Assignment 5
Semantics, WS 2009/10

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Hand in by 11.59am, Tuesday, November 24

Send your solutions to Exercises 5.2–5.5 in a file named `lastname.v` to doczkal@ps.uni-sb.de. Make sure that the entire file compiles without errors.

Exercise 5.1 (Contextual equivalence)

a) Show that the following terms are *not* contextually equivalent in PCF$^-$, by finding separating contexts.
   - $S\,0$ and $S(S\,0)$
   - $\lambda xy.x$ and $\lambda xy.y$
   - x and $\mathit{natcase}\ x\ 0\ (\lambda y.\, S\, y)$
   - f and $\lambda x.\, f\, x$

b) Find two terms $s \sim t$ such that $\emptyset \vdash s : \mathit{nat}$ and $\emptyset \nvdash t : \mathit{nat}$.

Exercise 5.2 (Typing relation) Prove the following facts about PCF$^-$ in Coq.

a) There exists a type T such that $\emptyset \vdash (\lambda x.x)\,0 : T$.

b) $S \neq S \to T$ for all types S and T.

c) There is no type T such that $\Gamma \vdash t\,t : T$.

Exercise 5.3 (Canonical forms) Show the following properties of PCF$^-$ in Coq.

a) If $\emptyset \vdash t : \mathit{nat}$ and t is a value, then either $t = 0$ or $t = S\,v$ for some value v.

b) If $\emptyset \vdash t : S \to T$ and t is a value, then $t = \lambda x{:}S.s$ for some x and s.

Exercise 5.4 (Closure semantics) Define the closure semantics for PCF$^-$ as a relation $\mathit{cl\_eval} : (\mathit{env}\ \mathit{sv}) \to \mathit{ter} \to \mathit{sv} \to \mathit{Prop}$ in Coq, where the type sv implements the semantic values.

Exercise 5.5 (Natrec) Extend the Coq formalization of PCF$^-$ with *natrec*, and adapt the proofs of unique types, progress, preservation and determinacy.