Assignment 8
Semantics, WS 2009/10

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www.ps.uni-sb.de/courses/sem-ws09/

Hand in by 11.59am, Tuesday, January 5

Send your solutions to Exercise 8.5 in a file named `lastname.v` to doczkal@ps.uni-sb.de. Make sure that the entire file compiles without errors. You can find a template file on the course webpage.

Recommended reading: B. Pierce, *Normalization*, Chapter 12 of "Types and Programming Languages."

Exercise 8.1 (Typing and normalization)

a) Find a term t such that $\mathit{halts}\, t$ holds even though t is not typable.

b) Find a PCF term t that is well-typed, but such that $\mathit{halts}\, t$ does not hold.

c) Explain why the normalization theorem implies that it's not possible to define the recursion operator *fix* in System T.

Exercise 8.2 (Failure of the naive normalization proof) Show that for terms t and t', $\mathit{halts}\, t$ and $\mathit{halts}\, t'$ does not in general imply $\mathit{halts}\,(t\ t')$.

Exercise 8.3 (Logical relation) Make sure that you can state the definition of the logical relation R (mathematical notation and in Coq).

Exercise 8.4 (Closure under reduction and expansion) Recall the statement of the reduction and expansion lemma that we used in the proof of the Basic Lemma: If $\emptyset \vdash t : T$ and $t \to^* t'$ then

$$R_T\, t' \;\Leftrightarrow\; R_T\, t\ .$$

Show that the condition $\emptyset \vdash t : T$ is necessary by finding terms t and t' such that $t \to^* t'$ and $R_T\, t'$ but not $R_T\, t$.

Make sure that you understand why "$\Leftarrow$" holds even without this condition.

Exercise 8.5 (Proving normalization in Coq) On the course webpages you can find a Coq formalization of the normalization proof for System T.

a) Familiarize yourself with the proof script. Make sure that you can state the closure properties (*R_stepmany_closed* and *R_expmany_closed*) of R.

b) Prove the preservation lemmas *BL_Zero* and *BL_Succ*.

c) Finish the normalization proof by proving the *normalization* theorem. Hint: The lemma *subst_id* may be useful.

Exercise 8.6 (Optional: Normalization with product types) If you like a challenge, you may enjoy to extend the normalization proof by adding product types to System T. You will need to extend the logical relation by a case for product types, $R_{T_1 \times T_2}$. Before attempting a Coq formalization, do this using pencil and paper first. Proceed as follows:

a) By looking at the typing rules for pairs and projections, state the corresponding preservation properties for the logical relation R.

b) From these requirements, try to synthesize a suitable definition of $R_{T_1 \times T_2}$ (in particular, $R_{T_1 \times T_2}$ must entail *halts t*).

c) Prove that the preservation lemmas that you stated in (a) indeed hold with respect to the proposed definition of R given in (b).

d) Check that the closure properties of R continue to hold.

e) Extend the proof of the Basic Lemma by the new cases for pairs and projections.