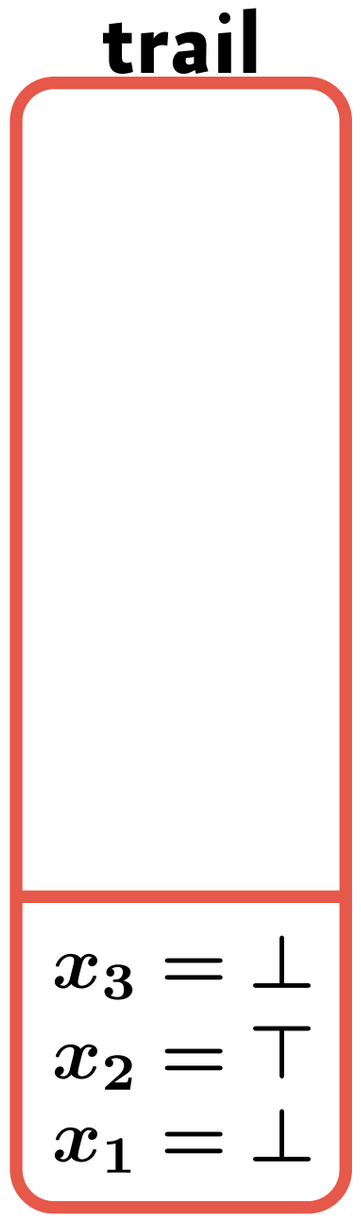

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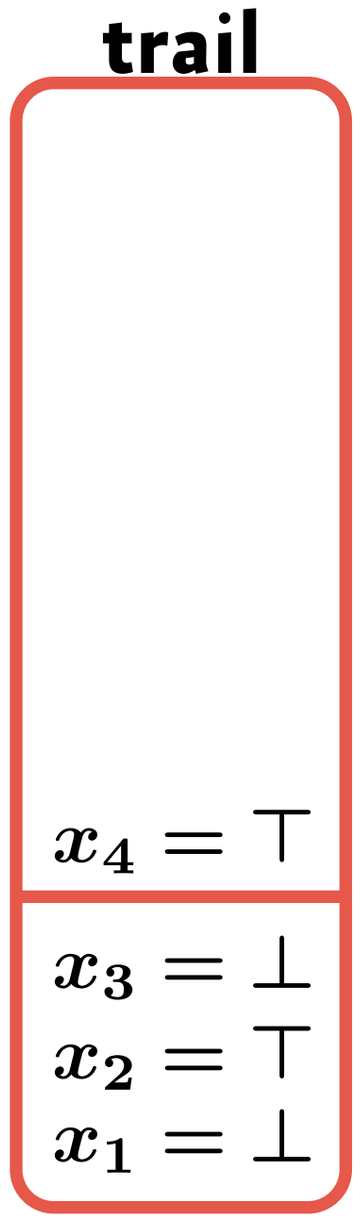

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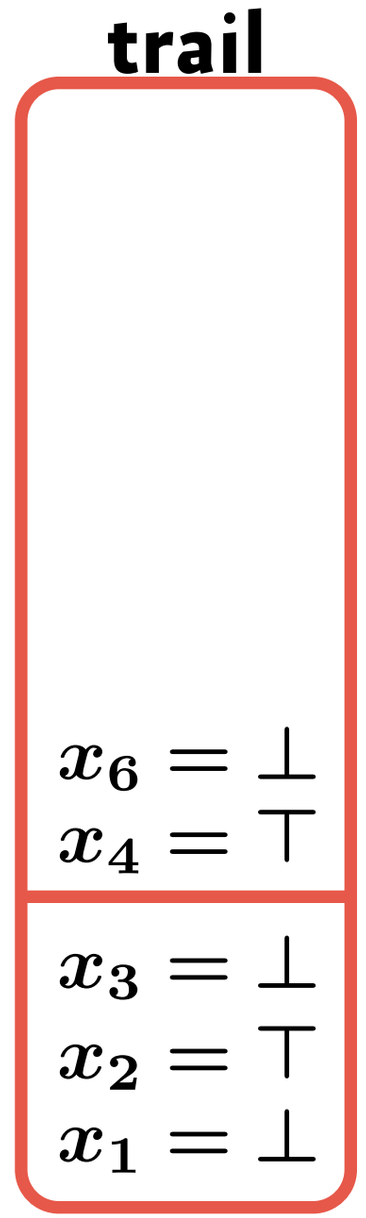

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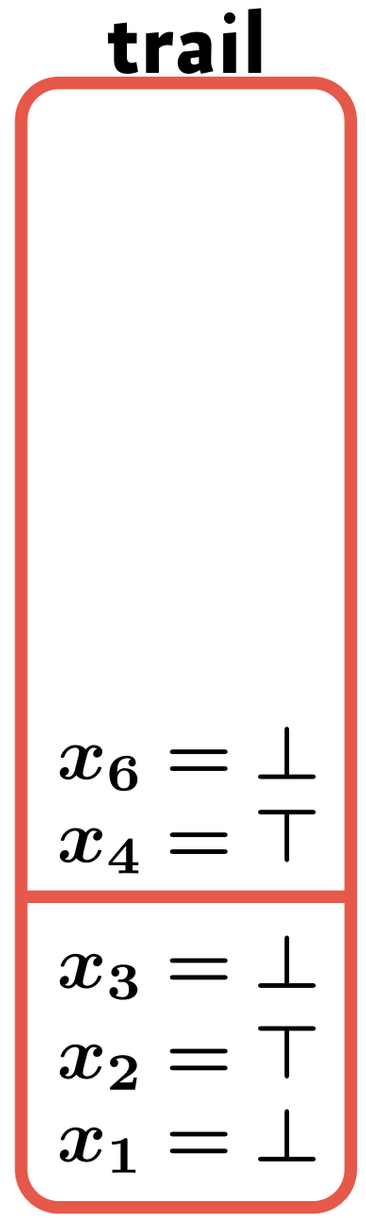

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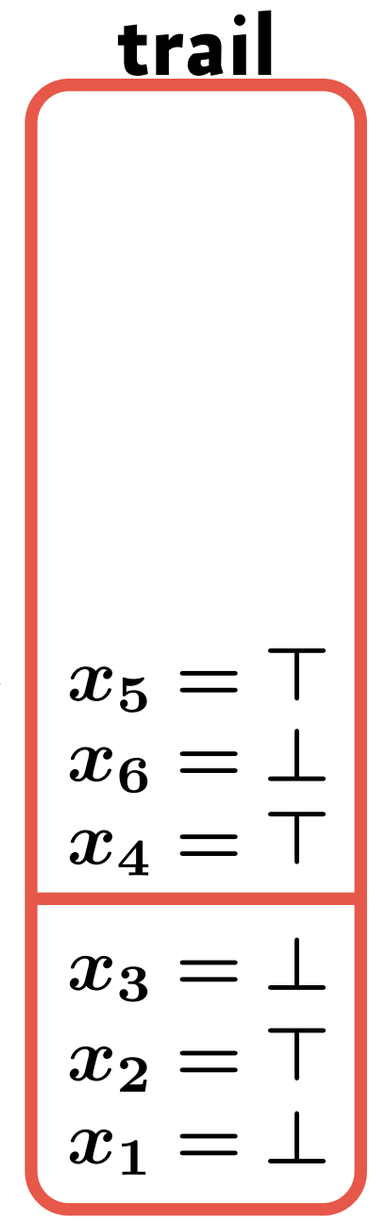

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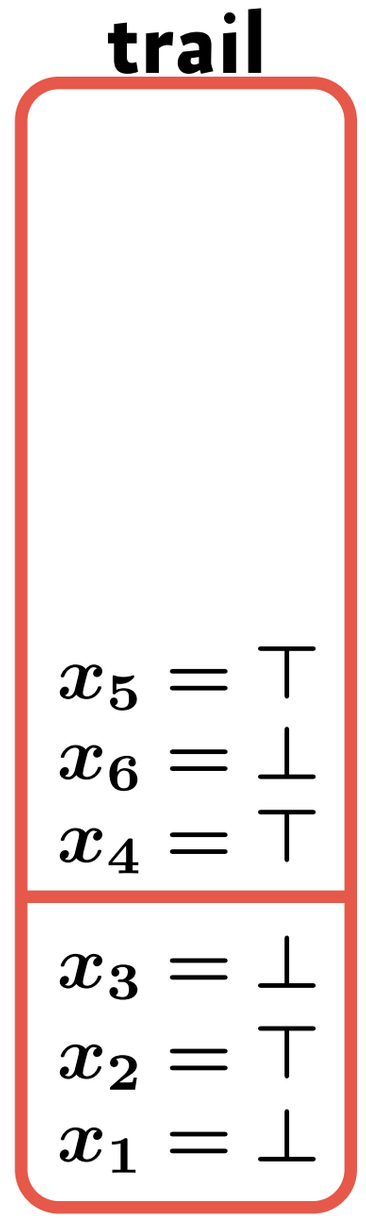
\end{aligned}
$$



Example: DPLL with trailing

$$
\begin{aligned}
\omega_1 &= x_1 \vee x_2 \\
\omega_2 &= \neg x_1 \vee \neg x_2 \\
\omega_3 &= x_1 \vee \neg x_3 \\
\omega_4 &= x_3 \vee x_4 \vee x_5 \\
\omega_5 &= x_4 \vee \neg x_5 \\
\omega_6 &= \neg x_4 \vee \neg x_6 \\
\omega_7 &= x_5 \vee x_6 \\
\omega_8 &= \neg x_5 \vee \neg x_6
\end{aligned}
$$

Solution!

trail

$\boldsymbol{x_5 = \top}$
$\boldsymbol{x_6 = \bot}$
$\boldsymbol{x_4 = \top}$

$\boldsymbol{x_3 = \bot}$
$\boldsymbol{x_2 = \top}$
$\boldsymbol{x_1 = \bot}$

Trailing and entailment

- **remember:**
  - unit propagator is entailed if one literal is true
  - entailed propagators can be removed
- **removal is a modification!**
  - put removal of unit propagator on the trail

Literature

- Davis, Putnam. *A Computing Procedure for Quantification Theory*. JACM 7(1).
- Davis, Logemann, Loveland. *A Machine Program for Theorem Proving*. CACM 5(7).

Making it fly

Event sets for unit propagation

- **consider an *n*-ary clause**
- **observation:**
  - no propagation when less than *n*-1 literals determined
  - no failure check when less than *n*-1 literals determined
- **required:**
  - find out when the (*n*-1)th literal is determined

Watched literals

- at any time, "watch" (subscribe to) **two non-false literals *p,q***

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may yield failure

Watched literals and trailing

- **observation:**

  watched literals are *backtrack-safe:*

  if clause *c* watches two non-failed literals, they will still be non-failed when backtracking!

- **consequence:**

  do not trail watches, keep them where they are!

- **benefit:**

  watches tend to move to "good" places and stay there

Example: watched literals

$$\overset{?}{\neg x_1} \vee x_4 \vee \neg x_7 \vee x_{11} \vee x_{12} \vee \overset{?}{x_{15}}$$

Example: watched literals

clause: $\neg x_1 \vee x_4 \vee \neg x_7 \vee x_{11} \vee x_{12} \vee x_{15}$

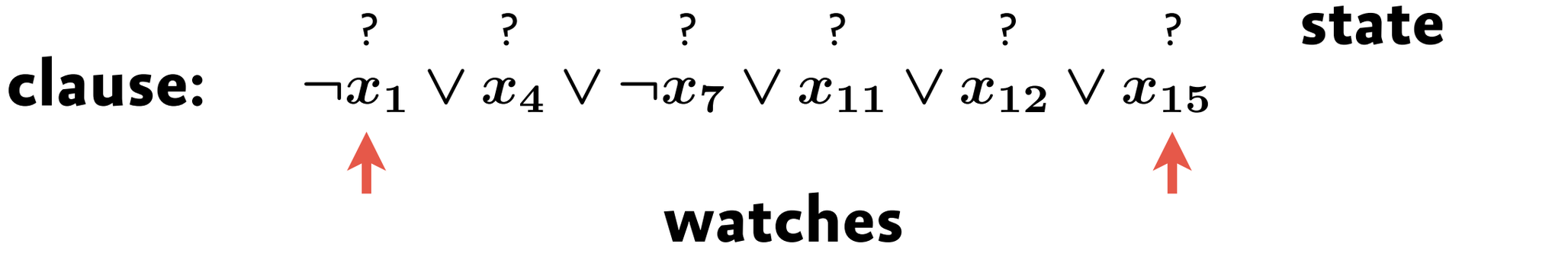
? ? ? ? ? ?

Example: watched literals

clause: $\overset{?}{\neg x_1} \vee \overset{?}{x_4} \vee \overset{?}{\neg x_7} \vee \overset{?}{x_{11}} \vee \overset{?}{x_{12}} \vee \overset{?}{x_{15}}$

watches

Example: watched literals



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$$x_1 = \top$$

Example: watched literals

$$\begin{array}{ccccccc} \bot & ? & ? & ? & ? & ? \\ \neg x_1 \vee x_4 \vee \neg x_7 \vee x_{11} \vee x_{12} \vee x_{15} \end{array}$$

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$$x_{15} = \bot, x_7 = \top, x_{11} = \bot$$

Example: watched literals

$$\underset{\neg x_1}{\bot} \vee \underset{x_4}{?} \vee \underset{\neg x_7}{\bot} \vee \underset{x_{11}}{\bot} \vee \underset{x_{12}}{?} \vee \underset{x_{15}}{\bot}$$

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$$x_4 = \bot$$

Example: watched literals

$$\begin{array}{cccccc} \bot & \bot & \bot & \bot & ? & \bot \\ \neg x_1 \vee & x_4 \vee & \neg x_7 \vee & x_{11} \vee & x_{12} \vee & x_{15} \end{array}$$

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Example: watched literals

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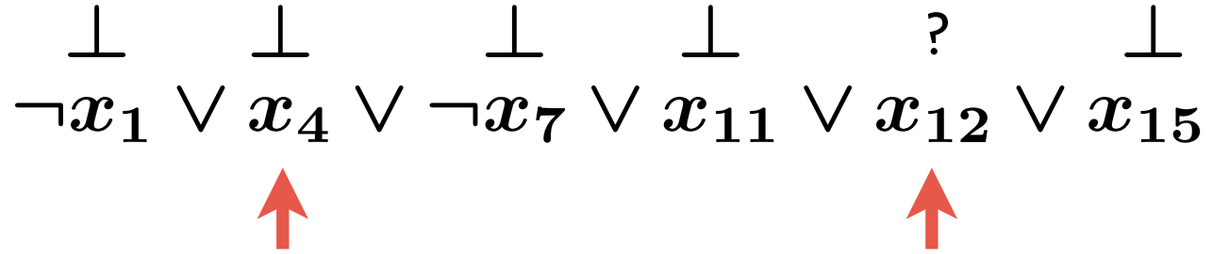
$x_4 = \bot$

only one left: $x_{12} = \top$

Example: watched literals



$x_4 = \bot$

only one left: $x_{12} = \top$

let's assume conflict, backtrack

Example: watched literals

$$\begin{array}{ccccccc} \bot & ? & ? & ? & ? & ? \\ \neg x_1 \vee & x_4 \vee & \neg x_7 \vee & x_{11} \vee & x_{12} \vee & x_{15} \end{array}$$

Example: watched literals

$$\begin{array}{cccccc} \bot & ? & ? & ? & ? & ? \\ \neg x_1 \vee & x_4 \vee & \neg x_7 \vee & x_{11} \vee & x_{12} \vee & x_{15} \end{array}$$

watches stay where they are!

Example: watched literals

$$\begin{array}{ccccccc} \bot & ? & ? & ? & ? & ? \\ \neg x_1 \vee & x_4 \vee & \neg x_7 \vee & x_{11} \vee & x_{12} \vee & x_{15} \end{array}$$

Example: watched literals

$$\begin{array}{ccccccccccc} \bot & & ? & & ? & & ? & & ? & & ? \\ \neg x_1 & \vee & x_4 & \vee & \neg x_7 & \vee & x_{11} & \vee & x_{12} & \vee & x_{15} \end{array}$$

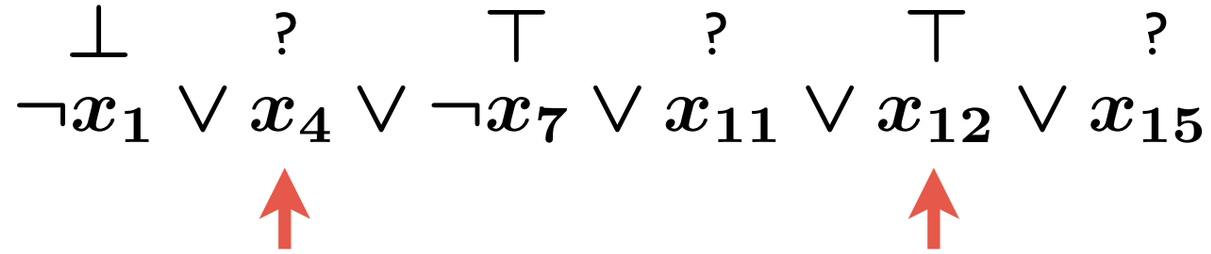
$$x_7 = \bot, x_{12} = \top$$

Example: watched literals

$$\begin{array}{ccccccccccc} & \bot & & ? & & \top & & ? & & \top & & ? \\ \neg x_1 & \vee & x_4 & \vee & \neg x_7 & \vee & x_{11} & \vee & x_{12} & \vee & x_{15} \\ & & \uparrow & & & & & & \uparrow & & \end{array}$$

$$x_7 = \bot, x_{12} = \top$$

Example: watched literals



Example: watched literals

$$\begin{array}{ccccccccccc} & \bot & & ? & & \top & & ? & & \top & & ? \\ \neg x_1 & \vee & x_4 & \vee & \neg x_7 & \vee & x_{11} & \vee & x_{12} & \vee & x_{15} \end{array}$$

$$x_4 = \bot$$

Example: watched literals

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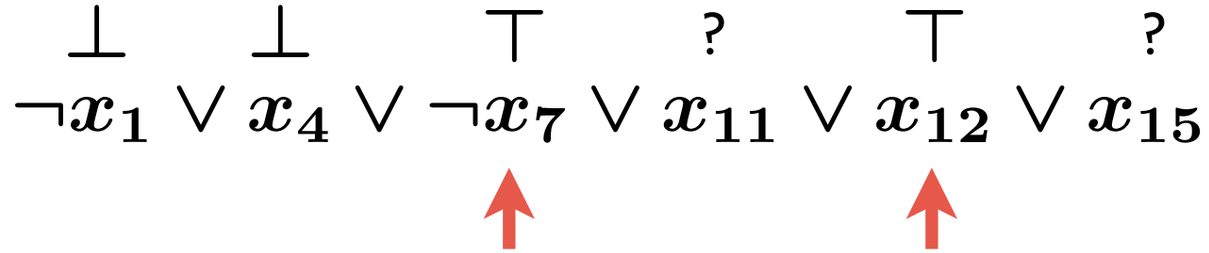
Example: watched literals



Conflicts

- **conflict = failure**
- **idea:**
  - analyze reason for conflict
  - add new (implied) constraint that prevents similar conflict during rest of search
- implied constraints: results of **resolution**

Recap: resolution

- **resolution rule:** $\dfrac{A \vee x \qquad B \vee \neg x}{A \vee B}$
- **trivial clause:** contains both x and $\neg x$
- **subsumed clause:** $A \leq B \Leftrightarrow A \wedge B \equiv A$
- **saturation:** F is saturated if no nontrivial, not subsumed clause can be added by resolution
- **satisfiability:** a saturated F is satisfiable iff it does not contain the empty clause

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$(x \vee y) \leq (x \vee y \vee \neg z)$

Resolution vs. DPLL

- **observation:**

  for a saturated formula *F*, no search is necessary to find a solution

- **why?**
  - if *F* unsatisfiable, it contains the empty clause
  - if *F* satisfiable, unit propagation achieves domain consistency **for *F***
- **why not saturate?**
  - adds exponential number of clauses

Conflict-clause learning

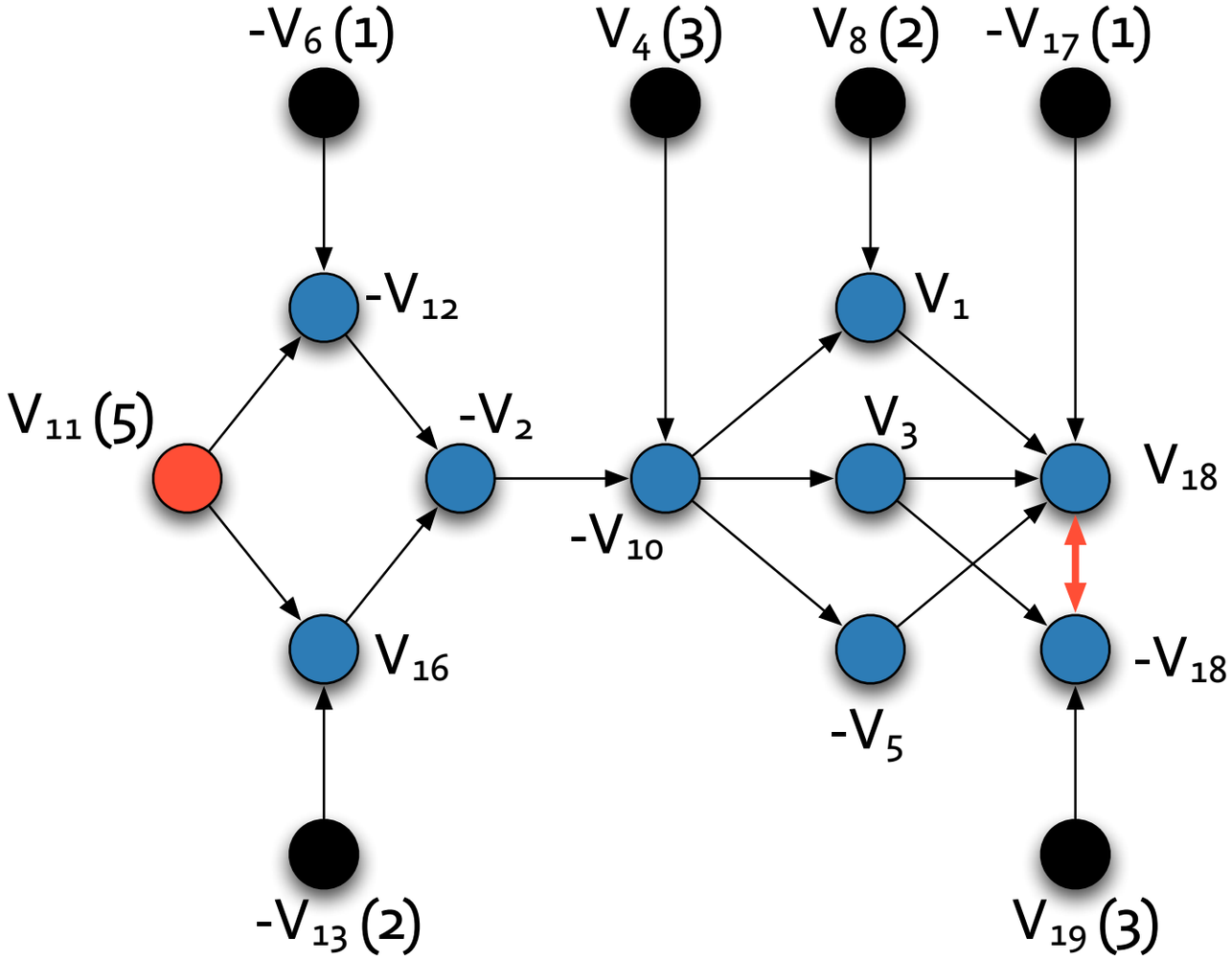
- **failure in DPLL = missing resolvent**

- **idea:**
  learn {the,a,some} missing resolvent(s) from failure!

