# Assignment 7
# Semantics, WS 2009/10

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www.ps.uni-sb.de/courses/sem-ws09/

Hand in by 11.59am, Tuesday, December 15

Send your solutions in a file named `lastname.v` to doczkal@ps.uni-sb.de. Make sure that the entire file compiles without errors. You can find a template file on the course webpage.

**Exercise 7.1 (Proof terms and proof scripts)** Find proofs for the following propositions both in the form of proof terms (*exact*) and in the form of proof scripts with *intros* and *apply*.

a) $\forall PQ : Prop, (P \to P \to Q) \to ((P \to Q) \to P) \to Q$

b) $\forall PQRST : Prop, (Q \to R \to T) \to (P \to Q) \to P \to R \to T$

c) $\forall PQR : Prop, (P \to Q) \to (Q \to R) \to P \to R$

d) $\forall PQ : Prop, P \to (P \to \forall R, R) \to Q$

**Exercise 7.2 (Conjunction)** Consider the definition

```
Inductive and (X Y: Prop) : Prop :=
| and_I : X -> Y -> and X Y.
```

a) Give the types of *and*, *and_I*, and *and_ind*.

b) Find proofs for the following propositions both in the form of proof terms with *exact* and in the form proof scripts with *intros* and *apply*.

   (i) $\forall XY : Prop, X \to Y \to and\ X\ Y$

   (ii) $\forall XY : Prop, and\ X\ Y \to X$

**Exercise 7.3 (Disjunction)** Consider the definition

```
Inductive or (X Y: Prop) : Prop :=
| or_L : X -> or X Y
| or_R : Y -> or X Y.
```

a) Give the types of *or*, *or_L*, *or_R*, and *or_ind*.

b) Find proofs for the following propositions both in the form of proof terms with exact and in the form proof scripts with *intros* and *apply*.

   (i) $\forall XY : Prop, X \to or\ X\ Y$

   (ii) $\forall XYZ : Prop, or\ X\ Y \to (X \to Z) \to (Y \to Z) \to Z$

**Exercise 7.4 (Classical tautologies)** Coq defines *False* inductively:

```
Inductive False : Prop := .
```

This yields the following identifiers:

```
False : Prop
False_ind : forall P, False -> P
```

We define some propositions that are true classically.

```
Definition DN := forall P : Prop, ~~P -> P.
Definition Contra := forall P Q : Prop, (~P -> ~Q) -> Q -> P.
Definition Peirce := forall P Q : Prop, ((P -> Q) -> P) -> P.
```

While none of these propositions is provable constructively, they are all equivalent constructively. Find proofs for the following implications both in the form of proof terms (*exact*) and in the form of proof scripts with *intros* and *apply*.

a) $DN \to Contra$
b) $DN \to Peirce$
c) $Contra \to DN$
d) $Contra \to Peirce$
e) $Peirce \to DN$
f) $Peirce \to Contra$

Hints: Apply the premise of the implication as early as possible. A single application of the premise always suffices. For most examples *False_ind* is needed.