Object Calculus LO

$T \in Ty$	$= \; T \to T \;\mid\; \mathrm{Obj}\, X.F$	type
$t \in Ter$	$= \; x \;\mid\; \lambda x{:}T.t \;\mid\; t\,t \;\mid\; \mathrm{obj}\, f \;\mid\; t.l$	term
$l \in Lab$		label
$x \in Var$		variable
$F \in Lab \overset{fin}{\rightharpoonup} Ty$		type record
$f \in Lab \overset{fin}{\rightharpoonup} Ter$		term record
$\Gamma \in Var \rightharpoonup Ty$		type environment

The **proper reduction rules** are as follows:

$(\lambda x{:}T.t)t' \;\to\; t[x := t']$ **beta reduction**

$(\mathrm{obj}\, f).l \;\to\; (f\, l)(\mathrm{obj}\, f) \quad \text{if } l \in Dom\, f$ **method invocation**

Object types are recursive record types. We use the notation

$$\mathrm{Obj}\, F \;\overset{\text{def}}{=}\; \mathrm{Obj}\, X.F \quad \text{if } \forall l \in Dom\, f:\; X \notin FV(F\, l)$$

We represent types as rational trees, that may be infinite due to recursion (e.g., $\mathrm{Obj}\, X.\{l : X\}$). Hence we always have

$$\mathrm{Obj}\, X.F \;=\; \mathrm{Obj}\{\, (l,\; T[X := \mathrm{Obj}\, X.F]) \mid (l, T) \in F \,\}$$

The **subtype order** is defined coinductively by the following rules:

$$\frac{T_1' \le T_1 \qquad T_2 \le T_2'}{T_1 \to T_2 \le T_1' \to T_2'} \qquad \frac{Dom\, F \supseteq Dom\, F' \qquad \forall l \in Dom\, F':\; F\, l \le F'\, l}{\mathrm{Obj}\, F \le \mathrm{Obj}\, F'}$$

The typing relations are defined as follows:

$$\Gamma \vdash t : T \;\overset{\text{def}}{\Longleftrightarrow}\; \exists T':\; \Gamma \mapsto t : T' \;\wedge\; T' \le T$$

$$\frac{\Gamma x = T}{\Gamma \mapsto x : T} \qquad \frac{\Gamma[x := T] \mapsto t : T'}{\Gamma \mapsto \lambda x{:}T.t : T \to T'} \qquad \frac{\Gamma \mapsto t : T' \to T \qquad \Gamma \vdash t' : T'}{\Gamma \mapsto t\, t' : T}$$

$$\frac{Dom\, f = Dom\, F \qquad \forall l \in Dom\, f:\; \Gamma \vdash f\, l : \mathrm{Obj}\, F \to F\, l}{\Gamma \mapsto \mathrm{obj}\, f : \mathrm{Obj}\, F}$$

$$\frac{\Gamma \mapsto t : \mathrm{Obj}\, F \qquad l \in Dom\, F}{\Gamma \mapsto t.l : F\, l}$$

As it comes to the properties of the typing relation I would hope that Least Type, Subsumption, Preservation and Progress are satisfied. As it comes to Least Type I'm not sure at all, so it makes sense to look for a counterexample. Another open question is decidability of the typing relations.