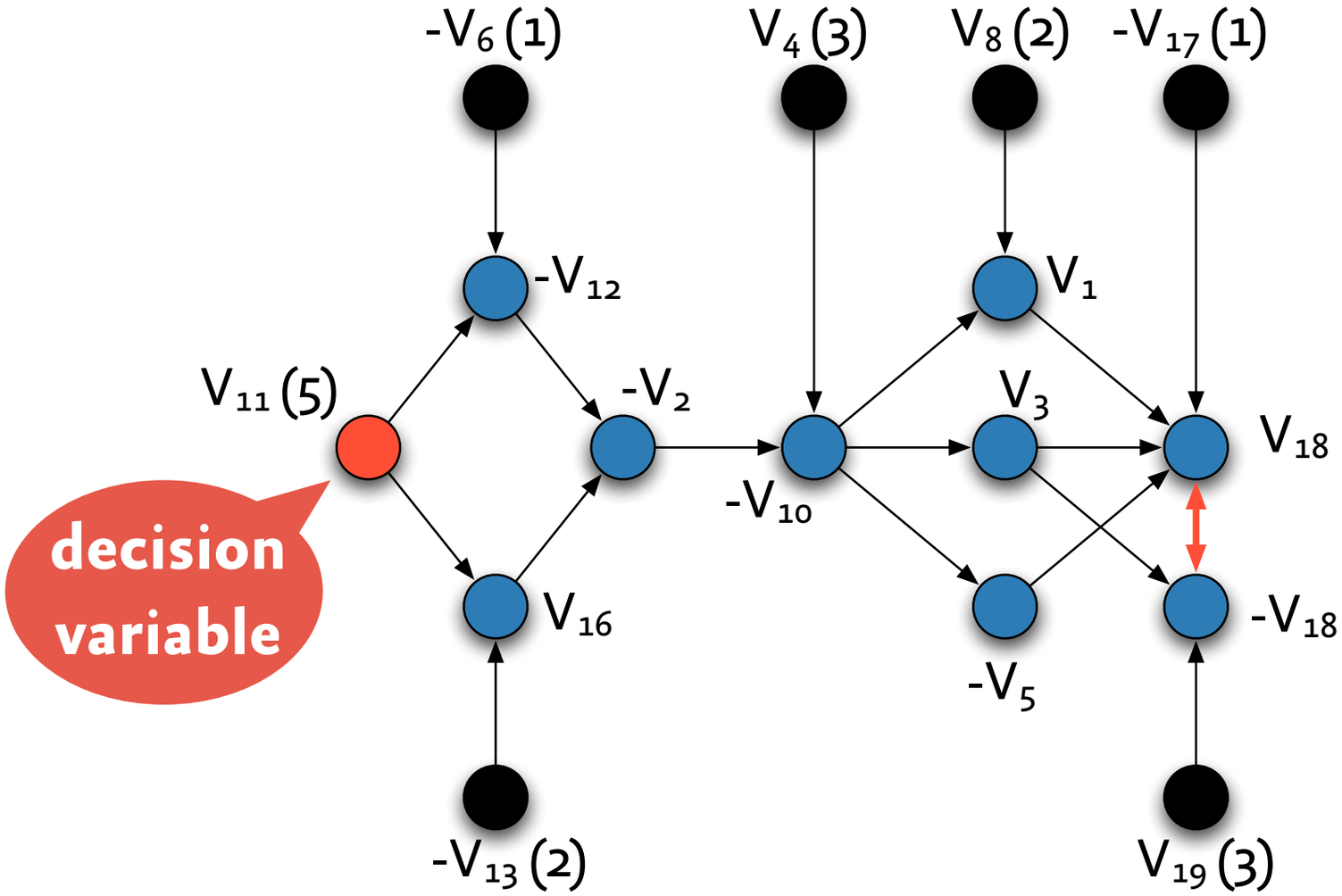
- **most simple version:**
  for inconsistent assignment $\{x \mapsto 1, y \mapsto 0, z \mapsto 1\}$
  learn clause $\neg(x \wedge \neg y \wedge z) \equiv \neg x \vee y \vee \neg z$

- **equivalent to exhaustive search!**

Implication graph



Implication graph



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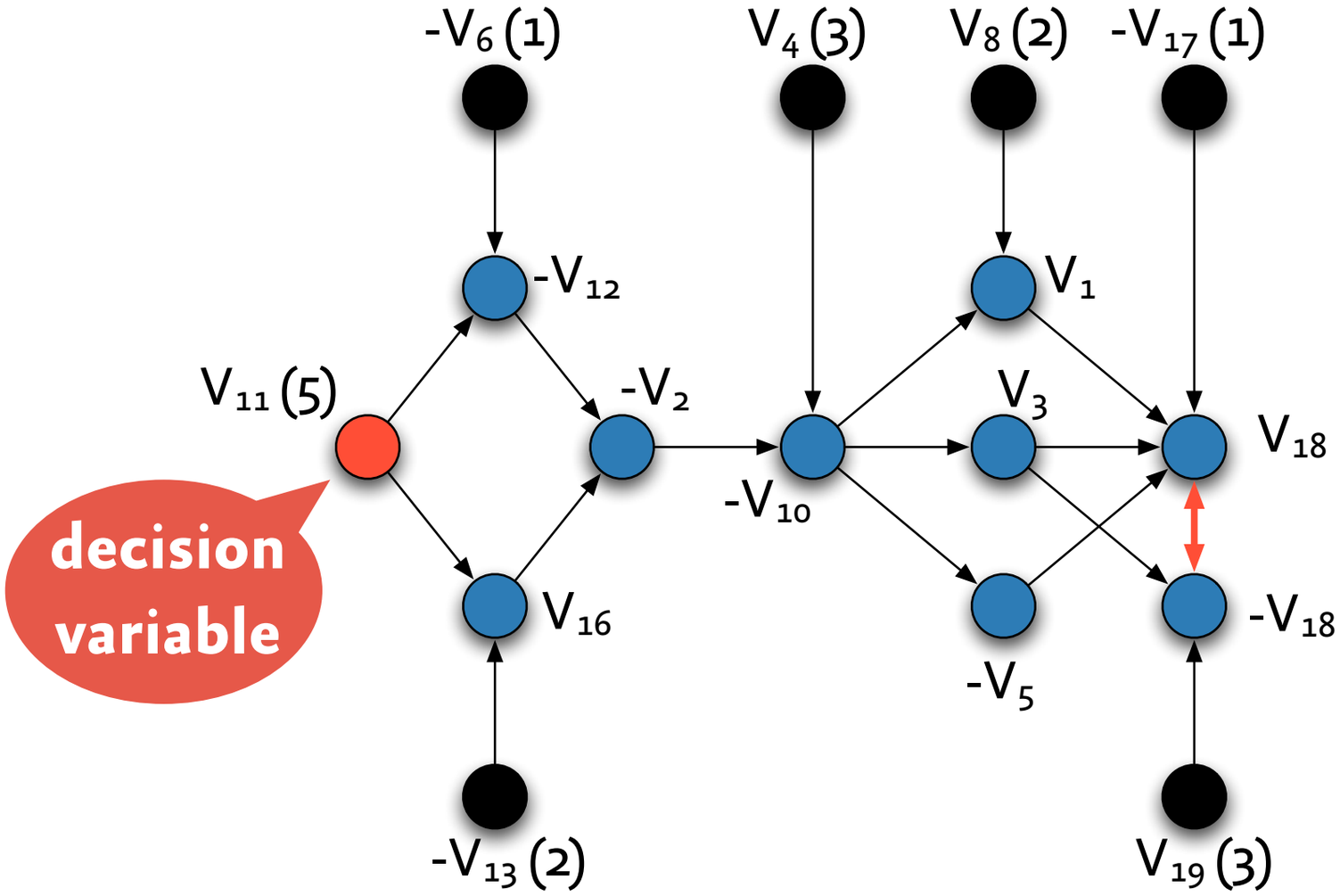
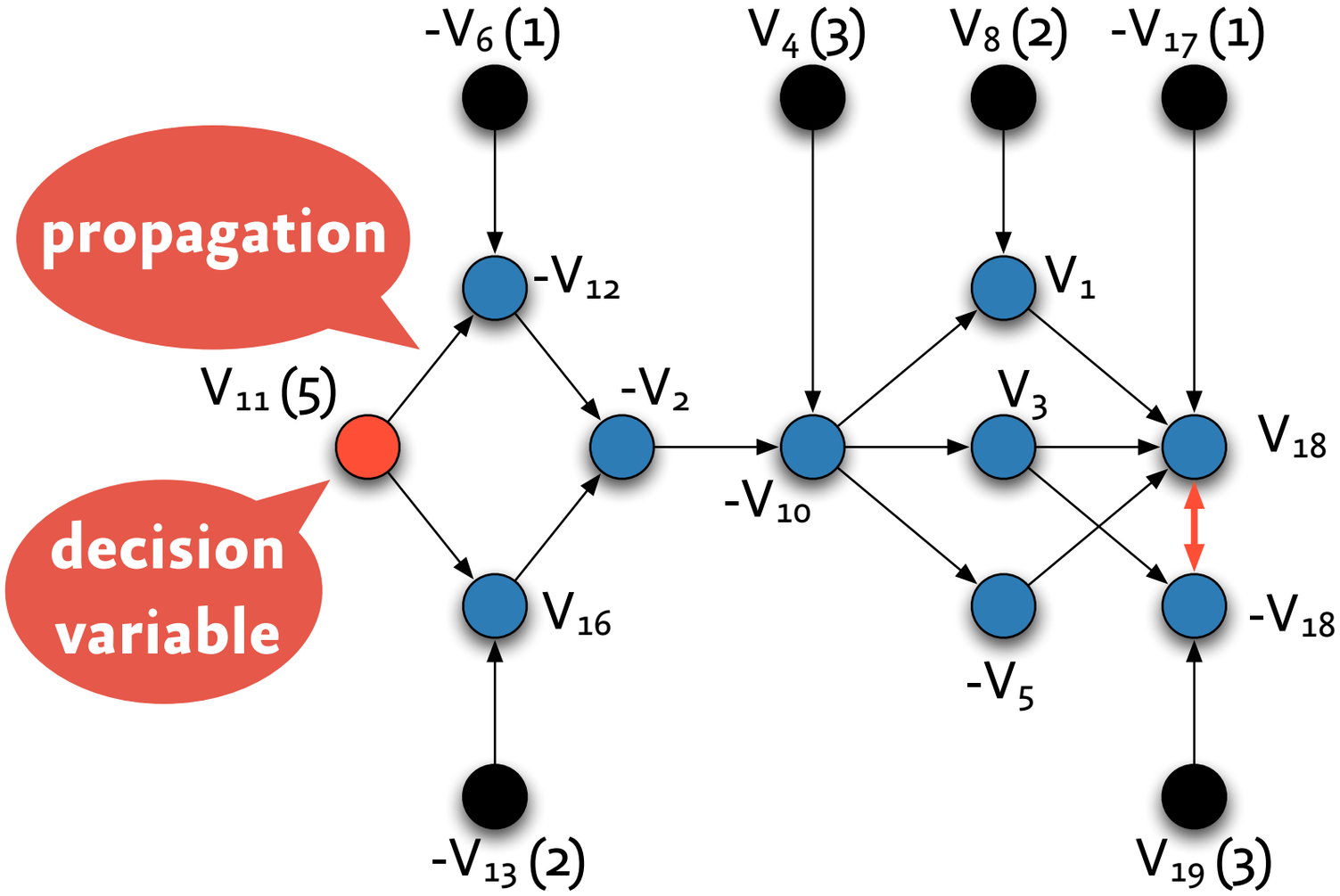


decision level: when was variable assigned? (depth in search)

Implication graph



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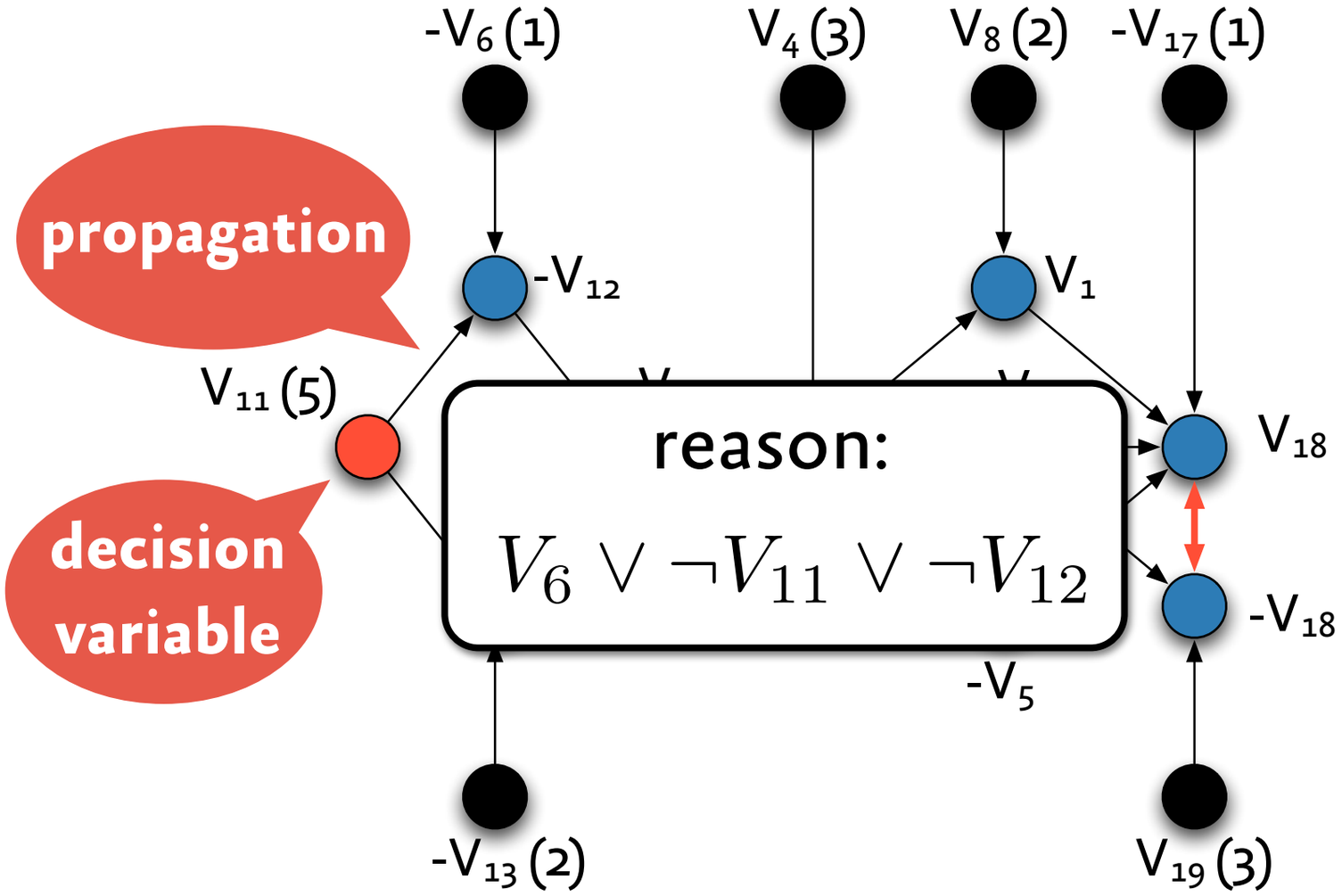

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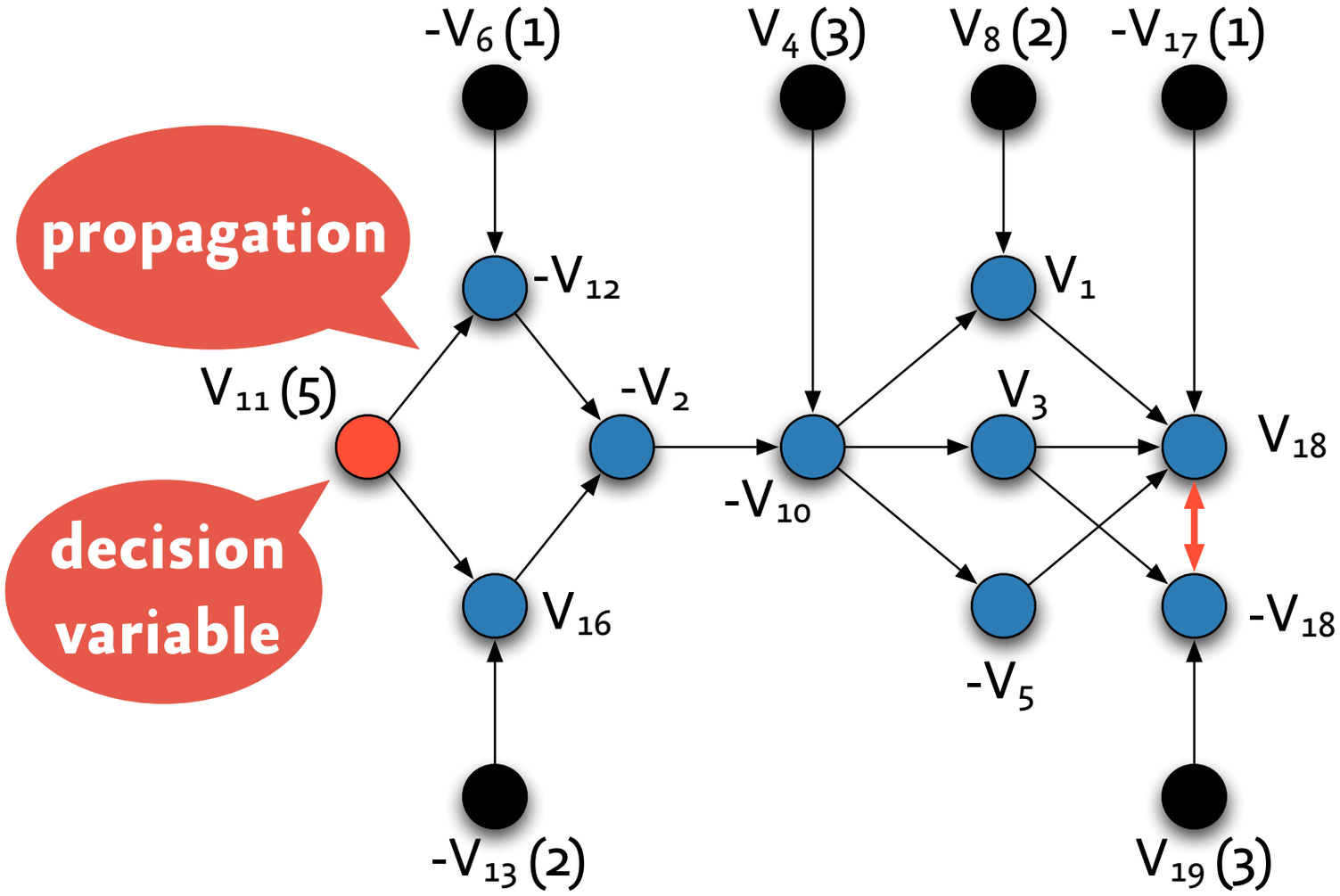

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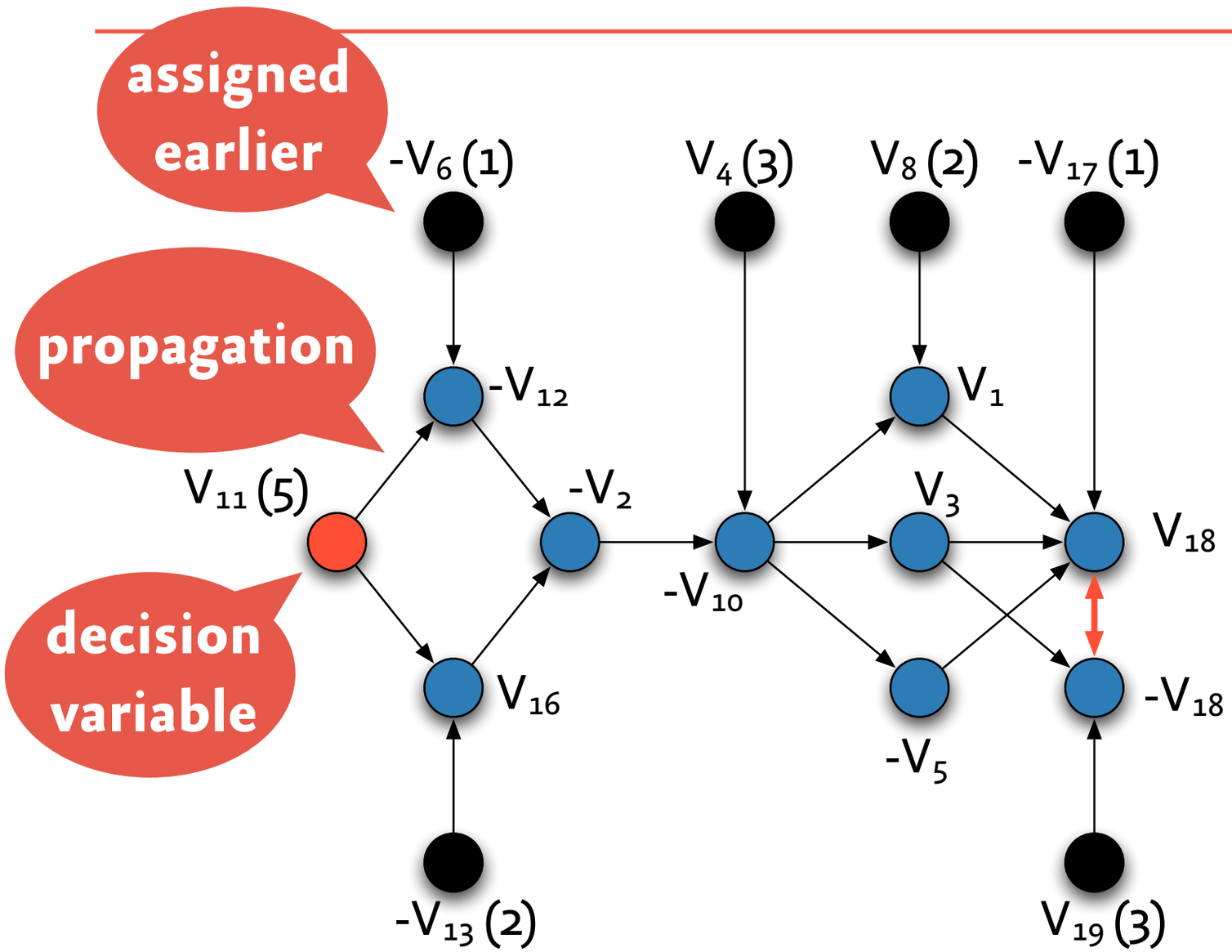
Implication graph



