Assignment 6
Semantics, WS 2011-2012

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Read in the lecture notes:

Read the new version of Chapter 4 of the lecture notes.

Exercise 6.1 Prove the following goals once using inversion and a second time without using inversion. Do not use induction.

a) Goal ~even 1.

b) Goal forall n, even (S (S n)) -> even n.

Exercise 6.2 Consider the inductive definition of *le* with one proper argument.

```
Inductive le (x:nat) : nat -> Prop :=
| lex : le x x
| leS : forall y, le x y -> le x (S y).
```

Prove the following by induction on *le*.

a) **Lemma** le_Sright x y : le x y -> le (S x) (S y).

b) Goal forall x, le x 0 -> x = 0.

Exercise 6.3 Consider the inductive definition of *le'* with two proper arguments.

```
Inductive le' : nat -> nat -> Prop :=
| lex' : forall x, le' x x
| leS' : forall x y, le' x y -> le' x (S y).
```

Prove the following two versions of $Sx \not\leq 0$ formulated using *le* and *le'*.

```
Goal forall x, ~ le (S x) 0.

Goal forall x, ~ le' (S x) 0.
```

Exercise 6.4 Consider the following inductively defined proposition.

```
Inductive F : Prop :=
| FI : F -> F.
```

Prove the following goal.

```
Goal F -> False.
```

Make sure you understand the goal you need to prove at each stage of the proof.

Exercise 6.5 Read the development of the abstract Imp language in the Coq file. Make sure you understand the definitions, theorems, and their proofs. Complete the proofs of *Seq_assoc*, *skip_div*, *monotone_while*, *optimization1*, *eval_monotone* and *eval_agrees_divergence*. **Note:** Two new tactics you may find helpful are *exfalso* and *case_eq*. *exfalso* strengthens the goal by changing the claim to *False*. This can be used when the current hypotheses are inconsistent. *case_eq t* can be used to replace the combination of tactics *remember t as x. destruct x*.