Assignment 1
Semantics, WS 2009/10

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Hand in by 11.59am, Tuesday, October 27

You can find a Coq template for the exercises on the course web page.

This exercise sheet counts towards your exam qualification. Send your solutions by email to doczkal@ps.uni-sb.de, and make sure that the entire file compiles without errors.

Exercise 1.1 (Commutativity of addition) Consider the following alternative definition of *plus* in Coq:

```
Fixpoint plus x y := match x with
  O => y
| S x' => S(plus x' y)
end.
```

State and prove a proposition *plus_comm* that asserts commutativity of *plus*.

Exercise 1.2 (Leq)

a) Define a procedure $leq : nat \to nat \to bool$ such that $leq\ x\ y$ returns *true* if and only if x is less than or equal to y.

b) State and prove a proposition *leq_refl* that asserts the reflexivity of *leq*, i.e., that $leq\ x\ x = true$ holds for all natural numbers x.

c) Prove the following two propositions:

$$leq\_l : \forall x\, y\, z,\ (leq\ x\ y) = true \to (leq\ (z+x)\ (z+y)) = true$$
$$leq\_r : \forall x\, y\, z,\ (leq\ x\ y) = true \to (leq\ (x+z)\ (y+z)) = true$$

Hint. To rewrite a particular subterm you may use the tactic *rewrite ... with ...*. There is an example in the template file that we provide.

Exercise 1.3 (iter)

a) Define $iter : (nat \to nat) \to nat \to nat \to nat$ so that $iter\ f\ n\ x = f^n\ x$.

b) Prove in Coq that $\forall f\, n\, x,\ iter\ f\ n\ (f\ x) = f\,(iter\ f\ n\ x)$ holds.

c) Prove a proposition *iter_plus* that asserts $\forall n\, x,\ iter\ S\ n\ x = n + x$.