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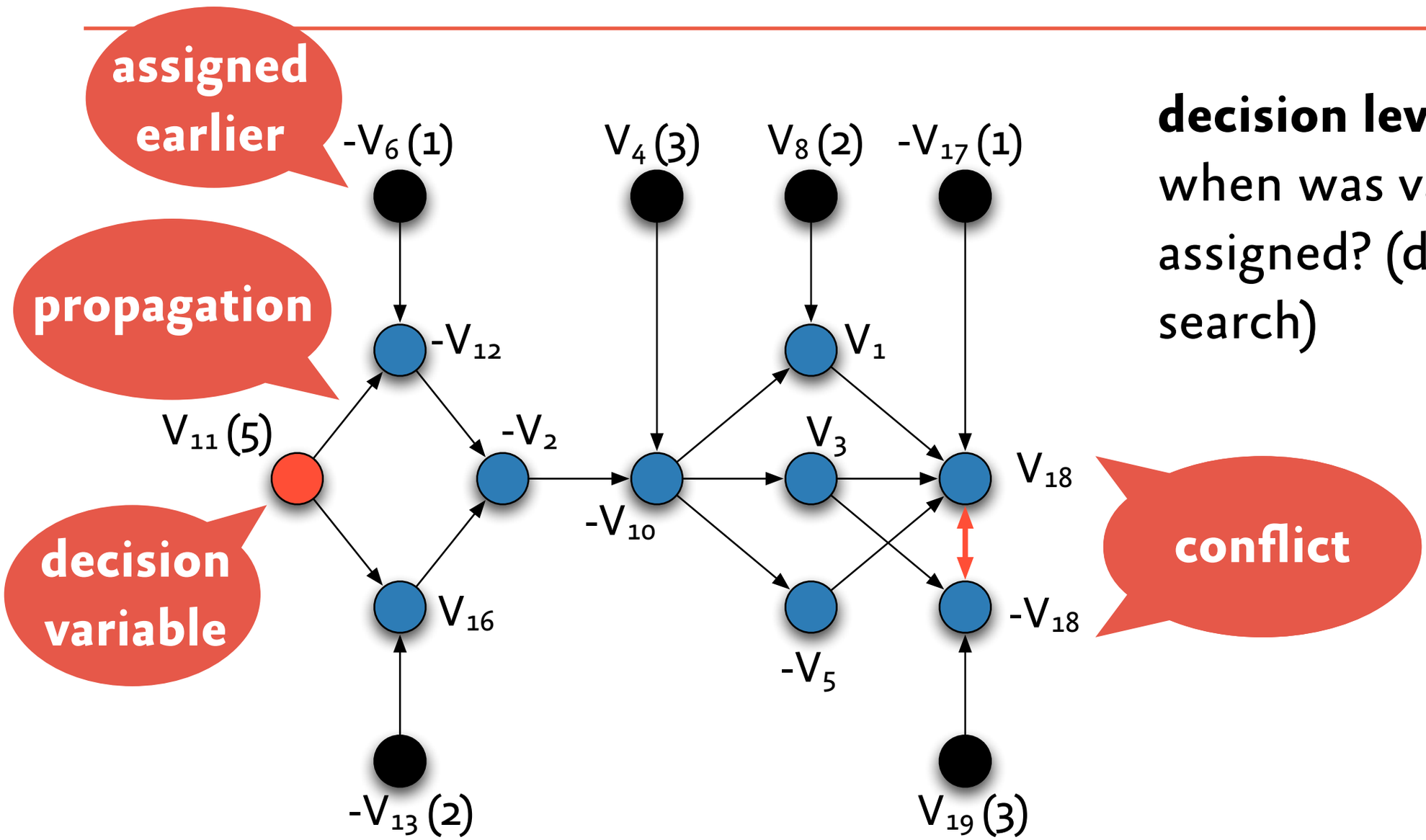
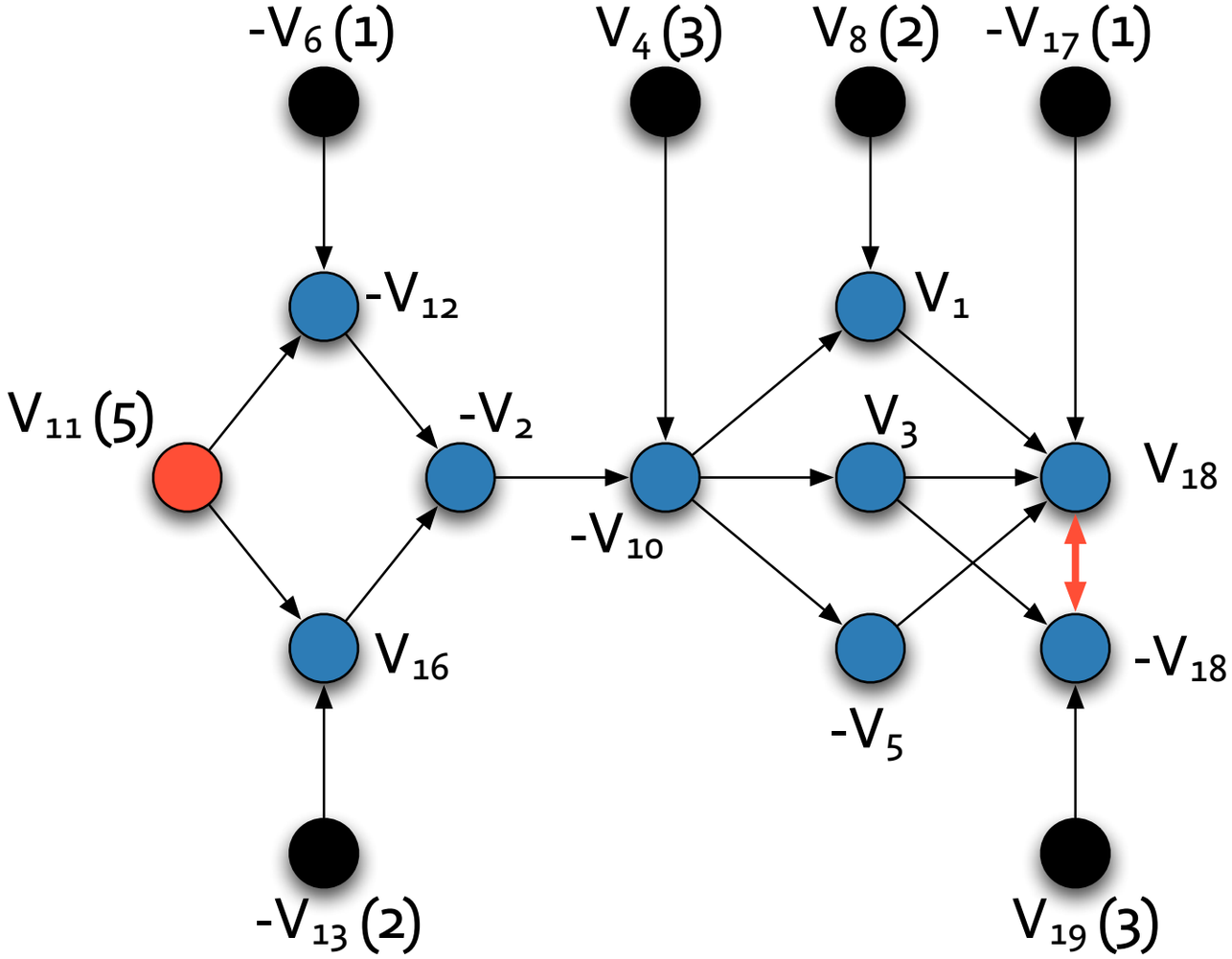
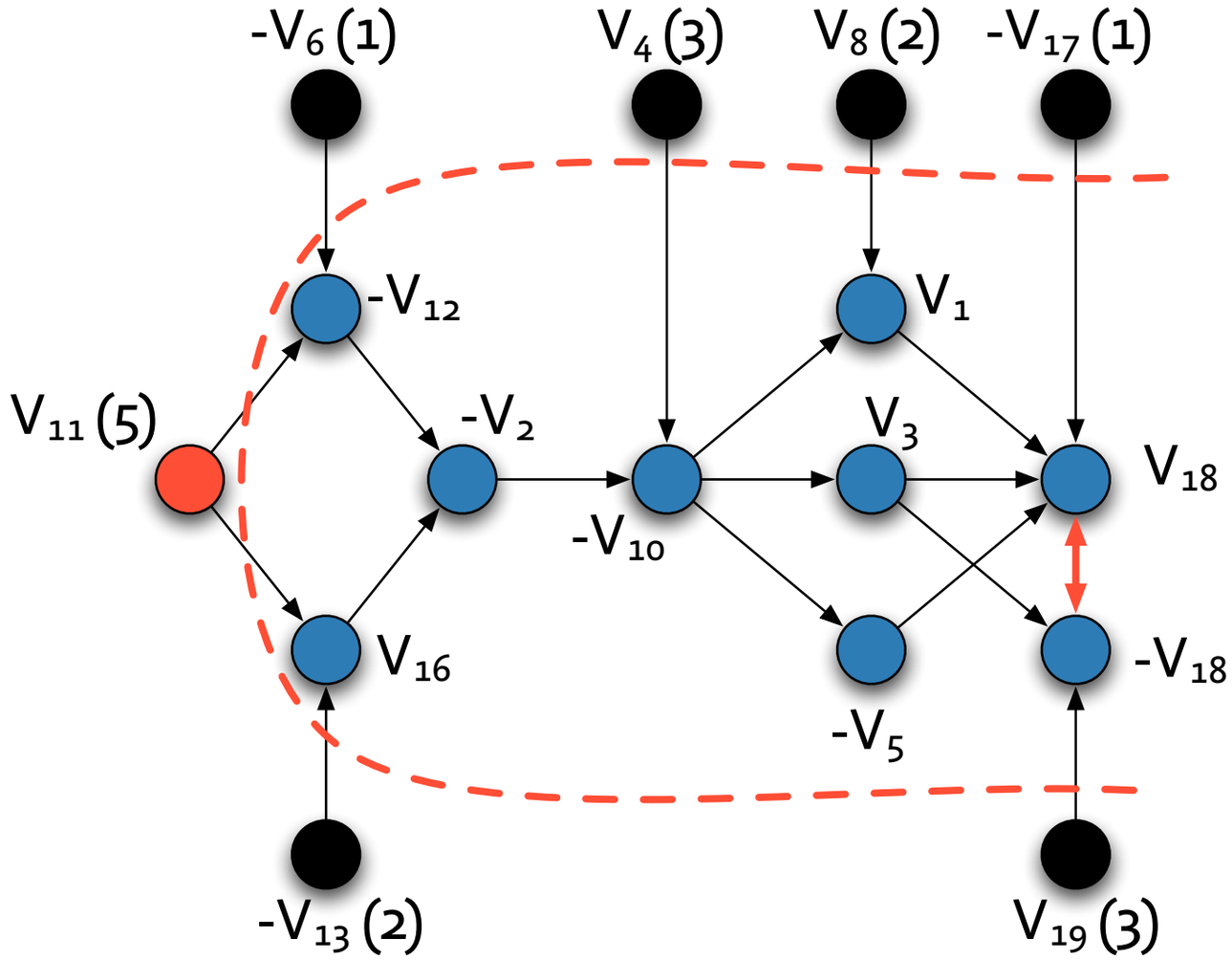


decision level: when was variable assigned? (depth in search)

