Assignment 12
Semantics, WS 2009/10

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Recommended reading: Chapter 7 of the lecture notes.

Exercise 12.1 (Leibniz equality) Prove with natural deduction that Leibniz equality is reflexive.

Exercise 12.2 (Leibniz equality) Prove that Coq's predefined equality agrees with Leibniz equality.

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Definition eq (X:Type) (x y : X) : Prop :=
    forall p : X -> Prop, p x -> p y.

Lemma eq_agrees : forall (X : Type) (x y : X),  eq X x y <-> x=y.
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Exercise 12.3 (Non-termination) Give a term s on which β-reduction does not terminate.

Exercise 12.4 (Bool) Give a term of type $\forall X{:}\mathbf{U}_0.\ \mathit{bool} \to X \to X \to X$ that acts as conditional.

Exercise 12.5 (Sum types) Express sum types in CC_ω and in Coq. That is, give closed terms of the following types.

$$\begin{aligned}
\mathit{sum} &: \mathbf{U}_0 \to \mathbf{U}_0 \to \mathbf{U}_0 \\
\mathit{inl} &: \forall X{:}\mathbf{U}_0.\ \forall Y{:}\mathbf{U}_0.\ X \to \mathit{sum}\,X\,Y \\
\mathit{inr} &: \forall X{:}\mathbf{U}_0.\ \forall Y{:}\mathbf{U}_0.\ Y \to \mathit{sum}\,X\,Y \\
\mathit{case} &: \forall X{:}\mathbf{U}_0.\ \forall Y{:}\mathbf{U}_0.\ \forall Z{:}\mathbf{U}_0.\ \mathit{sum}\,X\,Y \to (X \to Z) \to (Y \to Z) \to Z
\end{aligned}$$

Exercise 12.6 (Polymorphic lists) Express polymorphic lists in CC_ω and in Coq. That is, give closed terms of the following types.

$$\begin{aligned}
\mathit{list} &: \mathbf{U}_0 \to \mathbf{U}_0 \\
\mathit{nil} &: \forall X{:}\mathbf{U}_0.\ \mathit{list}\,X \\
\mathit{cons} &: \forall X{:}\mathbf{U}_0.\ X \to \mathit{list}\,X \to \mathit{list}\,X \\
\mathit{foldl} &: \forall X{:}\mathbf{U}_0.\ \forall Y{:}\mathbf{U}_0.\ (X \to Y \to Y) \to Y \to \mathit{list}\,X \to Y
\end{aligned}$$

Exercise 12.7 (Predecessor) Express the predecessor function for *nat* in CC_ω and in Coq. Recall that the trick is to iterate through the pairs $(0,0)$, $(1,0)$, $(2,1)$, $\ldots$, $(n, n-1)$.

Exercise 12.8 (Natrec) Express primitive recursion for *nat* in CC_ω and in Coq.