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http://www.ps.uni-sb.de/courses/sem-ws01/

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Disjunctive Completion

Consider the following program `twentyone` with program points $\{1, 2\}$ and variables $Var = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, whose values are integers (i. e., $Val = \mathbb{Z}$).

```
   x := 17; y := 4; z := 21;
1:
   x++; y--;
2:
   z := x + y; goto 1;
```

Exercise 9.1 (5) Perform a parity-analysis for the program `twentyone`: Specify the *acc*-equations, the abstract $acc^{\#}$-equations and their abstract operations, and compute the least fixpoint. Why does this analysis not find out that `z` is always odd?

Exercise 9.2 (5) Make the above parity-analysis more precise (in particular, it should yield that `z` is odd) by considering a larger abstract domain.
Hint: Think of a way to represent disjunctions of the abstract elements from exercise 9.1.

Exercise 9.3 (5) A more ambitious goal for anaylses is to find constant expressions in programs (constant propagation). Consider transitions from program point p to p' which are labeled by either of the following statements:

- $x := c$; for some $x \in Var$ and $c \in \mathbb{Z}$,
- $x := y + c$; for some $x, y \in Var$ and $c \in \mathbb{Z}$, or
- $x := y + z$; for some $x, y, z \in Var$.

For each of these transitions, give the collecting semantics *acc* and the abstract semantics $acc^{\#}$ for the abstract domain $Var \to D_{\mathbb{Z}}$, where $D_{\mathbb{Z}} = \mathbb{Z} \cup \{\bot, \top\}$ is the flat lattice of the integers. Argue that an analysis based on $acc^{\#}$ is not precise enough to find out that `z` is constant in program `twentyone`.

Exercise 9.4 (5) Outline a disjunctive version of constant propagation, i. e., specify an abstract domain and abstract operations analogously to exercise 9.2. Try to perform this analysis on program `twentyone`. What goes wrong?

Exercise 9.5 (5) Show that it is undecidable whether an abstraction is $\top$.

More precisely, prove that there is an abstraction α into an abstract domain $\langle D^{\#}, \sqsubseteq \rangle$ such that the following question is undecidable: Given a program P and a program point p in P, is $\alpha(acc(p)) = \top$?