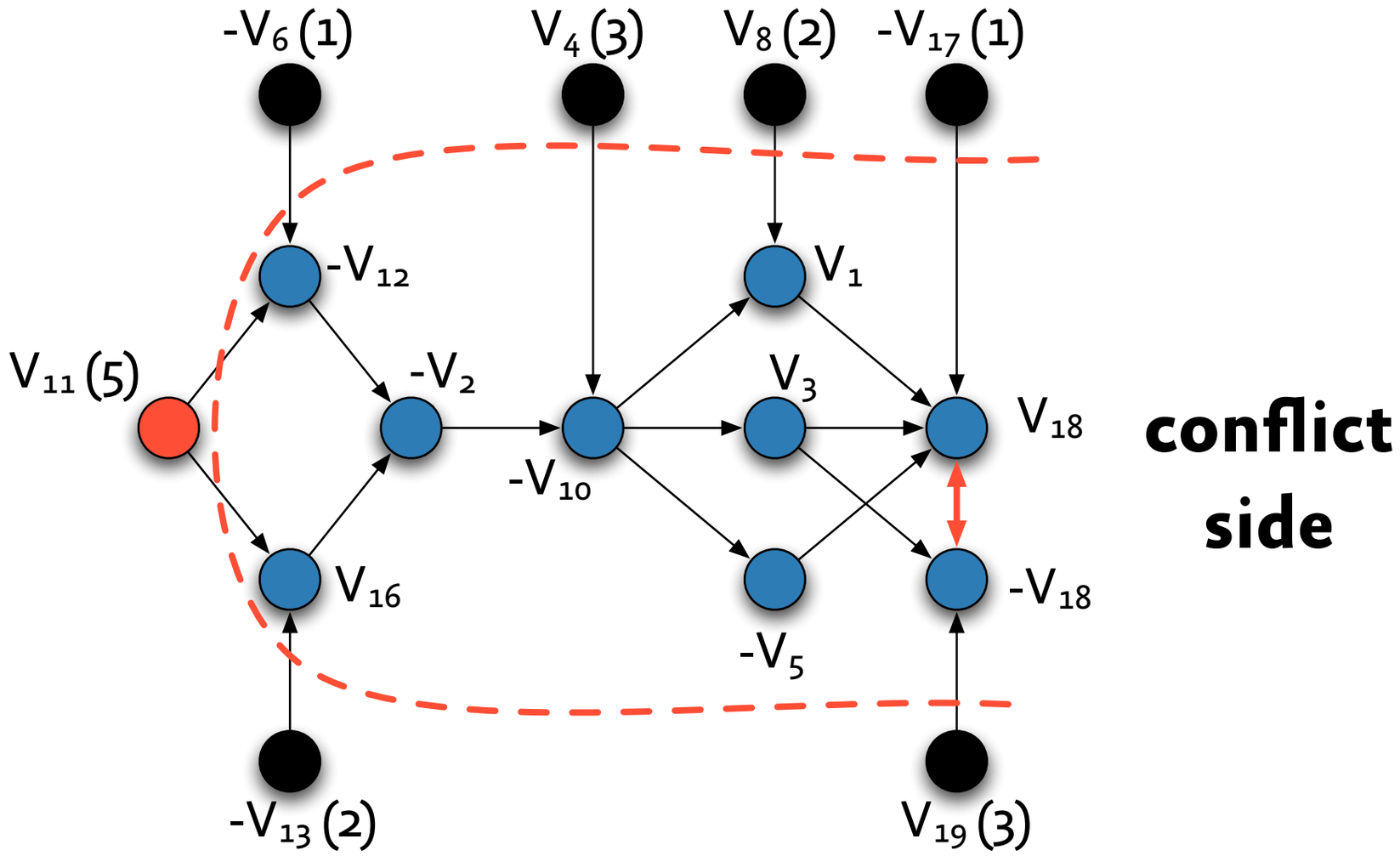
Implication graph: cuts



Implication graph: cuts

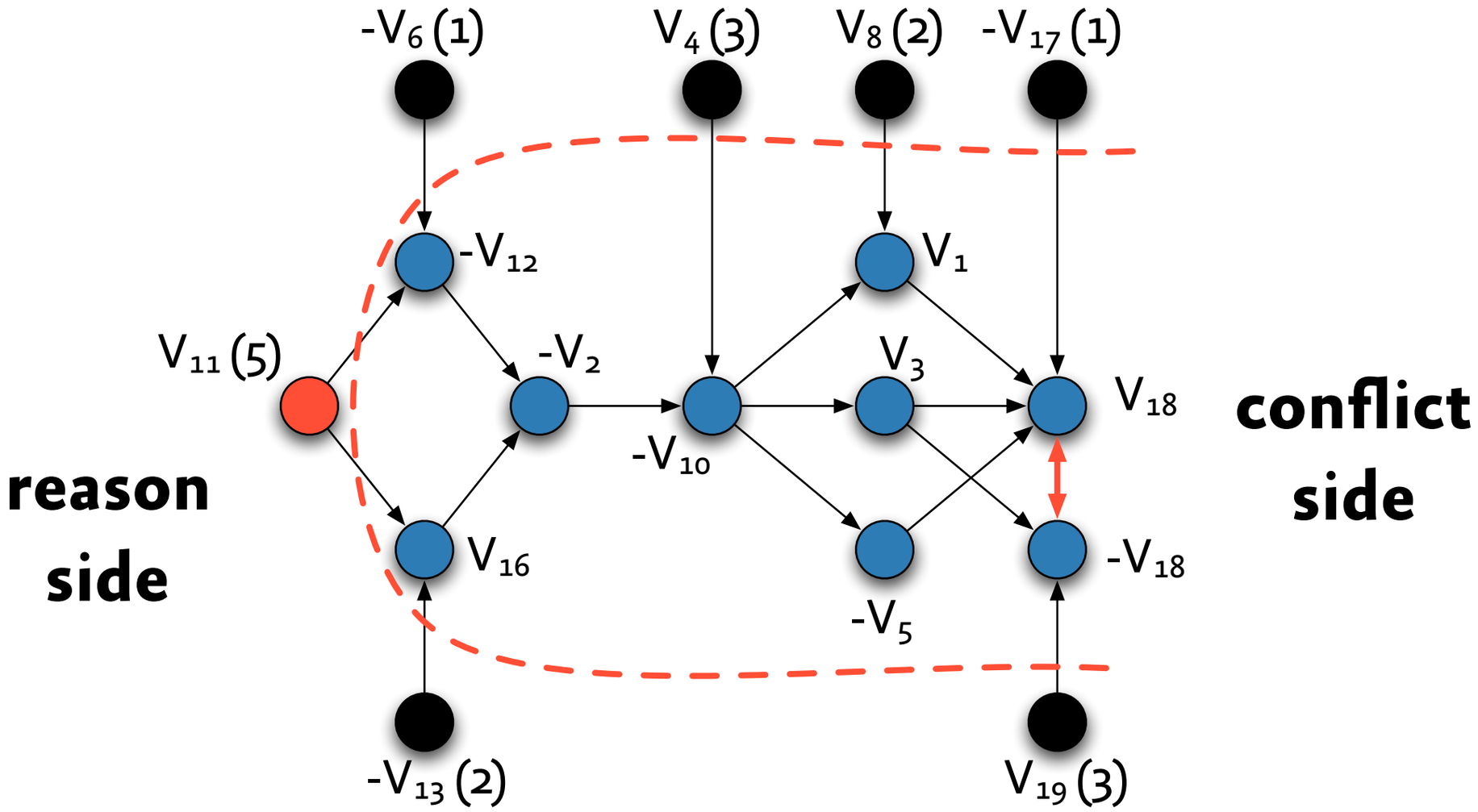


Implication graph: cuts



conflict side

Implication graph: cuts



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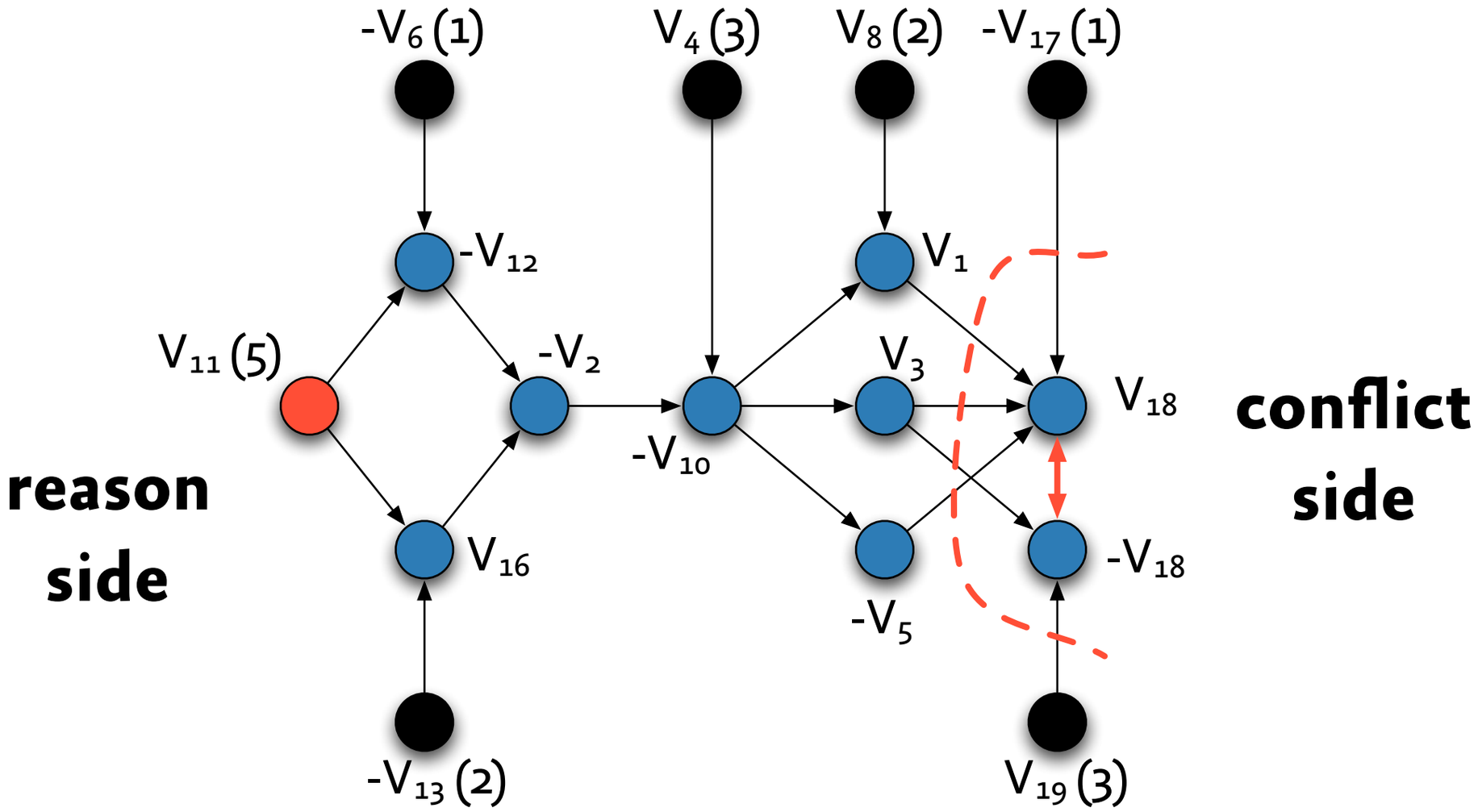
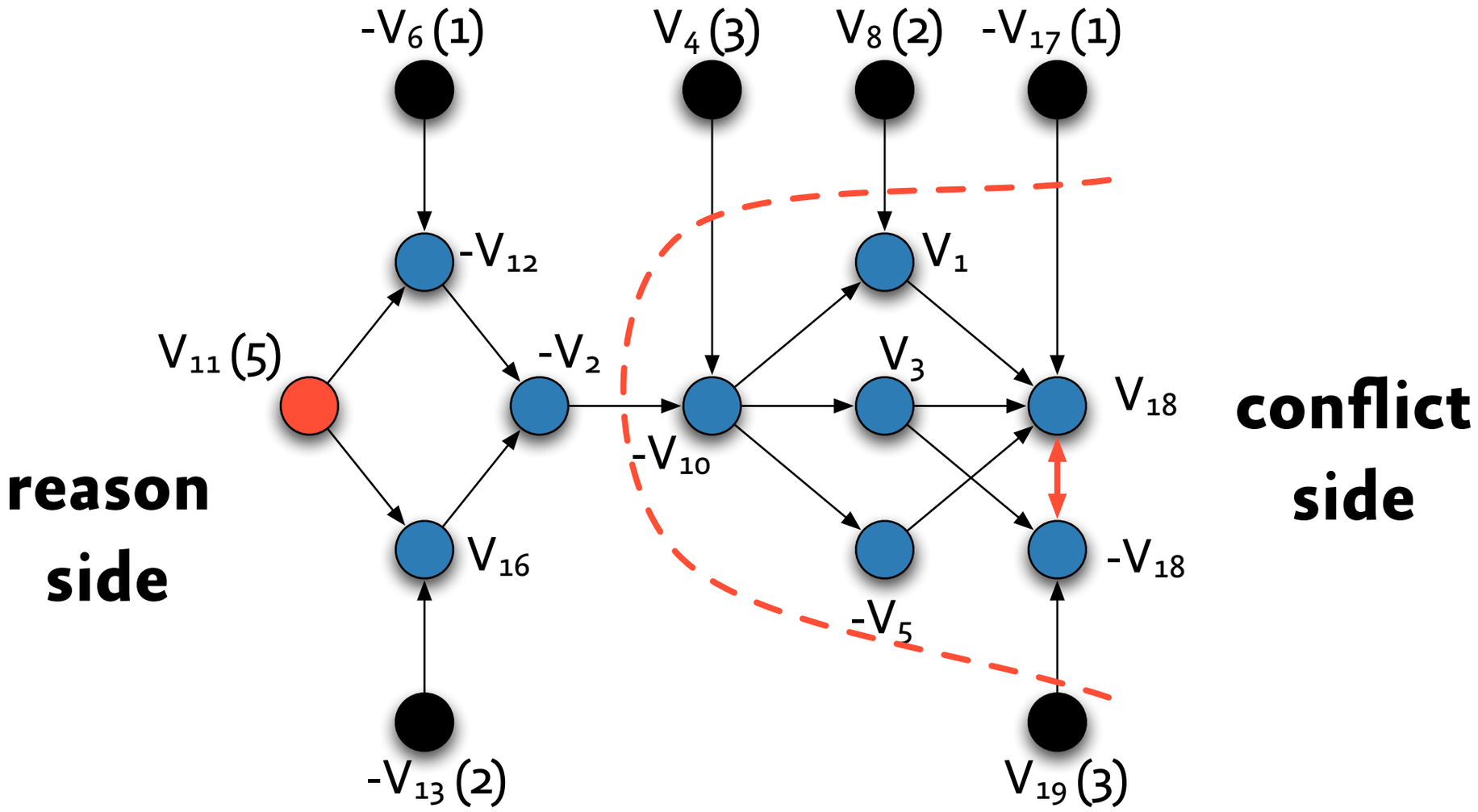
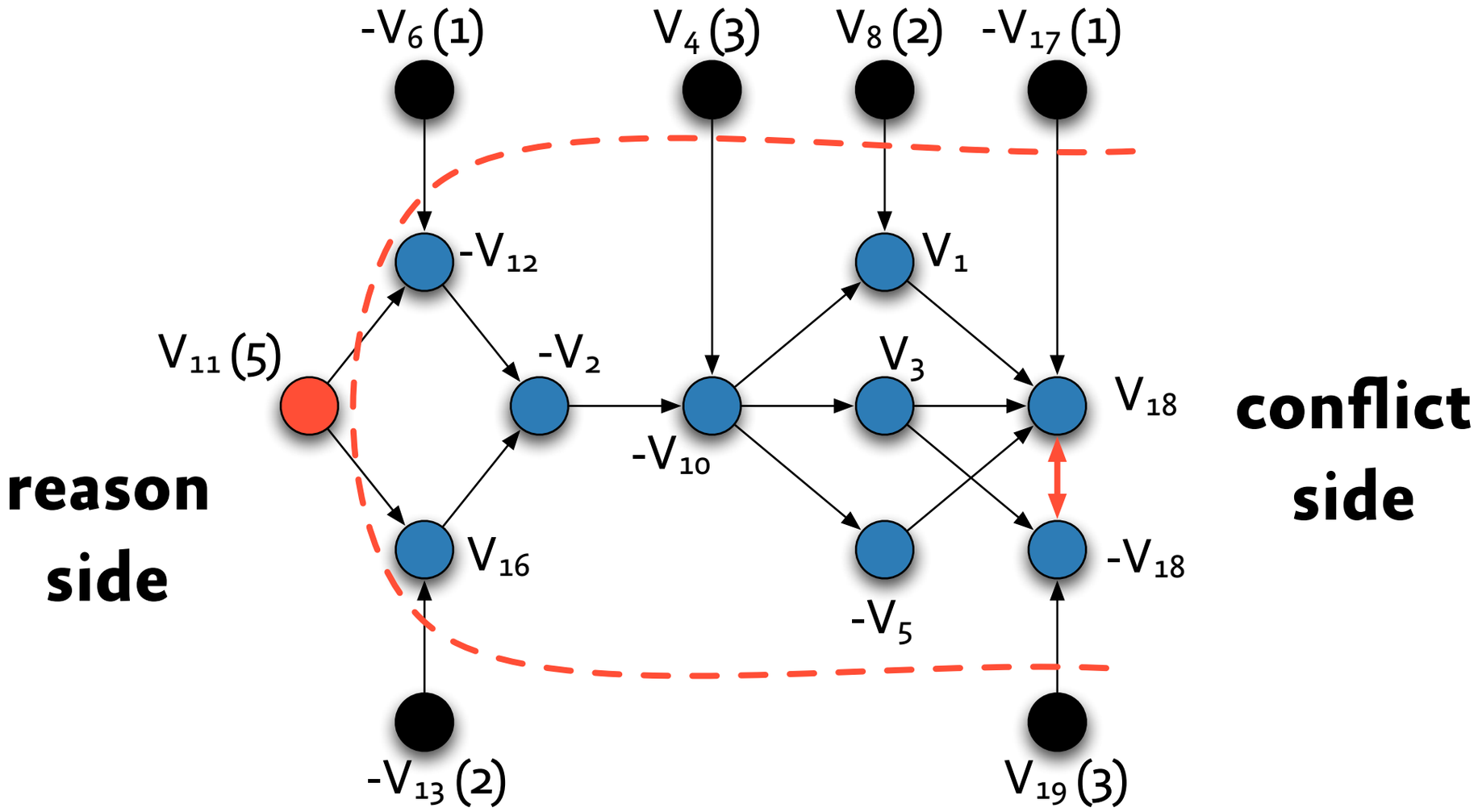


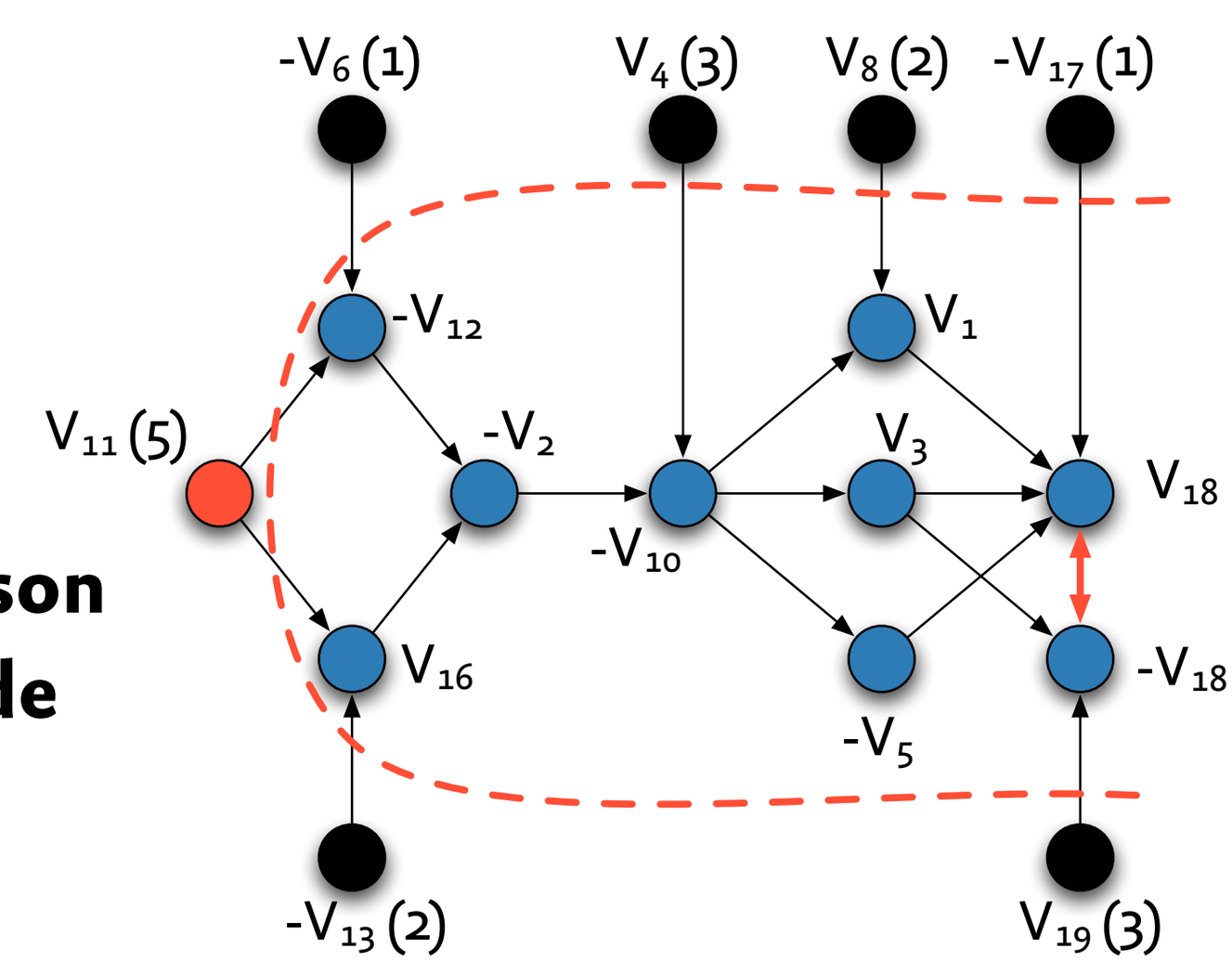
Implication graph: cuts



Implication graph: cuts



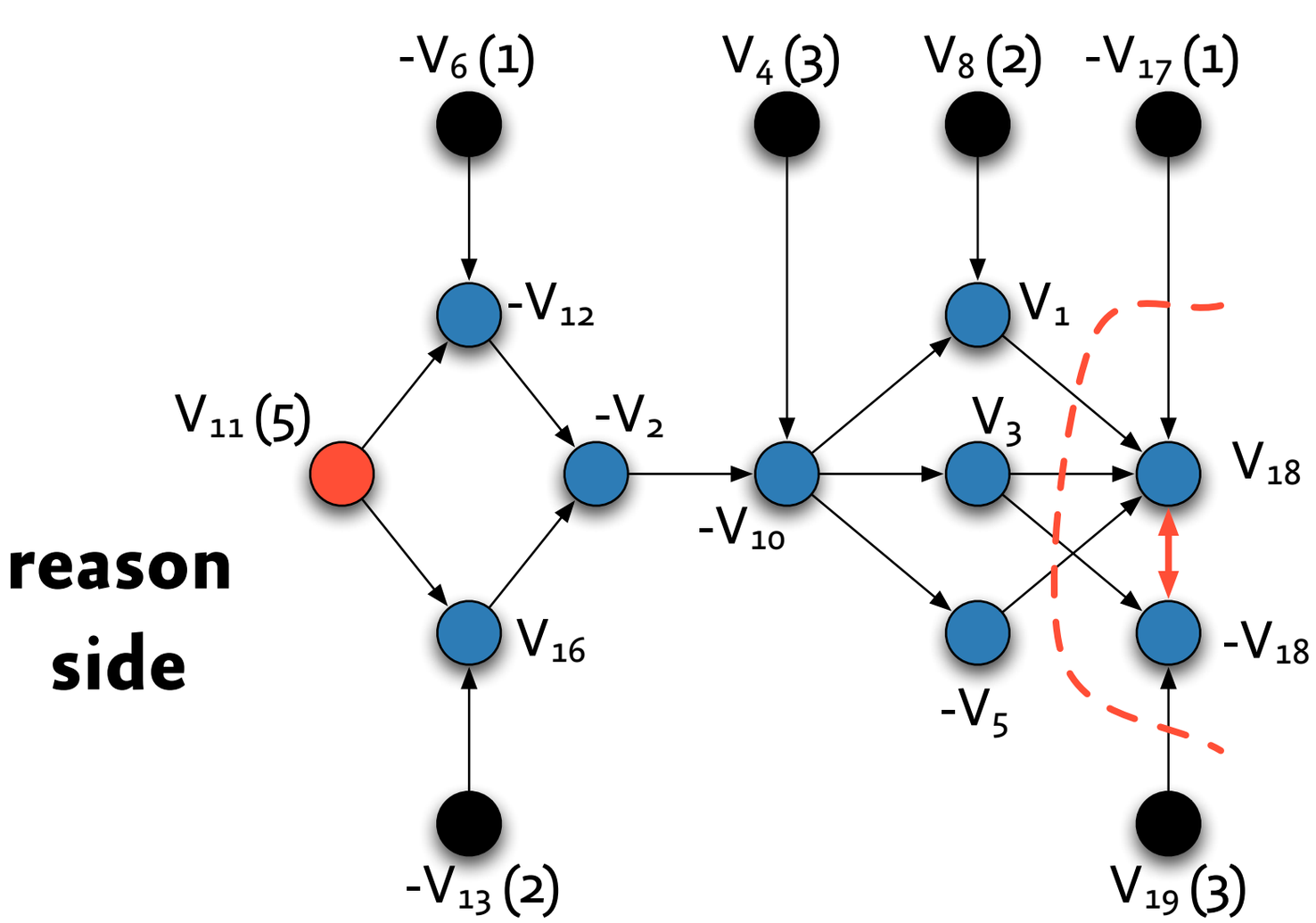
Implication graph: cuts



learned clause:

$$V_{17} \vee \neg V_8 \vee \neg V_4 \vee V_6 \vee \neg V_{11} \vee V_{13} \vee \neg V_{19}$$

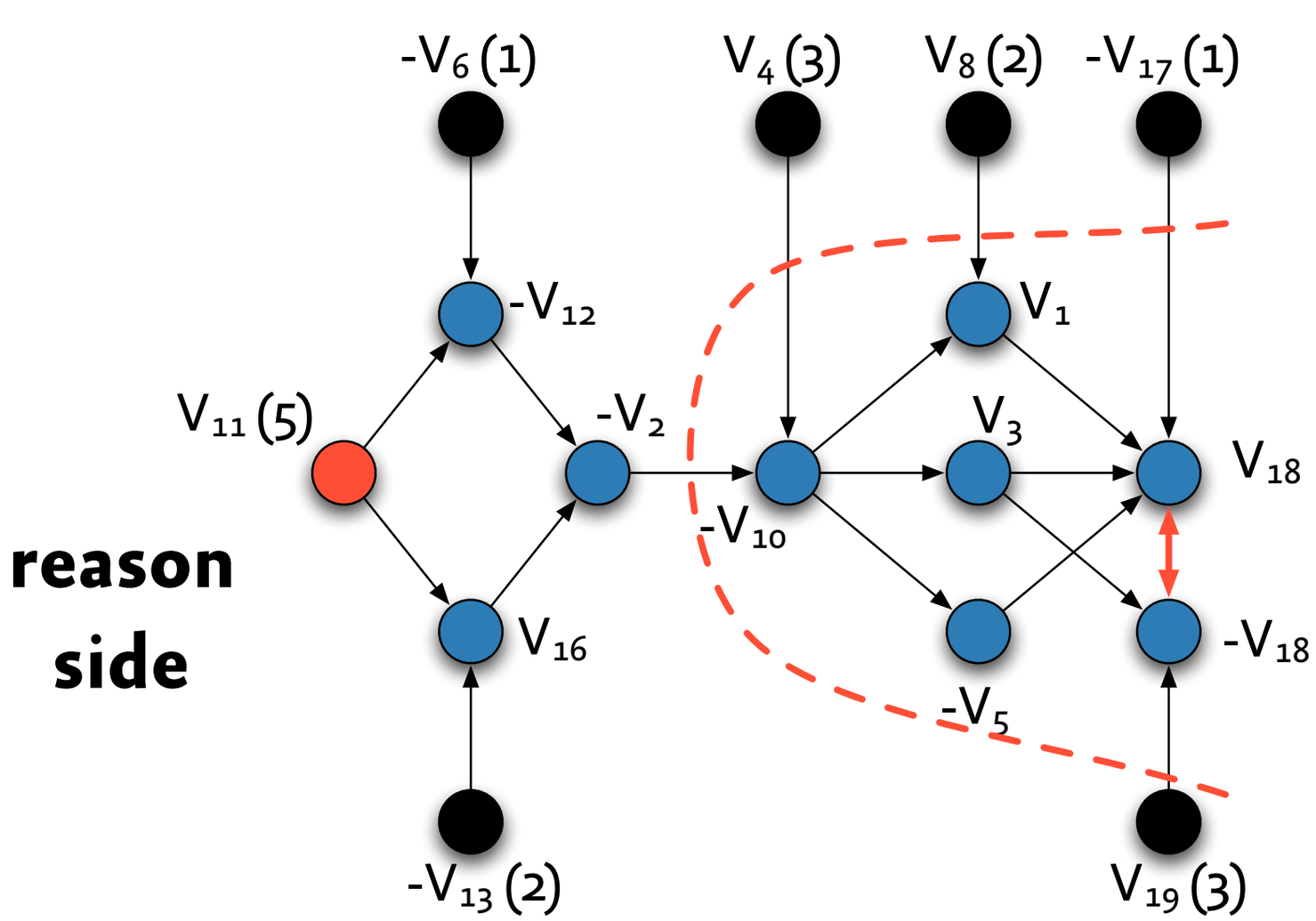
Implication graph: cuts



learned clause:

$$V_{17} \vee \neg V_1 \vee \neg V_3 \vee V_5 \vee \neg V_{19}$$

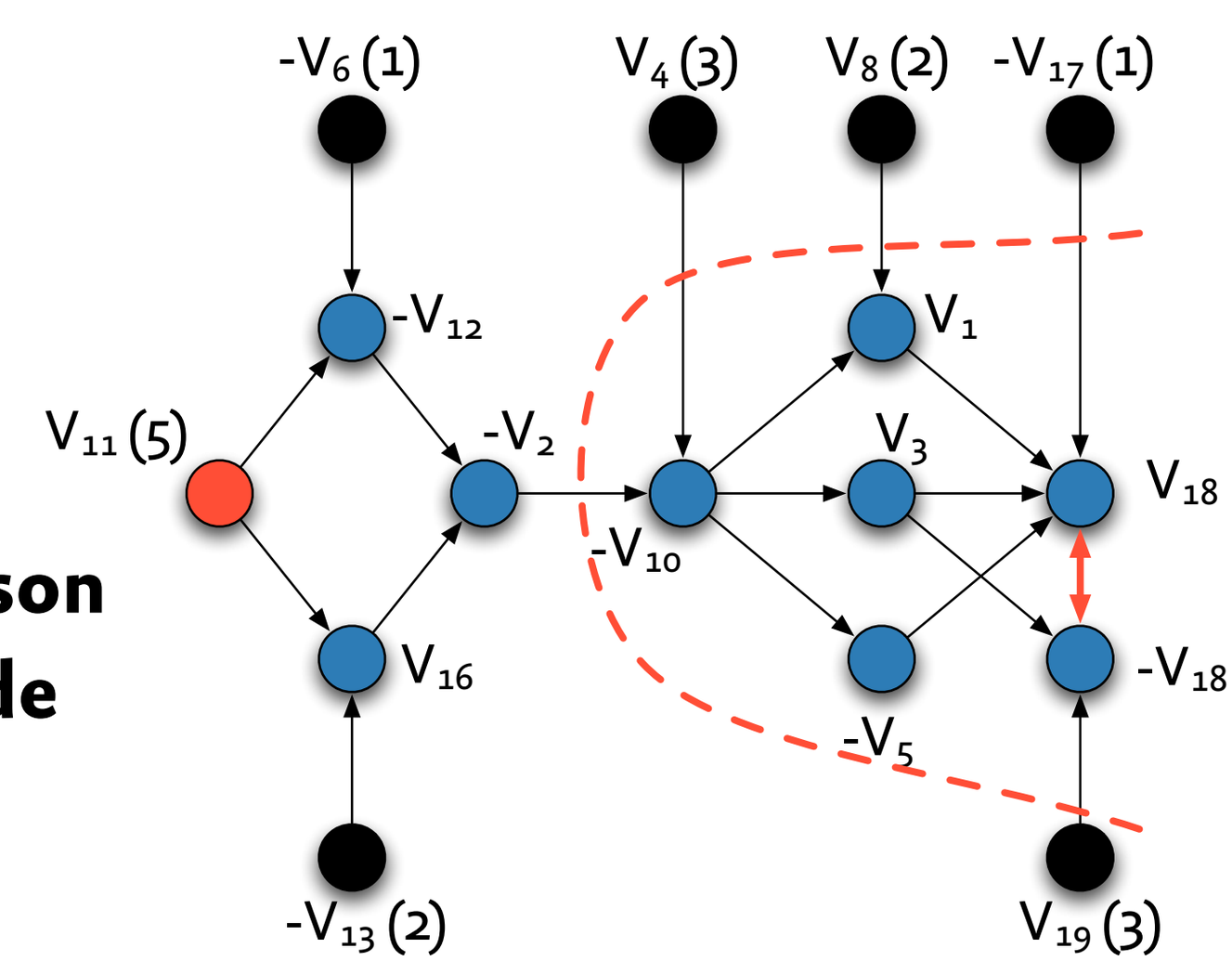
Implication graph: cuts



learned clause:

$$V_{17} \vee \neg V_8 \vee \neg V_4 \vee V_2 \vee \neg V_{19}$$

Implication graph: cuts



reason side

conflict side

learned clause:

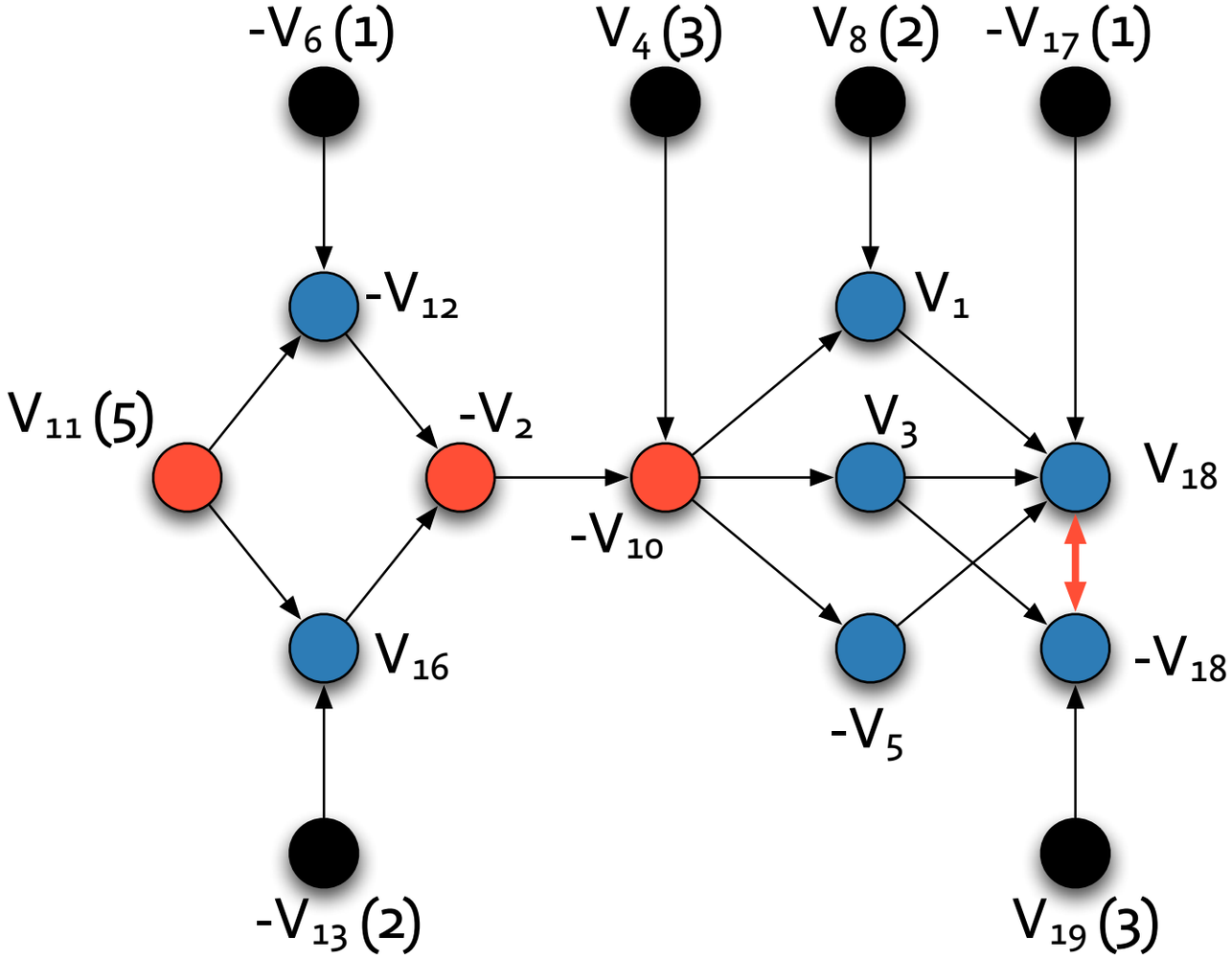
$$V_{17} \vee \neg V_8 \vee \neg V_4 \vee V_2 \vee \neg V_{19}$$

which cut to choose?

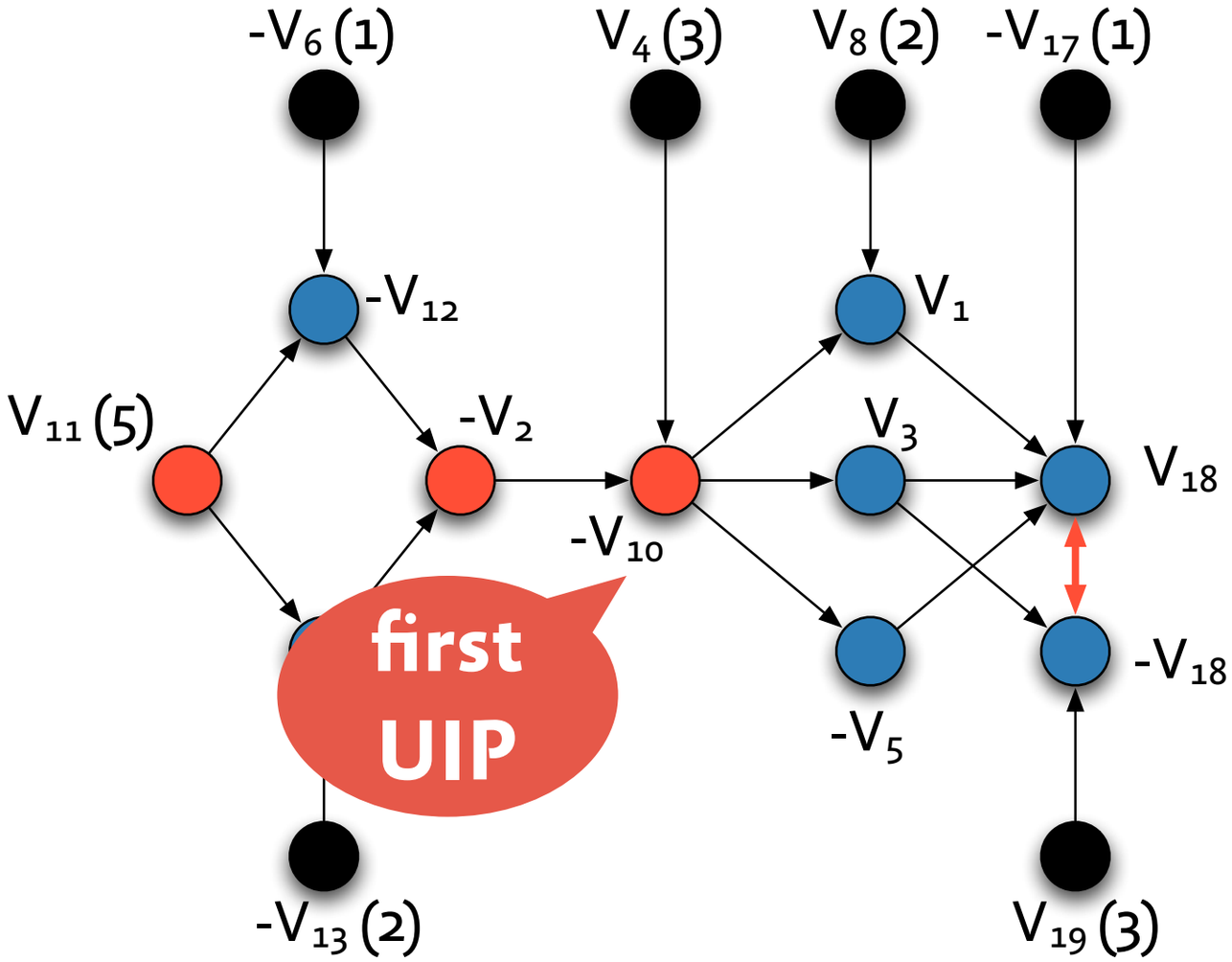
Unique implication points

- Variable V **dominates** V' iff any path from the decision variable to V' goes through V
- A **unique implication point** (UIP) dominates the conflict variable

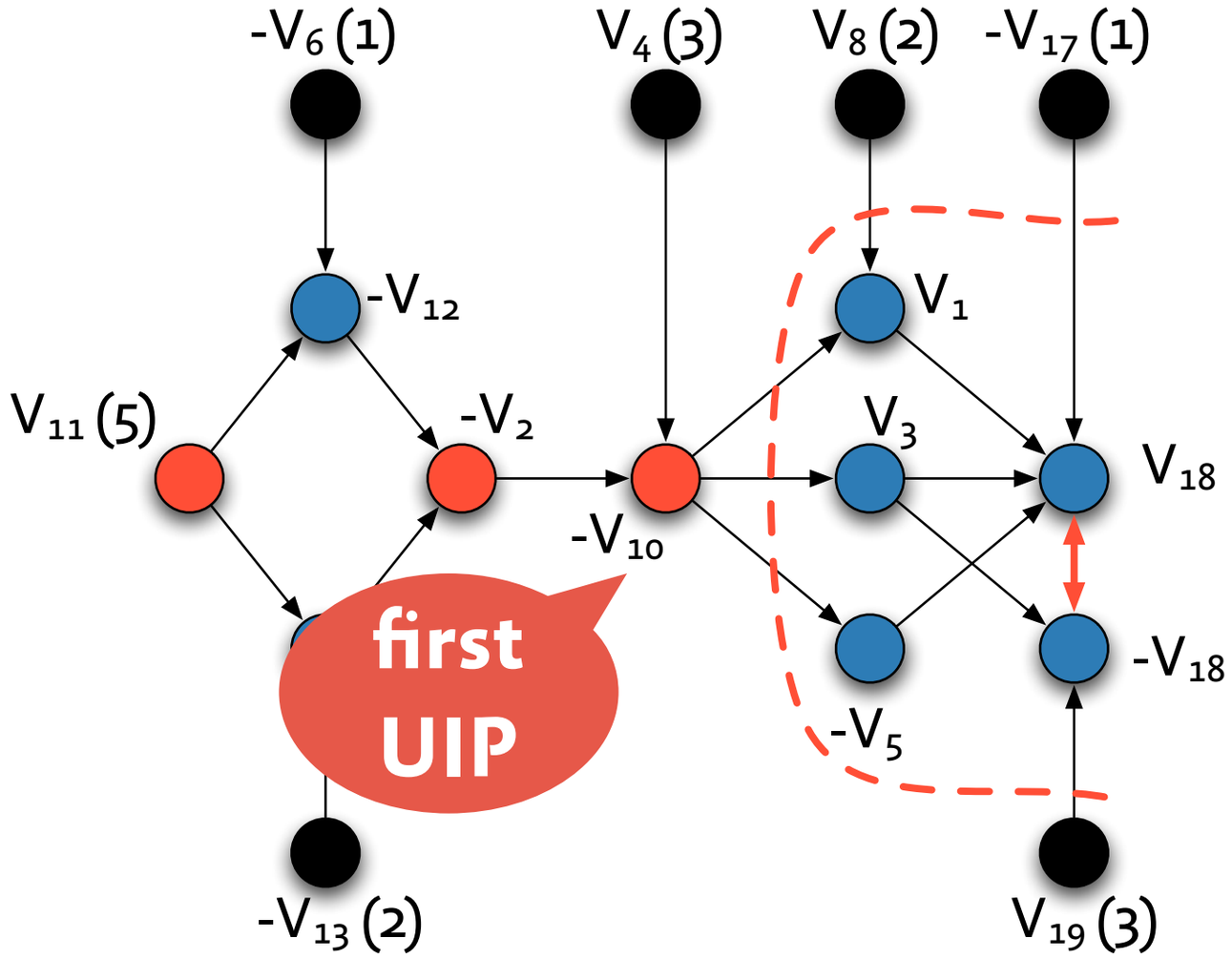
Unique implication points



Unique implication points



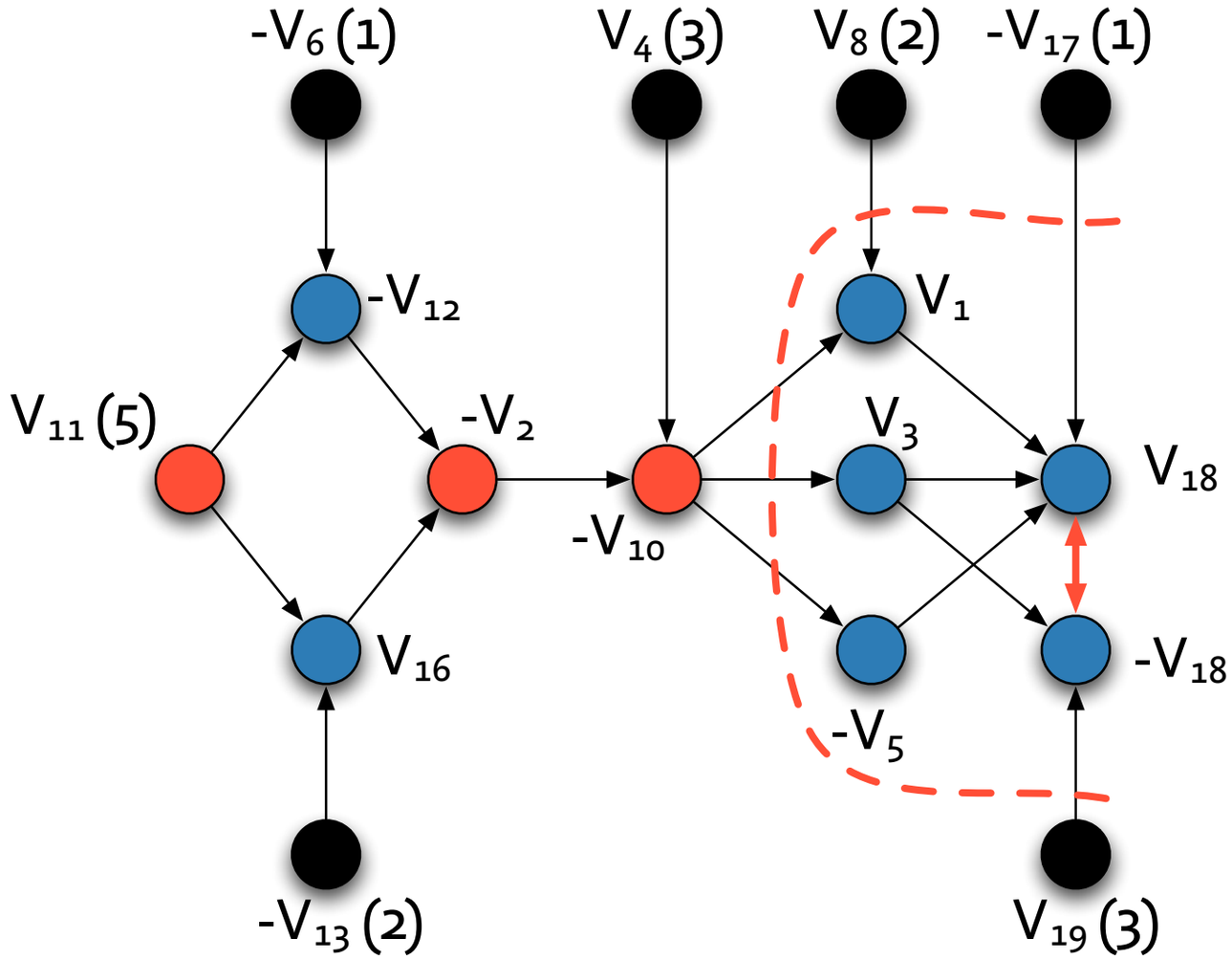
Unique implication points



1UIP cut

$$V_{17} \vee \neg V_8 \vee V_{10} \vee \neg V_{19}$$

Cuts and resolution



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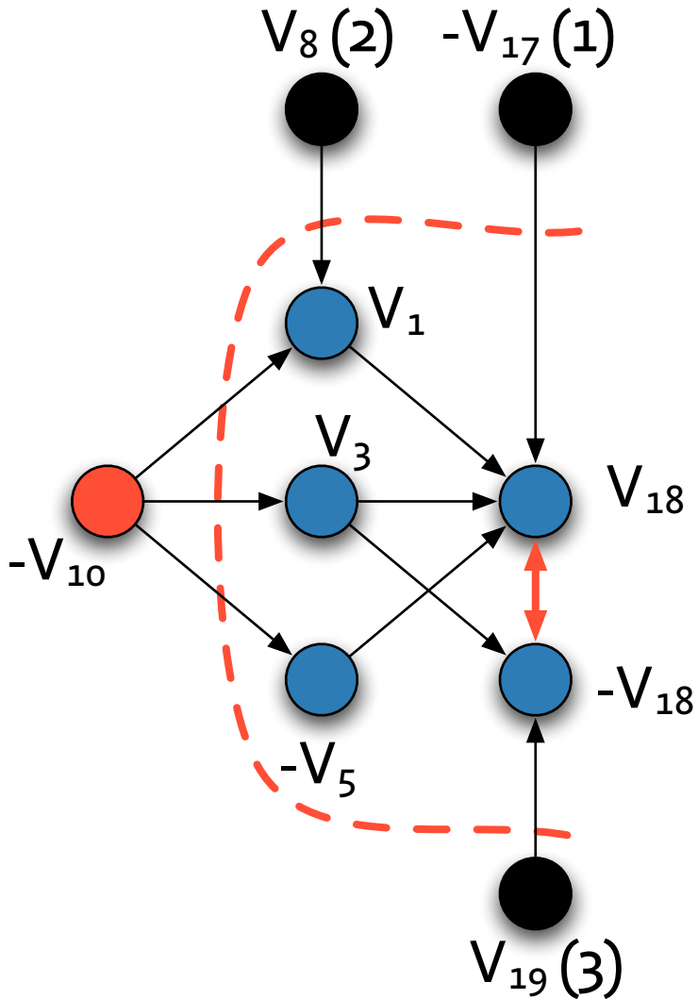
$$V_{17} \vee \neg V_8 \vee$$

$$V_{10} \vee \neg V_{19}$$

Cuts and resolution



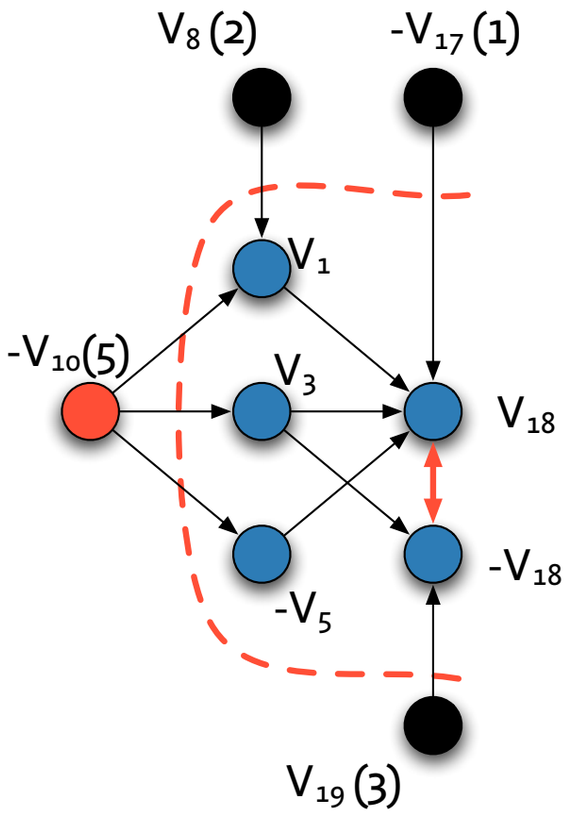
1UIP cut

$$V_{17} \vee \neg V_8 \vee$$

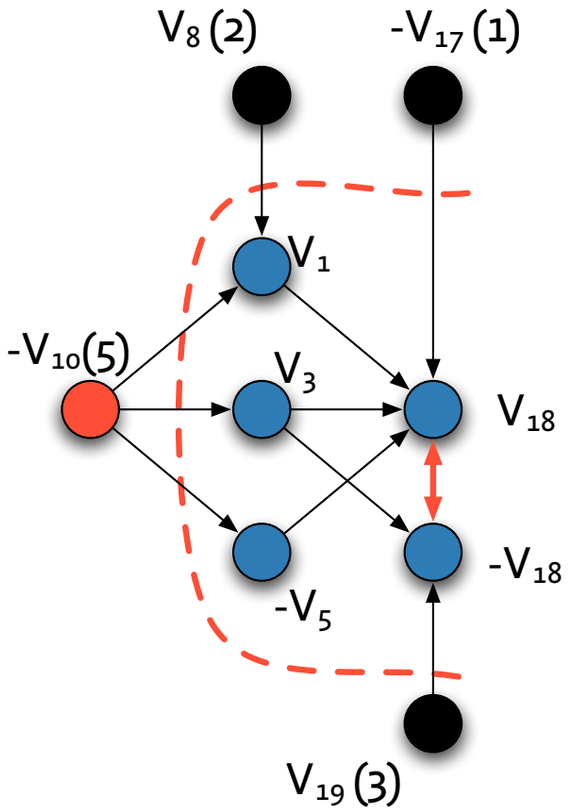
$$V_{10} \vee \neg V_{19}$$

Cuts and resolution

$\overline{V_3} \vee \overline{V_{18}} \vee \overline{V_{19}}$ $\overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee V_{18}$



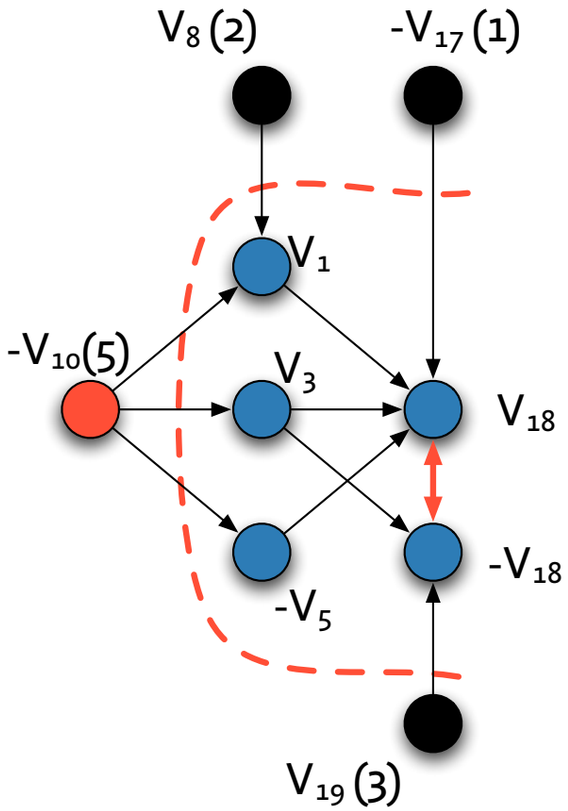
Cuts and resolution



$$\overline{V_3} \vee \overline{V_{18}} \vee \overline{V_{19}} \qquad \overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee V_{18}$$

$$\overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee \overline{V_{19}}$$

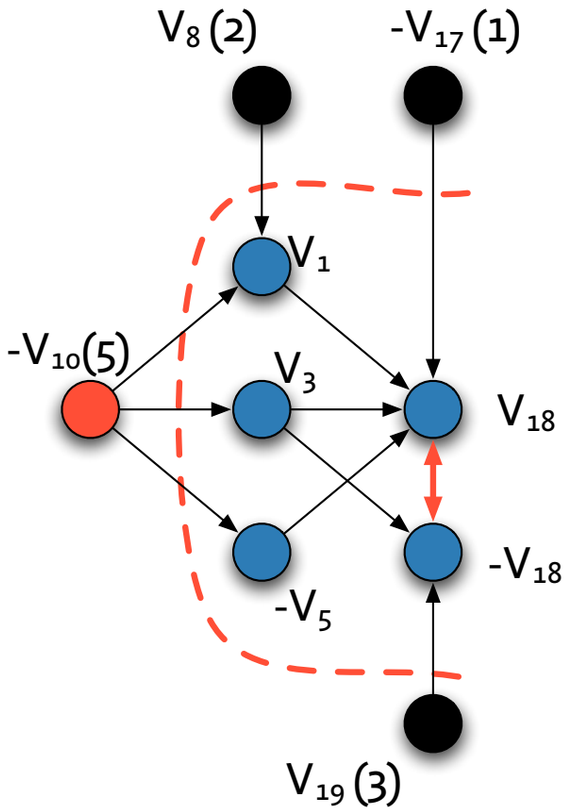
Cuts and resolution



$$\overline{V_3} \vee \overline{V_{18}} \vee \overline{V_{19}} \qquad \overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee V_{18}$$

$$V_3 \vee V_{10} \qquad \overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee \overline{V_{19}}$$

Cuts and resolution



$$\overline{V_3} \vee \overline{V_{18}} \vee \overline{V_{19}} \qquad \overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee V_{18}$$

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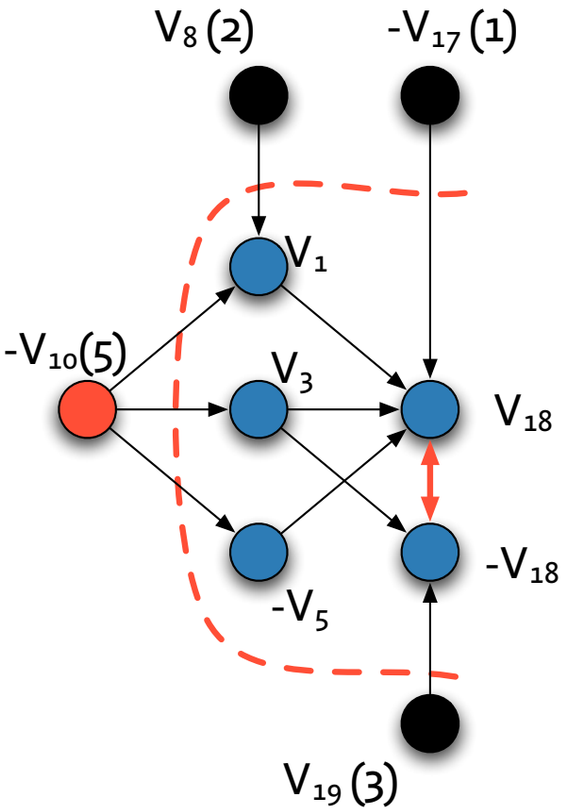
Cuts and resolution



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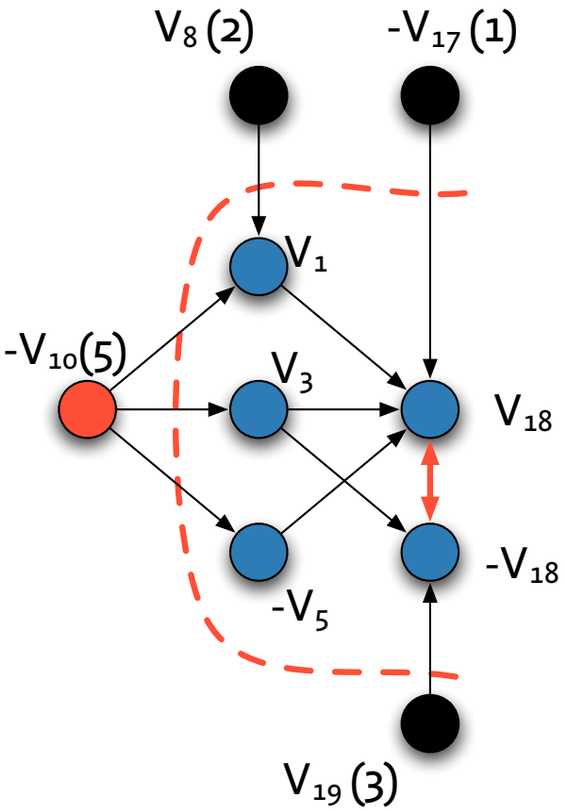
$$V_3 \vee V_{10} \qquad \overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee \overline{V_{19}}$$

$$V_1 \vee \overline{V_8} \vee V_{10} \qquad \overline{V_1} \vee V_5 \vee V_{10} \vee V_{17} \vee \overline{V_{19}}$$

Cuts and resolution



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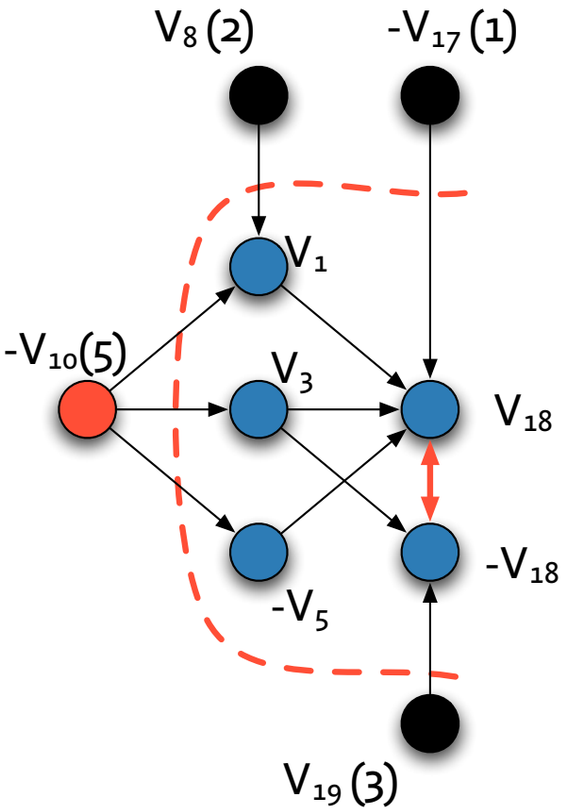
$$V_3 \vee V_{10} \qquad \overline{V_1} \vee \overline{V_3} \vee V_5 \vee V_{17} \vee \overline{V_{19}}$$

$$V_1 \vee \overline{V_8} \vee V_{10} \qquad \overline{V_1} \vee V_5 \vee V_{10} \vee V_{17} \vee \overline{V_{19}}$$

$$V_5 \vee \overline{V_8} \vee V_{10} \vee V_{17} \vee \overline{V_{19}}$$

Cuts and resolution



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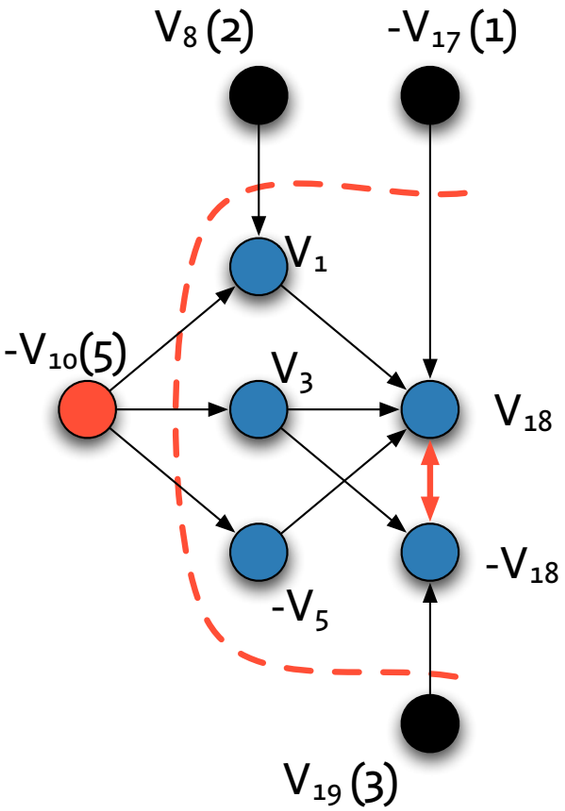
$$\overline{V_5} \vee V_{10} \qquad V_5 \vee \overline{V_8} \vee V_{10} \vee V_{17} \vee \overline{V_{19}}$$

Cuts and resolution



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$$\overline{V_5} \vee V_{10} \qquad V_5 \vee \overline{V_8} \vee V_{10} \vee V_{17} \vee \overline{V_{19}}$$

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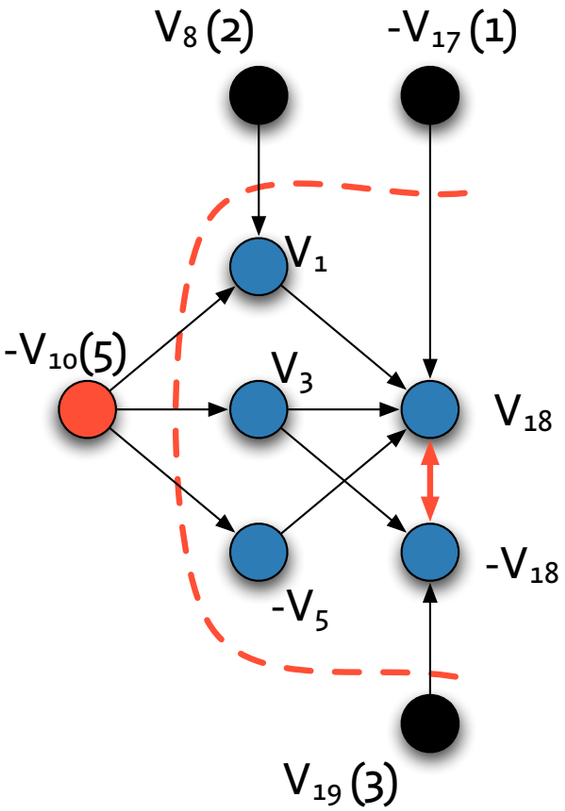
Cuts and resolution



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$$\overline{V_8} \vee V_{10} \vee V_{17} \vee \overline{V_{19}}$$

conflict clause!

Why 1UIP?

- **works well in practice**
- **conflict clause is *asserting***
  - only one literal of current decision level
  - backtracking causes unit propagation
  - search is forced in different direction
- **learned clause "close to conflict"**
  - slightly vague...

Finding the 1UIP

- **implication graph:**

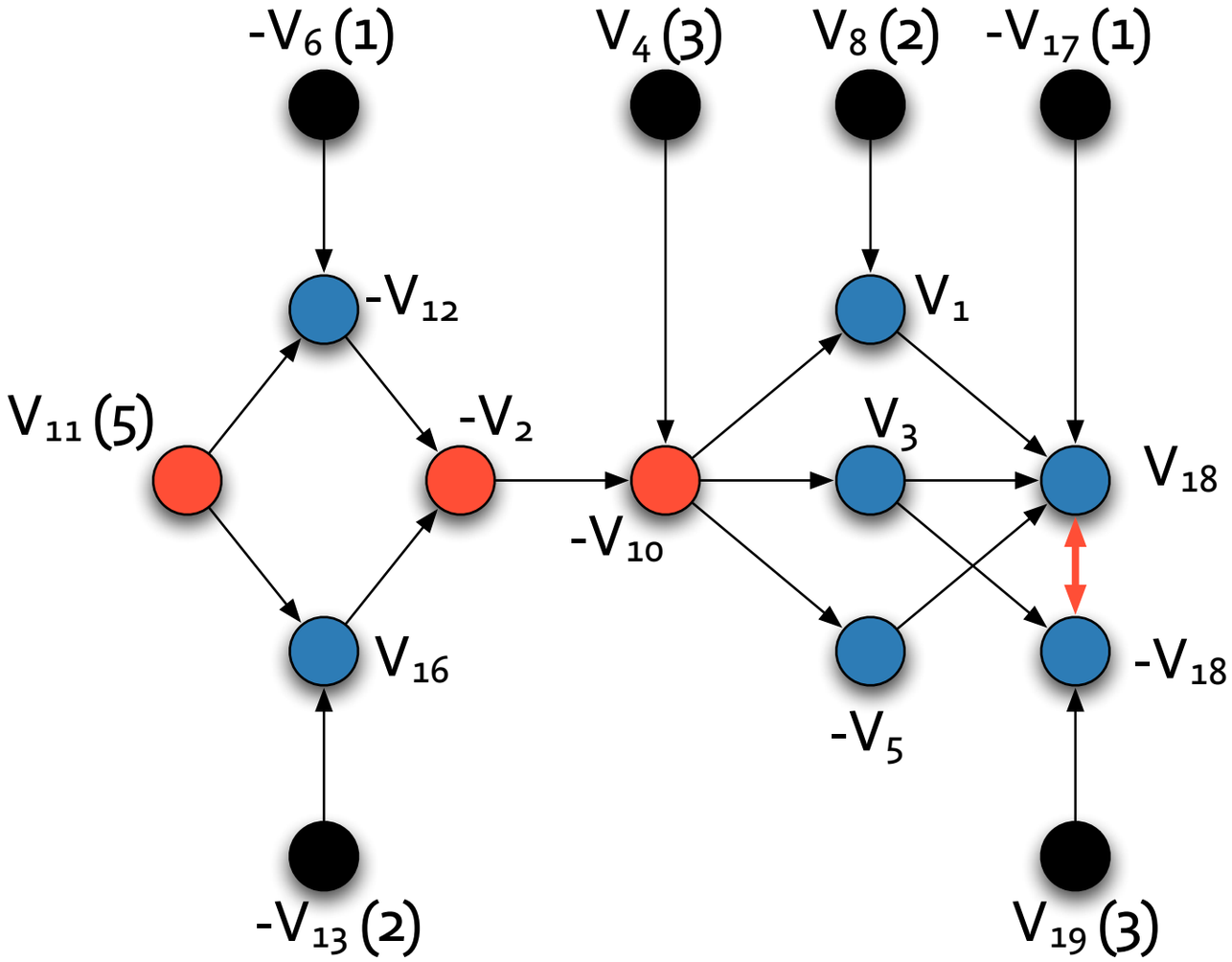
  trail the clause that caused assignment

- **breadth-first search**

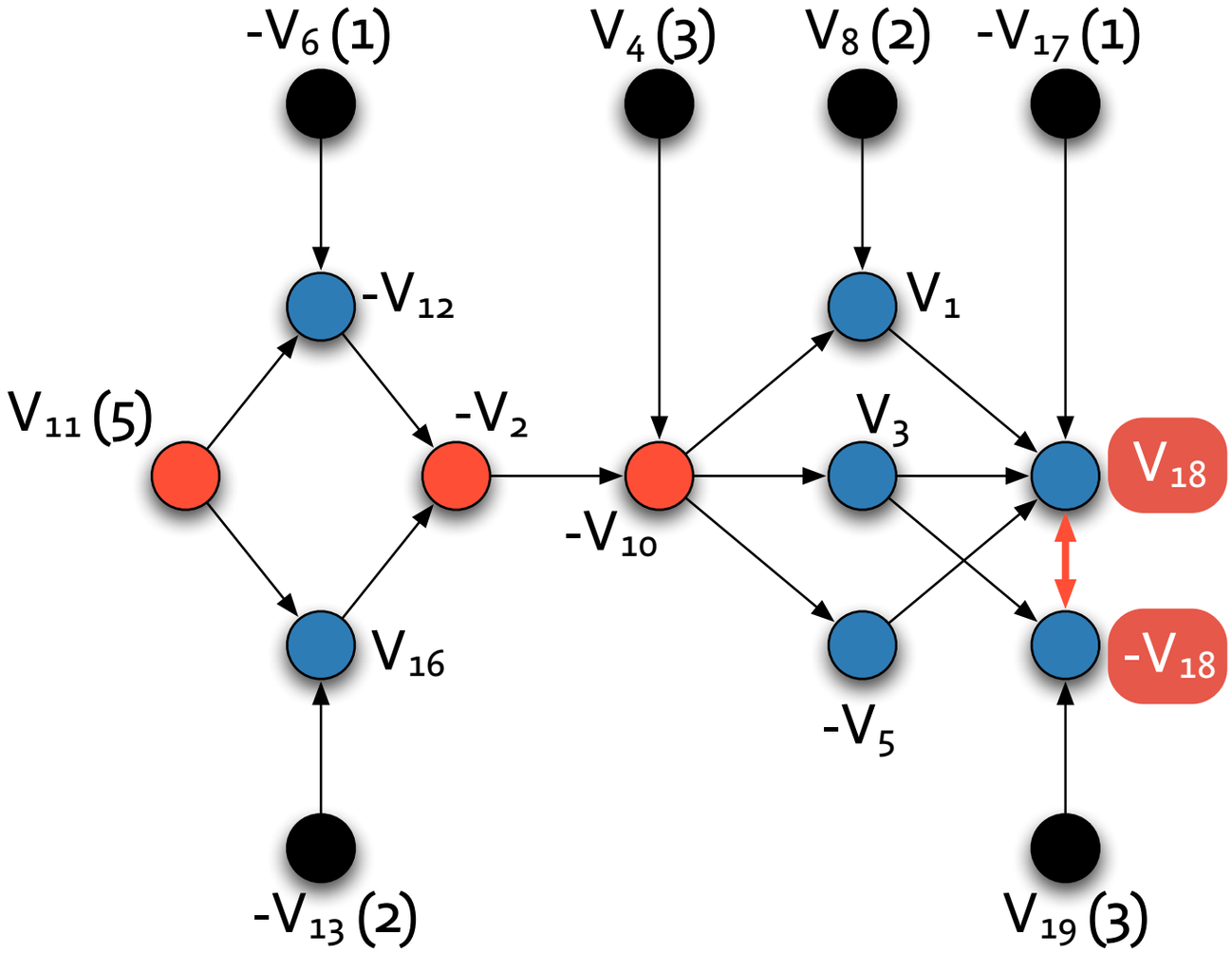
  - backwards from conflict

  - in the order of assignment (trail!)

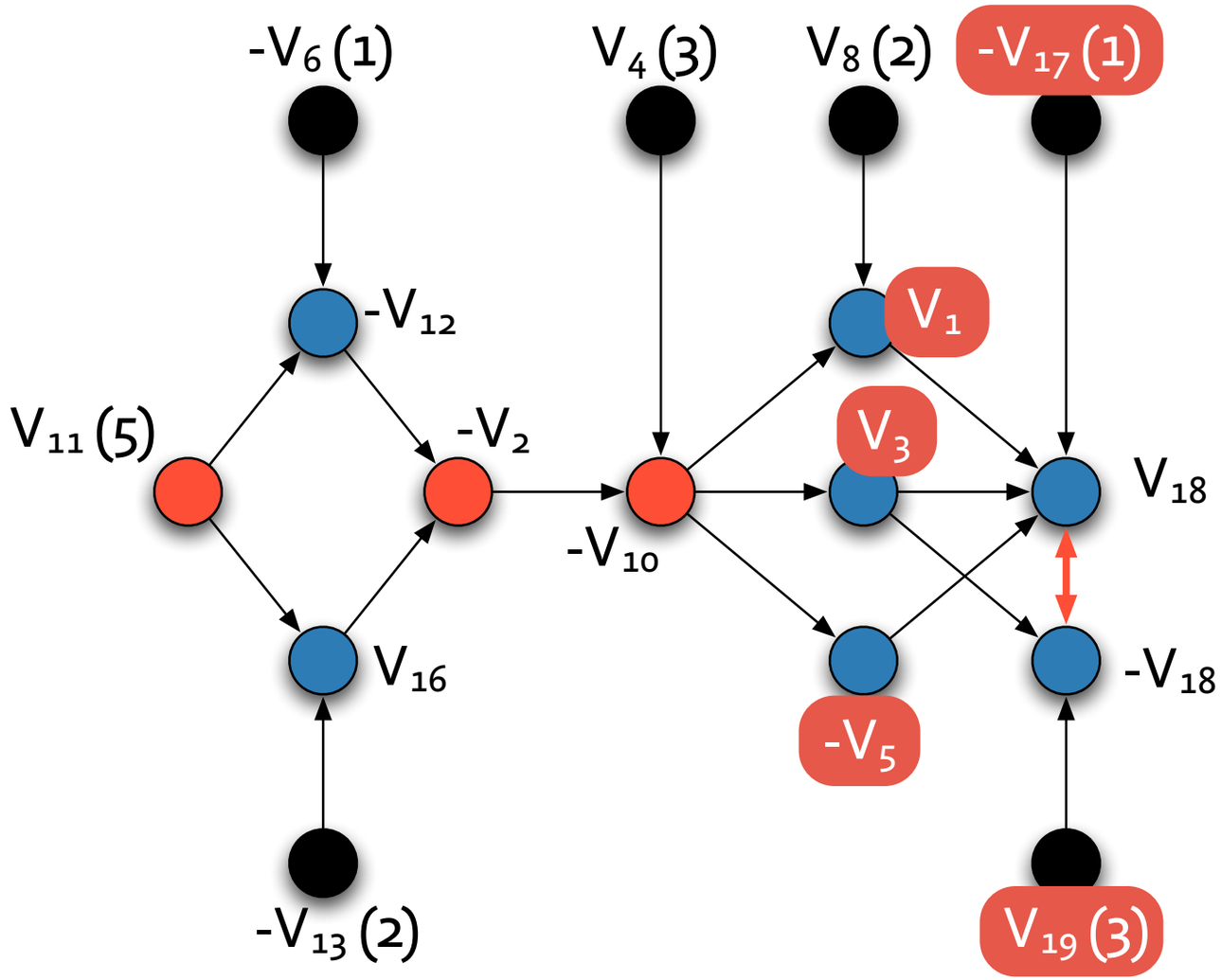
Finding the 1UIP



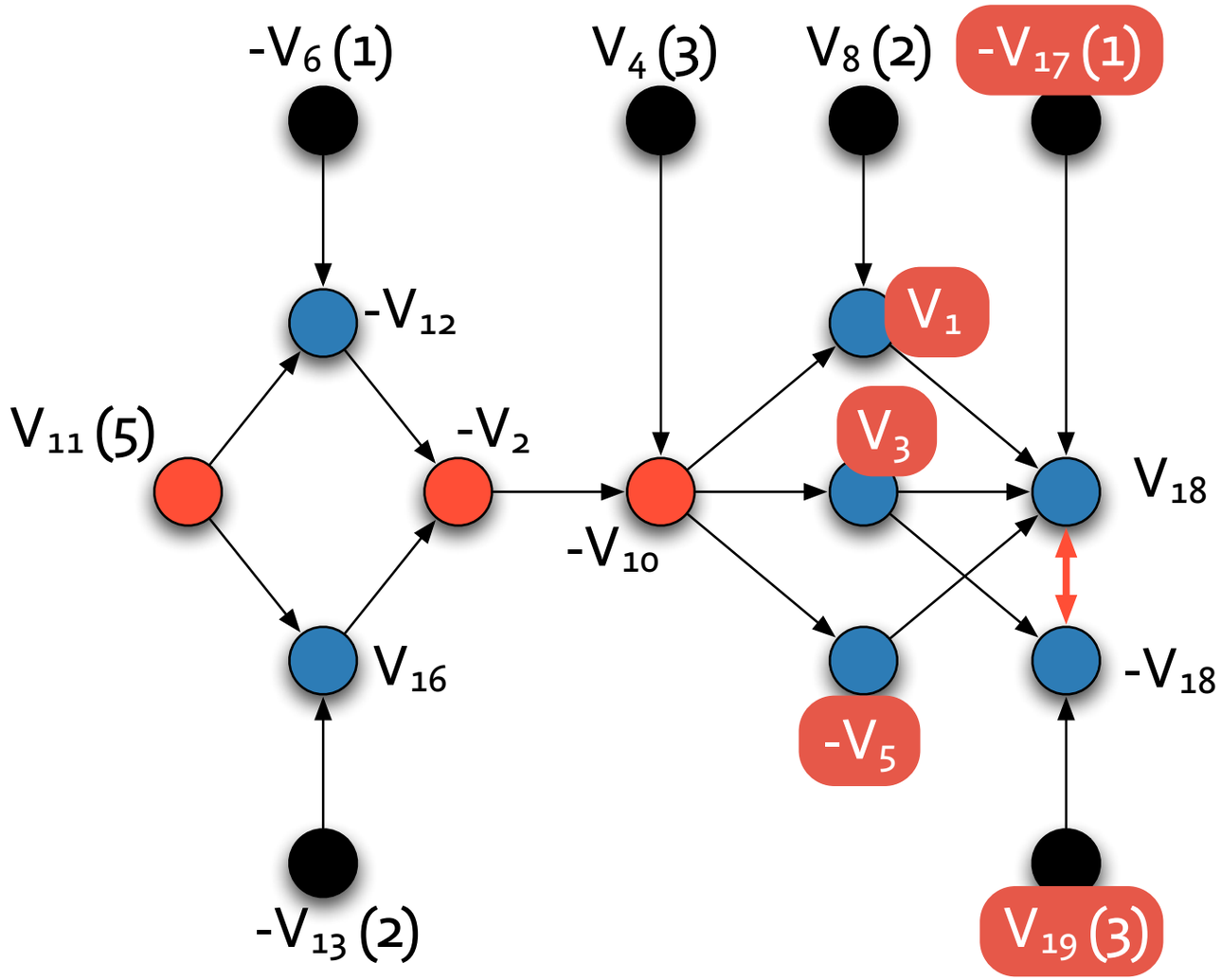
Finding the 1UIP



Finding the 1UIP

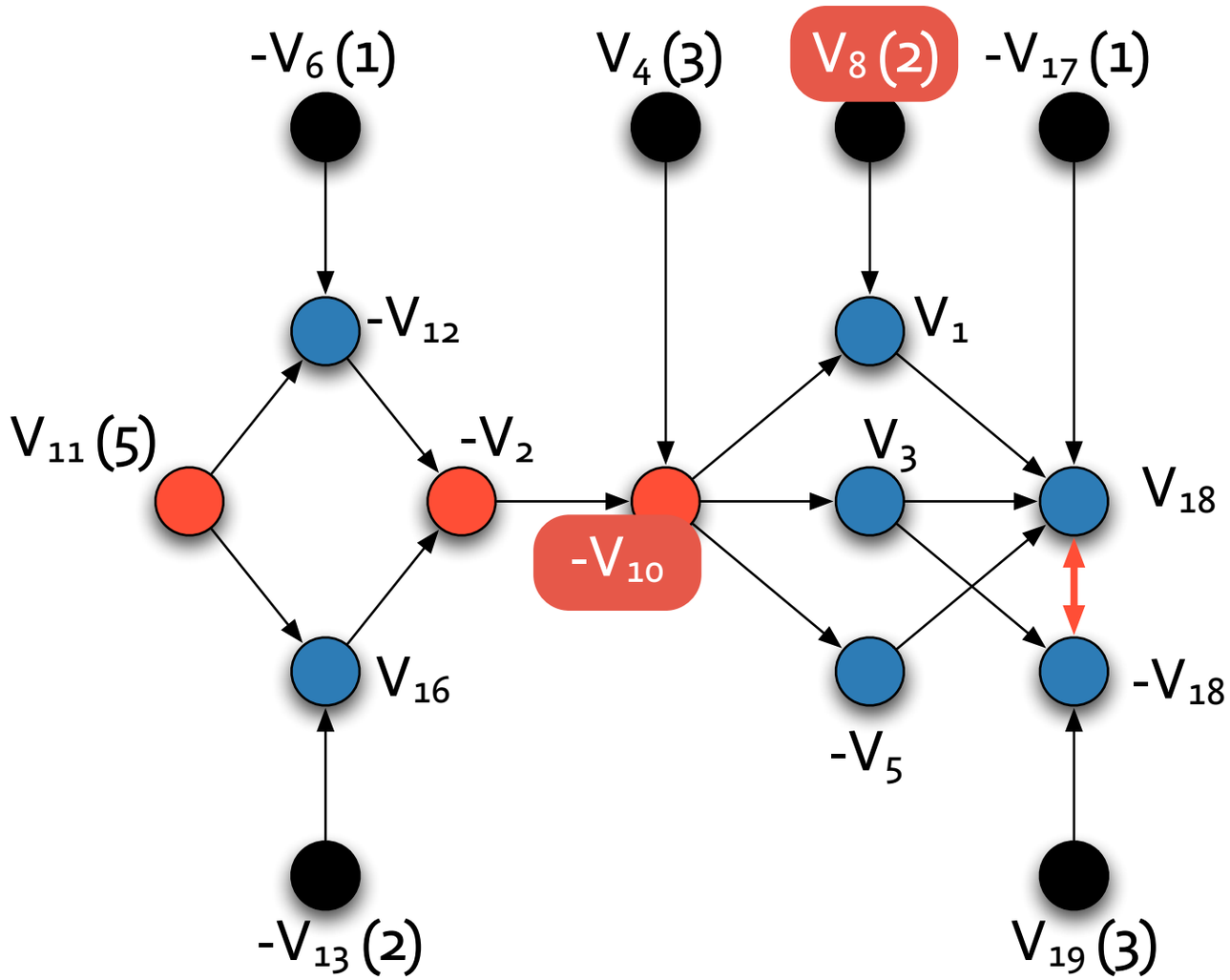


Finding the 1UIP



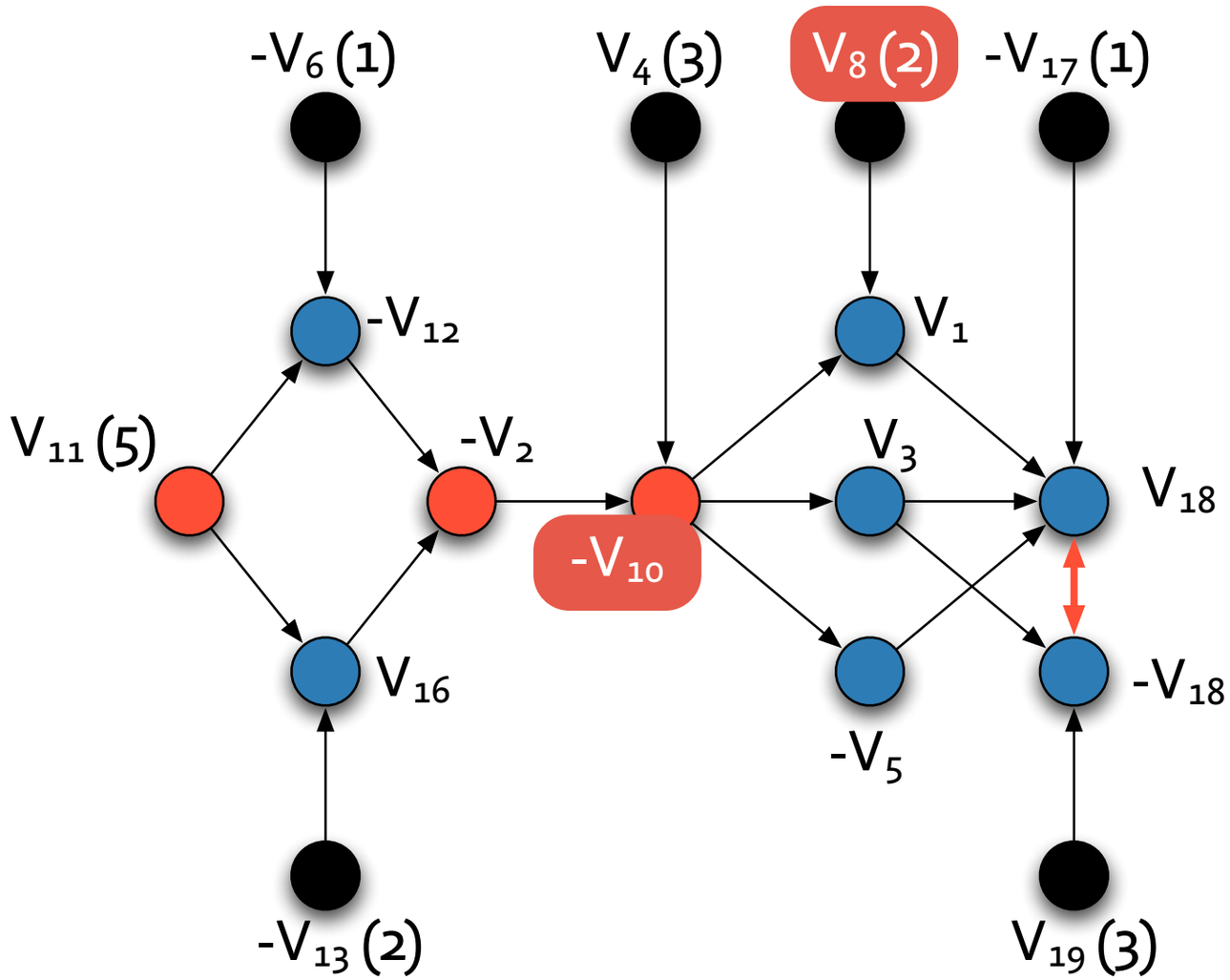
$$V_{17} \vee \neg V_{19}$$

Finding the 1UIP



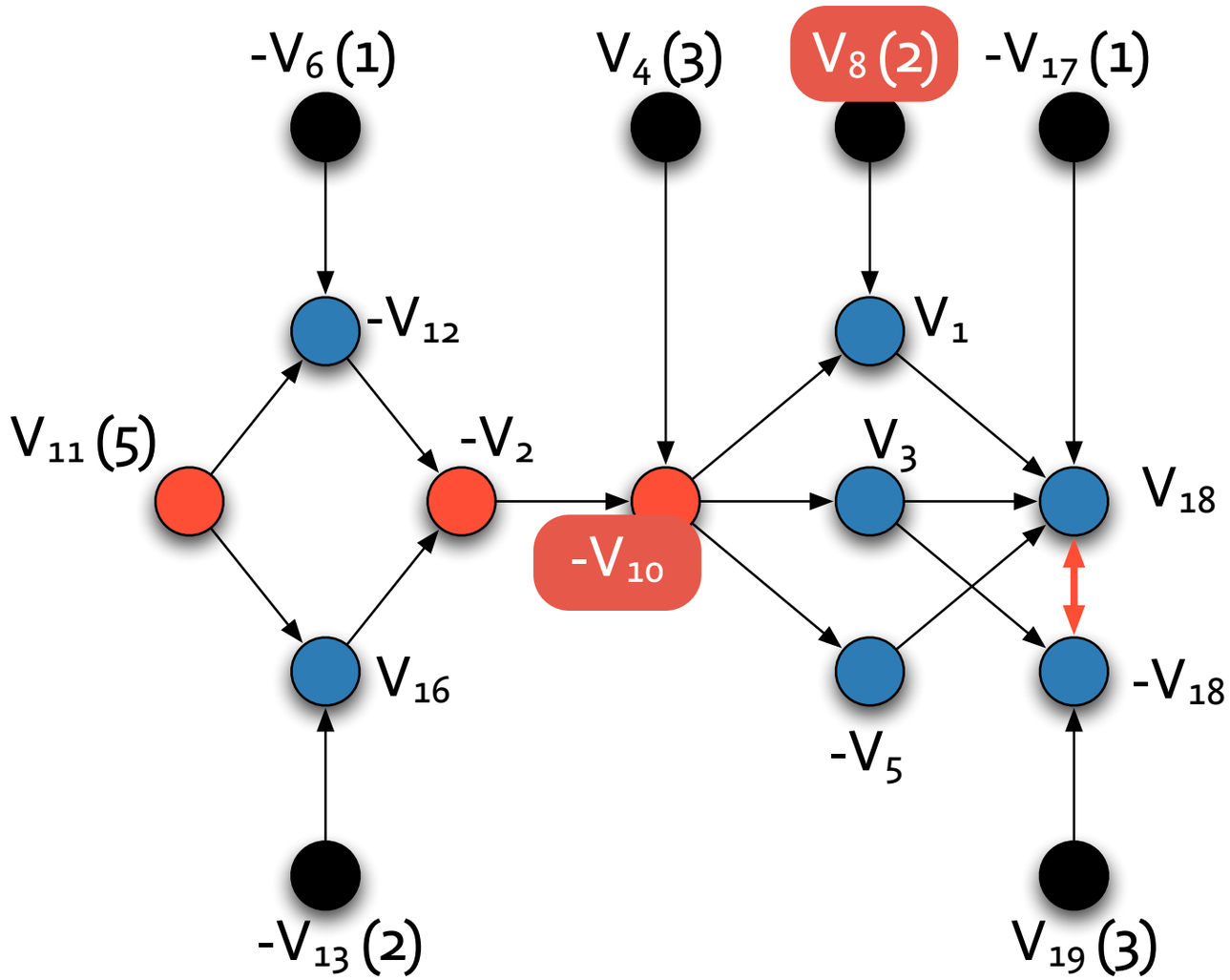
$$V_{17} \vee \neg V_{19}$$

Finding the 1UIP



$$V_{17} \vee \neg V_{19} \vee \neg V_8$$

Finding the 1UIP



$$V_{17} \vee \neg V_{19} \vee \neg V_8 \vee V_{10}$$

Conflict clauses, copying, trailing

- **Huge advantage of trailing:**
  - thousands of learned clauses make copying infeasible

    (possible solution: don't copy propagators)
  - decision level recorded in the trail

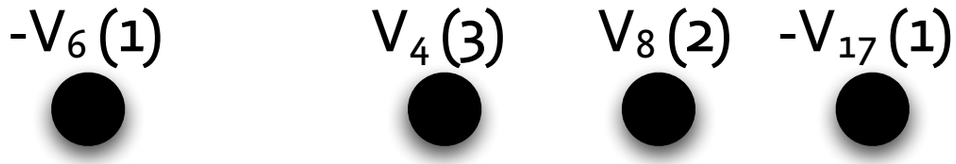
    (possible solution: record separately)
  - order of variable assignment recorded in the trail

    (possible solution: record separately)

Non-chronological backjumping

- **observation:**
  - conflict clauses tell us "how much went wrong"
  - backtracking only one step may lead to immediate failure
- **jump over several decision levels**

Non-chronological backjumping

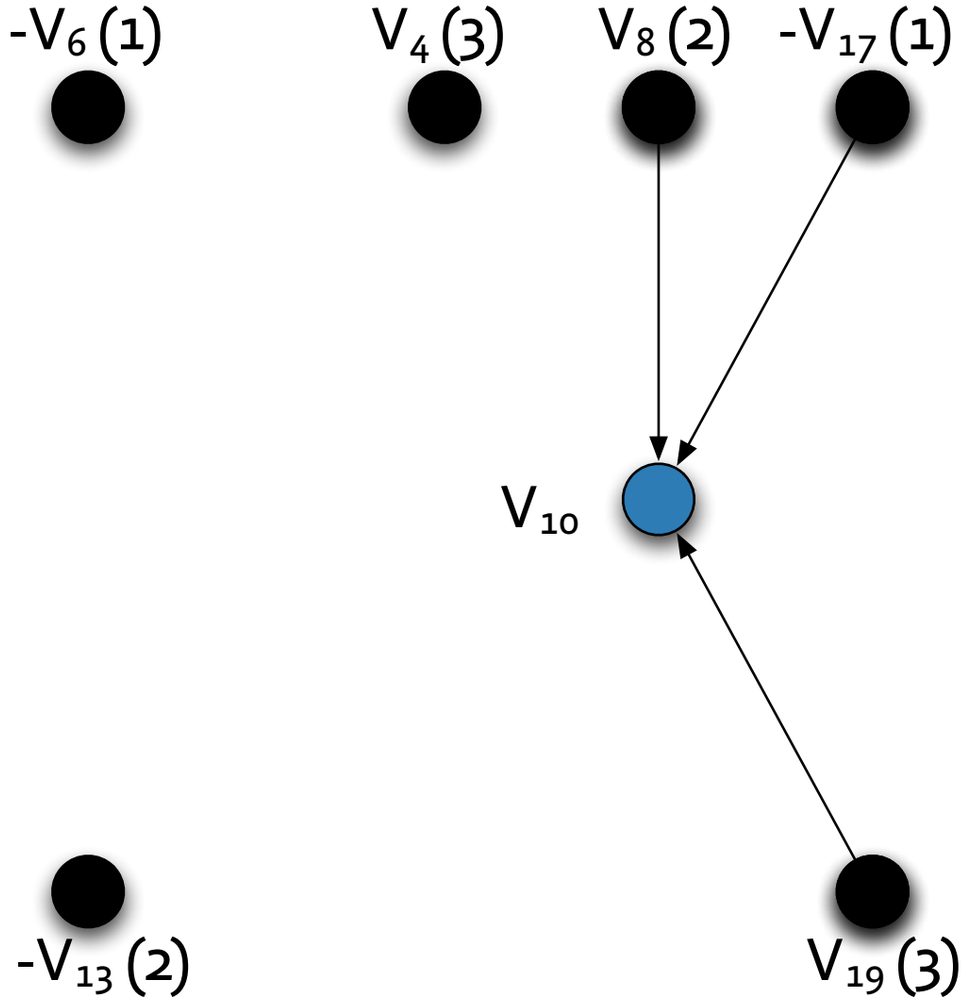


$V_{17} \vee \neg V_8 \vee$

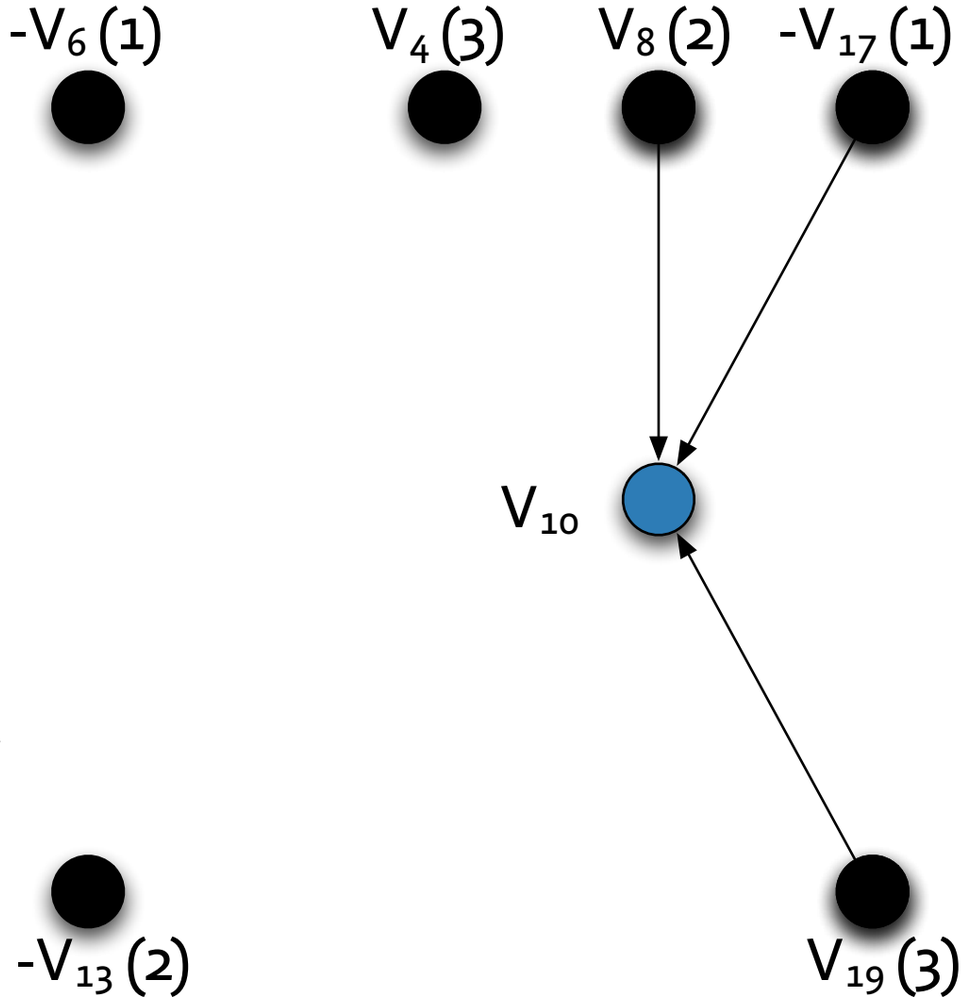
$V_{10} \vee \neg V_{19}$

Non-chronological backjumping

$V_{17} \vee \neg V_8 \vee$
$V_{10} \vee \neg V_{19}$



Non-chronological backjumping



$V_{17} \vee \neg V_8 \vee$
$V_{10} \vee \neg V_{19}$

$\neg V_{10} \vee V_{20}$

$\neg V_{10} \vee V_{22}$

$\neg V_{19} \vee \neg V_{20} \vee \neg V_{21}$

Non-chronological backjumping

$V_{17} \vee \neg V_8 \vee$
$V_{10} \vee \neg V_{19}$

$\neg V_{10} \vee V_{20}$

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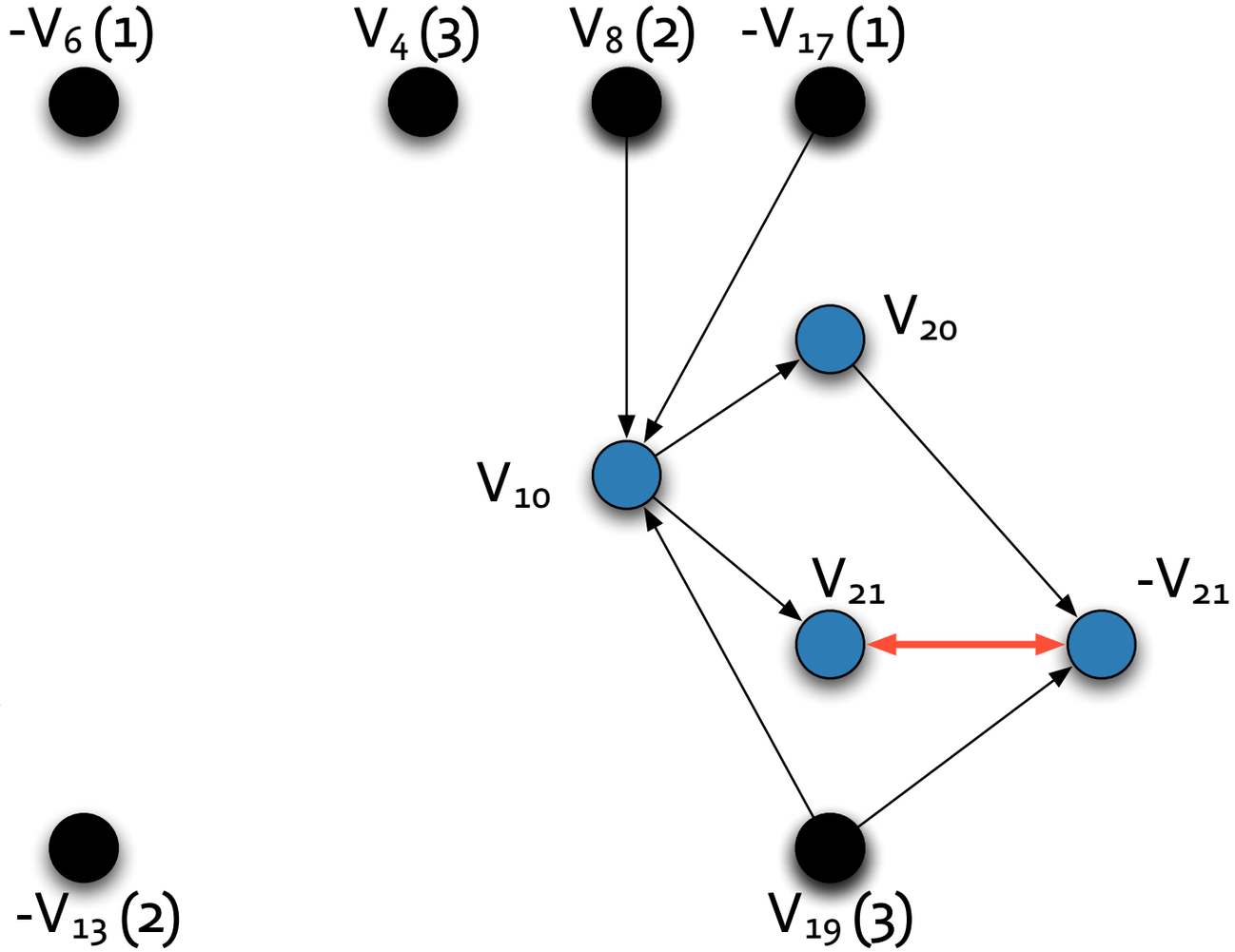

Non-chronological backjumping

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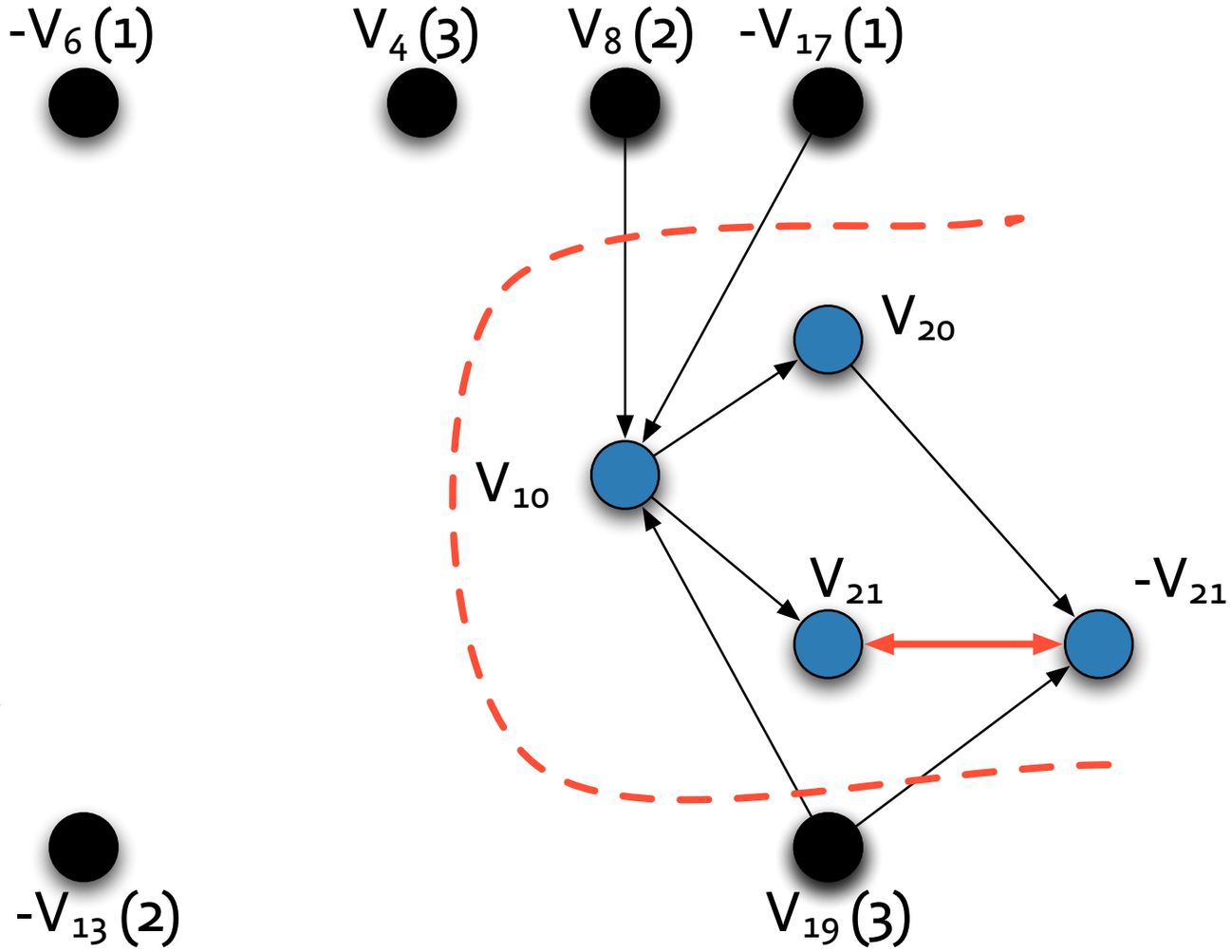


new conflict clause:

$\neg V_8 \vee V_{17} \vee \neg V_{19}$

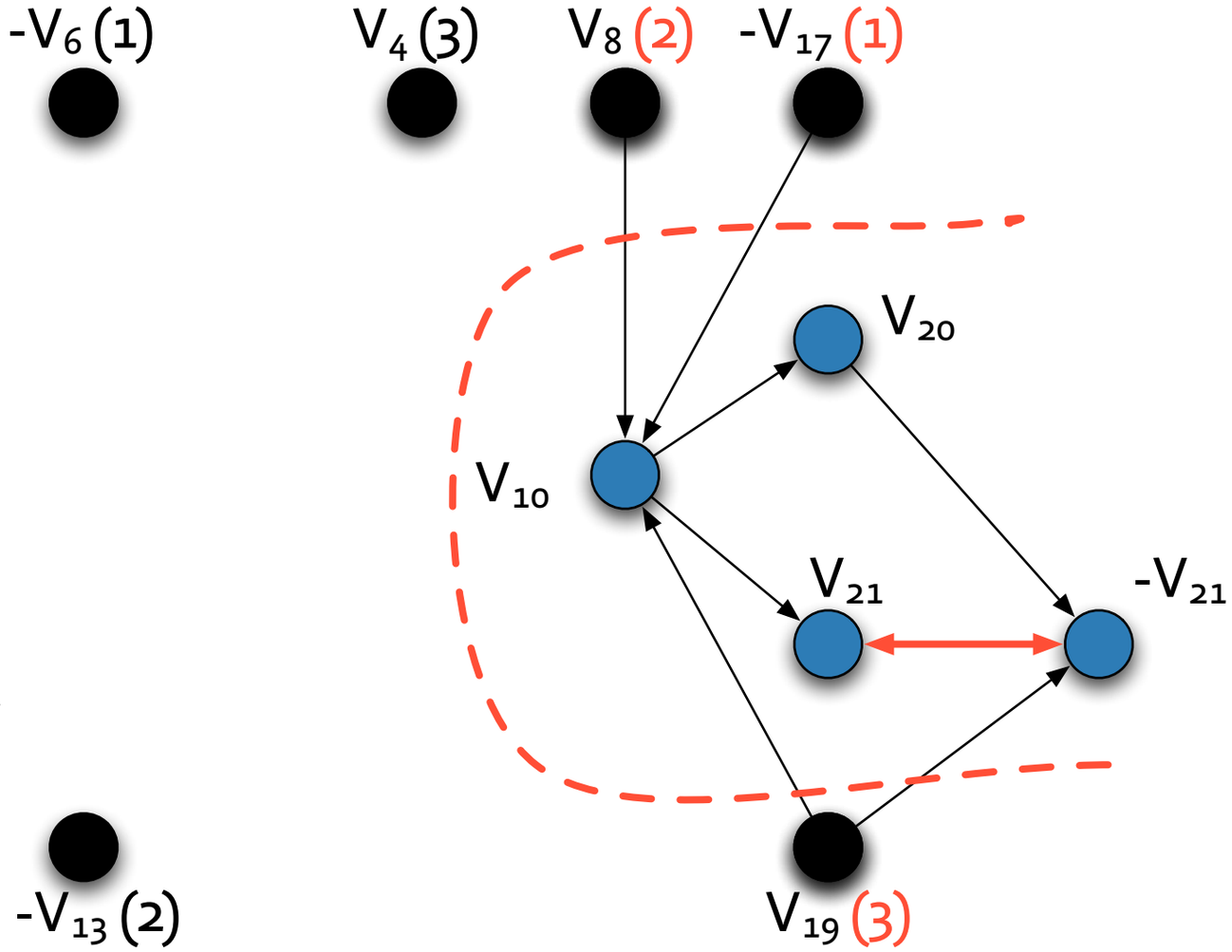
Non-chronological backjumping

$V_{17} \vee \neg V_8 \vee$
$V_{10} \vee \neg V_{19}$

$\neg V_{10} \vee V_{20}$

$\neg V_{10} \vee V_{22}$

$\neg V_{19} \vee \neg V_{20} \vee \neg V_{21}$



new conflict clause:

$\neg V_8 \vee V_{17} \vee \neg V_{19}$

Non-chronological backjumping

$V_{17} \vee \neg V_8 \vee$
$V_{10} \vee \neg V_{19}$

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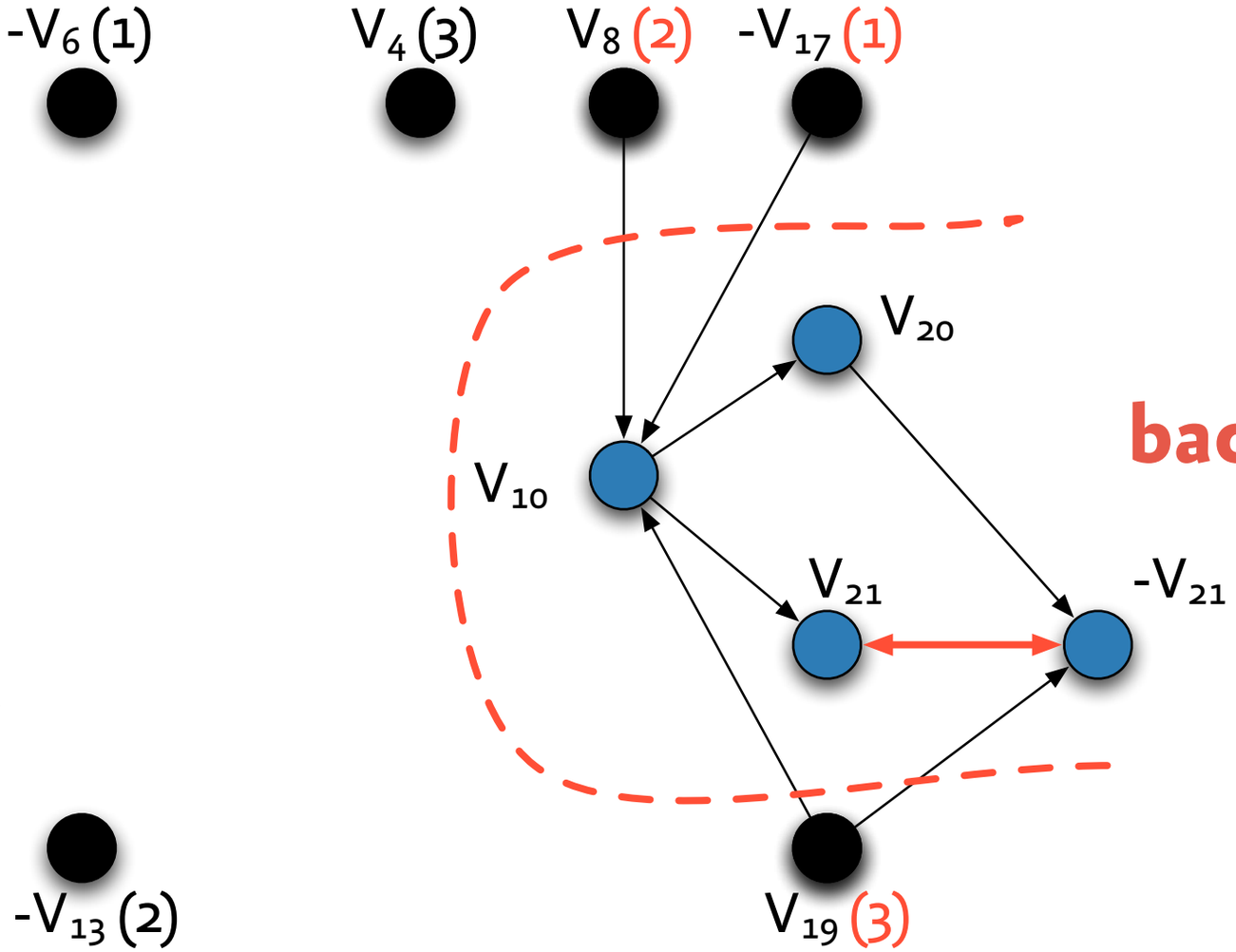
$\neg V_{10} \vee V_{22}$

$\neg V_{19} \vee \neg V_{20} \vee \neg V_{21}$



new conflict clause:

$\neg V_8 \vee V_{17} \vee \neg V_{19}$

backtrack to level 3!

Non-chronological backjumping

- conflict clauses cause new conflicts
- conflict clauses tell us how much went wrong
- cascades of backjumps
- prune large parts of the search

Branching heuristics

- **many proposals for variable selection:**
  - some static order
  - heuristic involving variable state and clause database
  - DLIS: most frequent literal (in non-determined clauses)
  - common: try to make fewer decisions
- **but:** fewer decisions does not mean less runtime!

Variable-State-Independent Decaying Sum (VSIDS)

- **each literal has a counter**
- **adding a clause:** increment counter for each literal
- choose literal with **highest counter**
- periodically **divide all counters** by a constant (decay)

- **heuristic adapts to learned clauses**
- **works extremely well in practice!**

Shuffling the punchcards

- **not really...**
- **but:**
  - randomize variable selection
  - cut-off for number of failures before restarting search with different random seed
  - learned clauses prevent visiting same state twice!
- **also applicable to CP!**

Literature

- Moskewicz et al. *Chaff: Engineering an efficient SAT solver.* DAC, 2001.
- Marques-Silva, Sakallah. *GRASP: A Search Algorithm for Propositional Satisfiability*. IEEE Transactions on Computers 48(5), 1999.

Solvers

MiniSAT

- **efficient**
  - solves industrial problems
  - incorporates all presented techniques
- **accessible**
  - open source
  - well documented
  - compact (~1K LOC)

http://www.cs.chalmers.se/Cs/Research/FormalMethods/MiniSat/Main.html

SATLib

- **collection of benchmarks**
  - all in DIMACS standard format
- **collection of solvers**
- **slightly out of date**

Other approaches

- **focus here:** complete solvers
- **very successful incomplete solvers**
  - local search techniques
  - stochastic algorithms

GSAT

```
GSAT(α,MAX_FLIPS,MAX_TRIES)
        for i := 1 to MAX_TRIES
            T := random truth assignment
            for j := 1 to MAX_FLIPS
                if T satisfies α return T
                p :=  variable such that changing its truth value
                      gives largest increase in number of clauses of
                      α that are satisfied by T
                T := T with truth assignment of p reversed
            end for
        end for
        return "no satisfying assignment found"
```

Literature

- Een, Sörensson. *An Extensible SAT-solver.* SAT 2003.
- Hoos, Stützle. *SATLIB: An Online Resource for Research on SAT.* SAT 2000.
- Selman, Levesque, Mitchell. *A New Method for Solving Hard Satisfiability Problems.* AAAI, 1992.

Planning

- **given:** initial state of the world + goal state
- **find a plan** that transforms the initial state into the goal state
- **plan = sequence of actions**

Planning: blocks world

- **simple example:**

  Two blocks, A and B. Initially, A is on top of B. Find a 2-step plan that results in B being on top of A.

- **formally:**

$$\mathrm{on}(A, B, 1) \wedge \mathrm{on}(B, \mathrm{Table}, 1) \wedge \mathrm{clear}(A, 1) \wedge \mathrm{on}(B, A, 3)$$

  + axioms about how the world behaves

Planning: blocks world

- **axioms:**

$$\forall x, y, z, i.\ \text{move}(x, y, z, i) \Rightarrow (\text{clear}(x, i) \wedge \text{clear}(z, i) \wedge \text{on}(x, y, i))$$

$$\forall x, y, z, i.\ \text{on}(x, y, i) \wedge \text{clear}(x, i) \wedge \text{clear}(z, i) \wedge \text{move}(x, y, z, i) \Rightarrow$$
$$(\text{on}(x, z, i + 1) \wedge \text{clear}(y, i + 1))$$

...

- **plan:** $\text{move}(A, B, \text{Table}, 1) \wedge \text{move}(B, \text{Table}, A, 2)$
- **problem:** not propositional!

Planning: blocks world

- **axioms:**

action

$$\forall x, y, z, i.\ \text{move}(x, y, z, i) \Rightarrow (\text{clear}(x, i) \wedge \text{clear}(z, i) \wedge \text{on}(x, y, i))$$

$$\forall x, y, z, i.\ \text{on}(x, y, i) \wedge \text{clear}(x, i) \wedge \text{clear}(z, i) \wedge \text{move}(x, y, z, i) \Rightarrow$$
$$(\text{on}(x, z, i + 1) \wedge \text{clear}(y, i + 1))$$

...

- **plan:** $\text{move}(A, B, \text{Table}, 1) \wedge \text{move}(B, \text{Table}, A, 2)$
- **problem:** not propositional!

Propositional model

- **assumption:** fixed, finite universe
- **variables:** $\mathrm{on}(x, y, i), \mathrm{clear}(x, i), \mathrm{move}(x, y, z, i)$ for all x, y, z, i
- **quantified formulae:** generate all instances

Propositional model

Propositional model

$$\forall x, y, z, i.\ \mathrm{move}(x, y, z, i) \Rightarrow (\mathrm{on}(x, z, i+1) \wedge \mathrm{clear}(y, i+1))$$

Propositional model

$$\forall x, y, z, i.\ \text{move}(x, y, z, i) \Rightarrow (\text{on}(x, z, i+1) \wedge \text{clear}(y, i+1))$$

$$\begin{aligned}
&\text{move}(A, B, \text{Table}, 1) \Rightarrow (\text{on}(A, \text{Table}, 2) \wedge \text{clear}(B, 2)) \\
&\text{move}(B, A, \text{Table}, 1) \Rightarrow (\text{on}(B, \text{Table}, 2) \wedge \text{clear}(A, 2)) \\
&\text{move}(A, \text{Table}, B, 1) \Rightarrow (\text{on}(A, B, 2) \wedge \text{clear}(\text{Table}, 2)) \\
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&\text{move}(\text{Table}, B, A, 1) \Rightarrow (\text{on}(\text{Table}, A, 2) \wedge \text{clear}(B, 2)) \\
&\text{move}(\text{Table}, A, B, 1) \Rightarrow (\text{on}(\text{Table}, B, 2) \wedge \text{clear}(A, 2))
\end{aligned}$$

Propositional model

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\end{aligned}$$

$$\forall 1 \leq i \leq N.\ \exists x, y, z.\ \mathrm{move}(x, y, z, i)$$

Propositional model

$$\forall x, y, z, i.\ \mathrm{move}(x, y, z, i) \Rightarrow (\mathrm{on}(x, z, i+1) \wedge \mathrm{clear}(y, i+1))$$

$$\begin{aligned}
&\mathrm{move}(A, B, \mathrm{Table}, 1) \Rightarrow (\mathrm{on}(A, \mathrm{Table}, 2) \wedge \mathrm{clear}(B, 2))\\
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\end{aligned}$$

$$\forall 1 \leq i \leq N.\ \exists x, y, z.\ \mathrm{move}(x, y, z, i)$$

$$\begin{aligned}
&\mathrm{move}(A, B, \mathrm{Table}, 1) \vee \mathrm{move}(B, A, \mathrm{Table}, 1) \vee\\
&\mathrm{move}(A, \mathrm{Table}, B, 1) \vee \mathrm{move}(B, \mathrm{Table}, A, 1) \vee\\
&\quad \mathrm{move}(\mathrm{Table}, A, B, 1) \vee \mathrm{move}(\mathrm{Table}, B, A, 1)
\end{aligned}
\quad \bigwedge \quad
\begin{aligned}
&\mathrm{move}(A, B, \mathrm{Table}, 2) \vee \mathrm{move}(B, A, \mathrm{Table}, 2) \vee\\
&\mathrm{move}(A, \mathrm{Table}, B, 2) \vee \mathrm{move}(B, \mathrm{Table}, A, 2) \vee\\
&\quad \mathrm{move}(\mathrm{Table}, A, B, 2) \vee \mathrm{move}(\mathrm{Table}, B, A, 2)
\end{aligned}$$

Propositional model

$$\forall x, y, z, i.\ \mathrm{move}(x, y, z, i) \Rightarrow (\mathrm{on}(x, z, i+1) \wedge \mathrm{clear}(y, i+1))$$

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&\mathrm{move}(A, B, \mathrm{Table}, 1) \Rightarrow (\mathrm{on}(A, \mathrm{Table}, 2) \wedge \mathrm{clear}(B, 2))\\
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&\mathrm{move}(\mathrm{Table}, A, B, 1) \Rightarrow (\mathrm{on}(\mathrm{Table}, B, 2) \wedge \mathrm{clear}(A, 2))
\end{aligned}$$

problem: quantification

$$\forall 1 \leq i \leq N.\ \exists x, y, z.\ \mathrm{move}(x, y, z, i)$$

$$\begin{aligned}
&\mathrm{move}(A, B, \mathrm{Table}, 1) \vee \mathrm{move}(B, A, \mathrm{Table}, 1) \vee\\
&\mathrm{move}(A, \mathrm{Table}, B, 1) \vee \mathrm{move}(B, \mathrm{Table}, A, 1) \vee\\
&\quad \mathrm{move}(\mathrm{Table}, A, B, 1) \vee \mathrm{move}(\mathrm{Table}, B, A, 1)
\end{aligned}
\quad \bigwedge \quad
\begin{aligned}
&\mathrm{move}(A, B, \mathrm{Table}, 2) \vee \mathrm{move}(B, A, \mathrm{Table}, 2) \vee\\
&\mathrm{move}(A, \mathrm{Table}, B, 2) \vee \mathrm{move}(B, \mathrm{Table}, A, 2) \vee\\
&\quad \mathrm{move}(\mathrm{Table}, A, B, 2) \vee \mathrm{move}(\mathrm{Table}, B, A, 2)
\end{aligned}$$

Propositional model

- **restrict to binary predicates:**

$$\text{move}(x, y, z, i) \equiv \text{object}(x, i) \wedge \text{source}(y, i) \wedge \text{dest}(z, i)$$

- **axiom:** $\forall 1 \leq i \leq N.\ \exists x, y, z.\ \text{move}(x, y, z, i)$

$$\forall i.\ x \neq y \Rightarrow \neg\text{object}(x, i) \vee \neg\text{object}(y, i)$$
$$\forall i.\ x \neq y \Rightarrow \neg\text{source}(x, i) \vee \neg\text{source}(y, i)$$
$$\forall i.\ x \neq y \Rightarrow \neg\text{dest}(x, i) \vee \neg\text{dest}(y, i)$$

- **dramatic reduction in size!**

Literature & Pointers

- Kautz, Selman. *Planning as Satisfiability*. ECAI, 1992.

  (authors of GSAT!)

- http://www.cs.rochester.edu/u/kautz/satplan/index.htm

Summary

- **SAT is an important sub-class of CP**
- **efficient solvers use DPLL with**
  - watched literals
  - conflict-clause learning
  - conflict-directed non-chronological backjumping
  - conflict-directed heuristics (VSIDS)

Next week

- SAT constraints

Next week

- SET constraints
- New constraint domain:
  - variables $x \in 2^{\mathbb{N}}$ (values = sets of integers)
  - constraints: $x \subseteq y, x = y \cap z, |x| = y$